

**Experimental and Analytical Evaluation of the Flexural and Axial Capacity of Rectangular  
Steel Hollow Structural Section (HSS) End-Plate Connections**

Edward J. Nelson

Civil and Architectural Engineering and Construction Management Department

Milwaukee School of Engineering

**Author Note**

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Correspondence concerning this article should be addressed to Edward J. Nelson, 1025 N. Broadway, Milwaukee, WI 53202. E-mail: [nelsonedwa@msoe.edu](mailto:nelsonedwa@msoe.edu)

### Abstract

There are many challenges involved in the design of hollow structural sections (HSS) steel end-plate connections. To solve these challenges, structural designers use design guides which have been developed based on several simplifying assumptions to streamline the design procedure, such as *Steel Design Guide 24* (Packer et al., 2010). The purpose of this research is to investigate if the current design procedures in *Steel Design Guide 24* for rectangular HSS steel end-plate connections are comprehensive and accurate. Twelve axial physical HSS end-plate connection specimens using square HSS designed with varying bolt patterns (side and corner) and weld patterns (all-around and workable-flats) were tested under axial tensile loading. Another twelve flexural specimens using rectangular HSS were designed with the varying bending axes (strong and weak) and weld pattern configurations (all-around and workable-flats) were tested under lateral eccentric loading. Linear strain gages, and strain rosettes were used to measure strains in the end-plates and HSS. Linear variable differential transformers (LVDTs) were used to measure end-plate and HSS displacement. The results for the axial specimens indicate that side bolt configurations provide more capacity than corner bolt configurations and the all-around weld provides more capacity than the workable-flats weld. The side bolts connections reached the 100-kip capacity of the test load cell without yielding, so the results were inconclusive. However, because of the stiff behavior of the end-plate, it is reasonable to suggest that the calculated connection capacity using the design procedure in Section 5.6 of *AISC Design Guide 24* (2010), governed by bolt strength, would have been reached. The corner bolt specimens underwent significant yielding at loads less than the calculated capacity, suggesting that the design procedures in *AISC Design Guide 24* are not applicable to the corner bolt configuration and would require a separate design procedure with adjusted equations. Results from the flexural tests indicate that the strong axis configurations provide more capacity than the weak axis configurations and the all-around weld provides more capacity than the workable-flats weld. Results were compared to calculated capacities using the design procedure from Example 4.1 in *AISC Design Guide 24* (2010) and the design procedure developed by Wheeler et al. (1998). The capacities from weld strength based on the *AISC Design Guide 24* (2010) procedure for the all-around weld specimens were conservative and showed good correlation; however, for the workable-flat welds specimens some capacities were unconservative. The calculated strength and serviceability capacities, based on plate and bolt strength from the Wheeler et al. (1998) design procedure, were conservative and showed good correlation with results for the all-around weld specimens. Therefore, it is recommended that the design procedure be considered for implementation into *AISC Design Guide 24*.

**Keywords:** hollow structural section (HSS), steel, end-plate, rectangular HSS, square HSS, flexural capacity, axial capacity, axial loading, lateral loading, fillet welds, American Institute of Steel Construction (AISC), shear, tension, moment, prying action

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### Nomenclature

*Symbols:*

$a$  – distance from the bolt hole centerline to edge of connecting element flange or angle leg, but not more than  $1.25b$ , in. (see Figures 4, 10, and 11)

$a'$  – equal to  $a + \frac{d}{2}$ , in.

$A_b$  – nominal unthreaded body area of bolt or threaded part, in.<sup>2</sup>, (mm<sup>2</sup>)

$A_w$  – effective throat area of weld

$A_{we}$  – effective area of the weld, in.<sup>2</sup> (mm<sup>2</sup>)

$b$  – for a tee type connecting element, the distance from the bolt centerline to the face of the tee stem; for an angle-type connecting element, the distance from the bolt centerline to centerline of the angle; for HSS, the distance from the bolt centerline to the face of the HSS wall, in. (see Figures 4, 10, and 11)

$b'$  – equal to  $b + \frac{d}{2}$ , in.

$b_e$  - effective plate width

$d$  – diameter of bolt, in

$d'$  – nominal width of the hole along the length of the connecting element from AISC

*Specification* Table J3.3, in

$f_{rv}$  – required shear stress using LFRD or ASD load combinations, ksi (MPa)

$F_n$  – nominal tensile stress,  $F_{nt}$ , or shear stress,  $F_{nv}$ , ksi (MPa)

$F_{nt}$  – nominal tensile stress from Table J3.2, ksi (MPa)

$F_{nt}'$  – nominal tensile stress modified to include the effects of shear stress, ksi (MPa)

$F_{nv}$  – nominal shear stress from Table J3.2, ksi (MPa)

$F_{nw}$  – nominal stress of the weld material, ksi (MPa)

$F_u$  – specified minimum tensile strength, ksi

$F_w$  – 60% weld electrode strength, ksi

$F_{wc}$  – design weld strength, ksi

$F_{yp}$  – yield strength of plate, ksi

$h_i$  - dimension of HSS, in. (see Figure 10)

$k_{ds}$  – directional strength increase factor

$L$  - distance from anchor rod to HSS corner

$L_w$  – total weld length, in.

$n$  – number of bolts

$M_1$  – moment at the stem line, kip-in

$M_2$  – moment at the bolt line, kip-in

$q_r$  – prying force per bolt at required design load, kips



$p$  – tributary length per bolt

$p_a$  – tributary bolt length for pattern A, in. (see Figure 10)

$p_b$  – tributary bolt length for pattern B, in. (see Figure 10)

$p_c$  – tributary bolt length for pattern C, in. (see Figure 10)

$p_i$  – tributary length of end-plate per bolt for the  $i$ th pattern, A, B or C, in.

$p_{i,max}$  – maximum bolt spacing for the  $i$ th pattern, in.

$P_a$  – ASD factored load, kips

$P_n$  – design/allowable strength

$P_u$  – LRFD factored load, kips

$P_y$  – yield load, kips

$Q$  – bolt factor

$R_{hss}$  - HSS thickness ratio

$R_n$  – design/allowable strength

$s$  – bolt hole spacing, in.

$t$  – thickness of tee flange or angle leg, in.

$t_b$  – wall thickness of HSS branch member

$t_c$  – connecting element thickness required to develop  $T_c$  in bolts with no prying action

$t_{min}$  – minimum connecting element thickness, in.

$t_{hss}$  – wall thickness of HSS

$t_{np}$  – minimum connecting element thickness that eliminates prying action

$t_p$  – plate thickness, in.

$t_w$  – weld effective throat thickness

$T_a$  – ASD factored tensile load, kips

$T_c$  – available tensile strength per bolt based on limit state of tension only as determined by

AISC *Specification* Section J3.7 or the combined limit states of tension and shear rupture as determined by AISC *Specification* Section J3.8, kips

$T_{c,adj}$  – adjusted bolt available tensile strength

$T_r$  – required tensile strength per bolt (exclusive of initial tightening and prying force), kips

$T_u$  – LFRD factored tensile load, kips

$w$  – weld size, in.

$w_i$  – dimension of HSS, in. (see Figure 10)

$W_t$  – throat width of the weld =  $w \times (0.707)$

$X_u$  – ultimate strength of weld metal

$\delta$  – ratio of the net tributary length at bolt line to gross tributary length at the face of the stem or

angle per leg =  $1 - \frac{d'}{p}$

$\alpha$  – ratio of the moment at the stem line, kip-in. =  $\frac{M_1}{\delta M_2}$

$\Theta$  – angle between the line of action of the required force and the weld longitudinal axis, degrees

$\Omega_b$  – 1.67, ASD bending resistance factor

$\Omega$  – ASD resistance factor

$\phi_b$  – 0.90, LRFD bending resistance factor

$\phi$  – LRFD resistance factor

$\rho$  – equal to  $\frac{a'}{b'}$

*Abbreviations:*

AACB – All-Around Corner Bolt

AASB – All-Around Side Bolt

AAST– All-Around Strong Axis

AAWK – All-Around Weak Axis

AISC – American Institute of Steel Construction

ASD – Allowable Stress Design

FE – Finite Element

FEA – Finite Element Analysis

HSS – Hollow Structural Sections

ksi – kips per square inch

LVDT – Linear Variable Differential Transformer

LFRD – Load and Resistance Factor Design

MSOE – Milwaukee School of Engineering

SG – Strain Gage

WFCB – Workable-Flat Corner Bolt

WFSB – Workable-Flat Side Bolt

WFST– Workable-Flat Strong Axis

WFWK– Workable-Flat Weak Axis

## **Experimental and Analytical Evaluation of the Flexural and Axial Capacity of Steel HSS End-Plate Connections**

Hollow Structural Sections (HSS) are widely used in the steel industry for steel construction because of their aesthetics and reduced weight compared to equivalent open steel sections; however, one of the drawbacks of using HSS members is that the design and fabrication of HSS connections is more challenging than open steel sections (Packer et al., 2010). Because of their closed shape, accessing the inside of the members is limited, which can reduce the strength of welded connections or introduce stress concentrations at the interface of HSS members and other steel members. Furthermore, determining the stress distribution around closed HSS sections and determining limit states in HSS connections is more complex compared to open steel sections. Because of the complexity of the analysis of HSS connections, it is very difficult to directly determine modes of failure and limit states and eventually the nominal capacity of HSS connections. To solve this problem, structural designers have been using design guides which have been developed based on several simplifying assumptions to streamline the design procedure, such as *Steel Design Guide 24* (Packer et al., 2010). It is important to verify the level of safety, reliability, and efficiency associated with these design guides and their applicability during different stages of construction through further experimental and analytical studies.

*Steel Design Guide 24*, published by the American Institute of Steel Construction (AISC), includes design examples for various HSS connections, including welded HSS to bolted end-plate connections (Packer et al., 2010). Figure 1 illustrates one configuration of this common connection, which consists of an HSS member, fillet welded at its end to a perpendicular steel plate that is then connected to another structural element via bolts or anchor rods. It is used for

various framing connections such as column end, beam-to-column, and splice connections. In these various configurations and loading, the connection can be subjected to shear, flexural (moment), and axial (tension and compression) forces; the connection may see only one force or a combination of the forces (Kanvinde et al., 2024). These forces should be carefully determined through structural analysis of all possible loading as they can cause the connection to have several limit states that must be checked during design. These limit states include the yielding and rupture limits of the different elements of the connection, which can be affected by complex phenomenon such as prying action, yield lines of the plate, and fillet weld behavior (Packer et al., 2010).

### Figure 1

*HSS End-Plate Connection*



The purpose of this Milwaukee School of Engineering Master of Science in Architectural Engineering (MSAE) capstone project was to investigate if the current design procedures in *Steel Design Guide 24* for rectangular HSS steel end-plate connections are comprehensive and accurate. This investigation was aided by literature research followed by physical testing and

analysis of results. For the physical testing, the capacities of HSS end-plate connections were tested under two types of loading, in two major phases. Phase 1 involved the testing of the connections under pure axial loading, subjecting the connections to only tension; this phase involved square HSS and investigated two-bolt patterns: corner bolts and side bolts. Phase 2 involved testing of the connection with lateral loading, subjecting the connection to bending moment and shear; this phase involved rectangular HSS and investigated bending about the weak and strong axes. Both phases investigated two weld patterns: all-around welds and workable-flat welds. Load cells, strain gages, and linear variable differential transformers (LVDTs) were used to collect load, strain, and displacement data, respectively.

This paper is organized in several sections to define the purpose and methods of this research and support any conclusions and recommendations made. First, background information is provided to introduce the key subtopics behind the research. Second, a literature review is presented consisting of several studies involving the key subtopics of this research to further define relevant knowledge and equations behind them. Third, the methods of the experimental testing conducted in this research are presented. Fourth, the results of the experimental tests are presented, analyzed, and discussed. Fifth, conclusions are made based on the results. Finally, recommendations are made based on the conclusions.

## Background

Structural steel connections often have various limit states, or ways an element can fail, that need to be checked. Different loading configurations can apply shears, moments, and axial forces on the connection that cause various limit states to occur. Tamboli (2017) describes significant challenges associated with the design of structural connections.

Connection design is an interesting subject because it requires a great deal of rational analysis in arriving at a solution. There are literally an infinite number of possible connection configurations, and only a very small number of these have been subjected to physical testing. Even within the small group that has been tested, changes in load directions, geometry, material types, fastener type, and arrangement very quickly result in configurations that have not been tested and therefore require judgment and rational analysis on the part of the designer... Connection design is both an art and a science. The science involves equilibrium, limit states, load paths, and the lower bound theorem of limit analysis. The art involves the determination of the most efficient load paths for the connection, and this is necessary because most connections are statically indeterminate.

(Chapter 2, 2.1 Introduction, 2.1.1 Philosophy)

Connections are an intricate part of structures that require in depth analysis and experimental testing to develop models and procedures for design.

Axial tension loading is one configuration of loading that an HSS end-plate could be subjected to; for example, it is common in splice connections (Willibald et al., 2002) and can occur in column base plate connections when wind causes net uplift on a structure (Kanvinde et al., 2024). Figure 2 shows the two configurations of axially loaded end-plates in this project. The

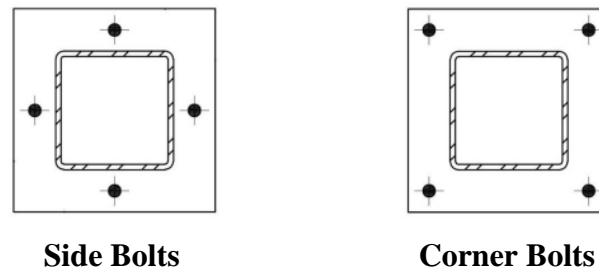


configuration on the left includes four bolts, one centered on each side of the HSS member.

Various studies have investigated this configuration; for example, Thornton (2017) developed a design method based of testing conducted by Willibald et al., Kato and Mukai (1985), Caravaggio (1988), and Willibald et al. (2003). The configuration on the right also includes four bolts, but they are instead located at the corners of the HSS member. This case was investigated by Christensen (2010). Only the side bolted configuration is considered in *AISC Steel Design Guide 24*.

## Figure 2

### *Configurations of Axially Loaded HSS End-Plate Connections*



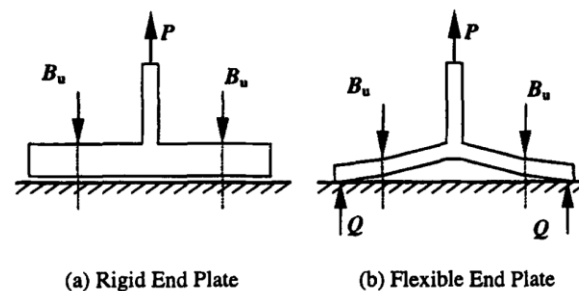
Lateral loading is another loading configuration that an HSS end-plate connection could be subjected to; for example, it can be caused by wind and earthquake loading. Lateral loading causes bending (moment) and shear on the connection. Literature on HSS moment connections is limited (Packer et al., 2010). Wheeler et al. (1998) developed an analytical model to determine the moment capacity of rectangular HSS end-plate connections. Heinisuo et al. (2012) investigated yield mechanisms of rectangular HSS end-plate connections. *AISC Steel Design Guide 24* provides limited design for HSS end-plate moment and shear connections.

One limit state for both axial tension and lateral loading is the bolt tensile capacity, or how much tension the bolts can undergo before yielding or rupturing; all four bolts in axial

loading and two bolts in lateral loading will be in tension. Both axial tension loading, and lateral loading can cause prying action to occur in the connection, which increases the bolt tension force. The *AISC Steel Construction Manual* defines prying action as “...a phenomenon that occurs in bolted joints with tensile bolt forces. The tensile force in the bolt is increased due to the deformation of the connecting element” (American Institute of Steel Construction, 2022a). In other words, the deformation of the end-plate causes it to lever against the connected element to increase the tensile force on the bolts. See Figure 3(b) for a diagram of prying action where the deformation of the plate results in the prying forces,  $Q$ , which increase the bolt force,  $B_u$ .

**Figure 3**

*Forces Diagrams for (a) Rigid Plates with No Prying Action and (b) Flexible Plates with Prying Action*



*Note.* Adapted from “Design Model For Bolted Moment End-Plate Connections Using Rectangular Hollow Sections” by A. T. Wheeler et al., 1998, *Journal of Structural Engineering*, 124(2), p. 165 ([https://doi.org/10.1061/\(ASCE\)0733-9445\(1998\)124:2\(164\)](https://doi.org/10.1061/(ASCE)0733-9445(1998)124:2(164))).

The deformation of the end-plate, which affects the prying force in the bolt, depends on how the plate yields about yield lines. The tributary length ( $p$  in Figure 4) is based on yield line theory (American Institute of Steel Construction, 2022a). Prying action and yield lines complicate the design of HSS steel end-plate connections; *AISC Steel Design Guide 24* states

“due to the complexity of the analysis accounting for prying action and the position of yield lines in the plate, it is difficult to directly determine nominal capacities according to the AISC *Specification* provisions” (Packer et al., 2010, p. 51).

The thickness, configuration, and loading of the end-plate can affect its behavior and whether yield lines and prying action occur before failure. Wheeler et al. (1998) observe that

As outlined by Nair et al. (1974), the ultimate strength of the connection may be reached either before or after yielding has occurred in the end-plate. In the former case, the end-plate is said to be "rigid," but in the latter case, the end-plate is said to be "flexible." The design of rigid end-plate connections may be less difficult than design of flexible end-plate connections due to the need to consider prying effects in the latter, but the flexible end-plate provides a substantially more economical and ductile connection. (p. 164)

The difficulty in design due to prying effects in flexible plate connections is mentioned but is justified because of better economics and ductility. Ductility is important in connections for several reasons. For example, it increases strength, leads to more robust structures, provides warning of failure, and helps structures survive earthquake loading (Engelhardt, 2007).

Wheeler et al. (1998) note that the behavior of the end-plate can be classified further into three categories depending on plate thickness.

As suggested by Kennedy et al. (1981), the behavior of the end-plate can be divided into three distinct categories based on the plate thickness and magnitude of loading. The first mode is termed thick plate behavior and is characterized by the absence of prying effects and yield lines, resulting in a direct relationship between the bolt loads and the applied moment. At the other extreme, the third mode is termed thin plate behavior and is

characterized by yield lines through the bolt positions and a maximum value of the prying force. The resulting bolt loads are the superposition of the bolt pretension, the prying forces, and the forces induced in the bolts from the applied moment. The second mode, termed intermediate plate behavior, falls between the thick and thin plate behavior and is characterized by the prying force ranging from zero to the maximum attainable value. (p. 164)

For thin and intermediate plates, the resultant bolt force has three components: the bolt pretension, the prying forces, and the tension forces resulting from an applied moment or axial load.

Another limit state for both loading configurations is the fillet weld capacity; the capacity of HSS fillet weld connections have been investigated recently by Wilsmann (2012), Packer et al. (2016), and Tousignant and Packer (2017). Depending on the end-plate configuration and loading, the entire length of weld length may not be effective and only the effective portions should be considered for design (Kanvinde et al., 2024). The applicability of the directionality factor, which can allow for a strength increase of up to one and a half times depending on the direction of the loading relative to the direction of the weld, must also be considered for HSS end-plate connections (Packer et al., 2016).

Packer et al. (2016) subsequently performed a large number of laboratory tests on HSS-to-rigid end-plate connections to investigate the applicability of the  $\sin \theta$  factor to single-sided fillet welds to HSS, joined to a rigid end-plate. These experiments removed the influence of a flexible landing surface for the fillet weld, and hence removed the weld effective length phenomenon. It was shown that HSS-to-plate fillet welds still did not

provide the adequate structural reliability if the  $\sin \theta$  factor was implemented.

(Tousignant & Packer, 2017)

### **Literature Review**

The purpose of this literature review was to find relevant sources of information and guidance on the design of HSS end-plate connections. To find relevant literature beyond theses from past research conducted at MSOE and AISC design guides and specifications, keywords were used in online search engines and databases. The literature review is organized in four main categories: design guides, axial, flexural, and fillet welds. Relevant knowledge and equations from the sources in each category pertaining to the design of the components of HSS end-plate connections are highlighted.

## Design Guides and Specifications

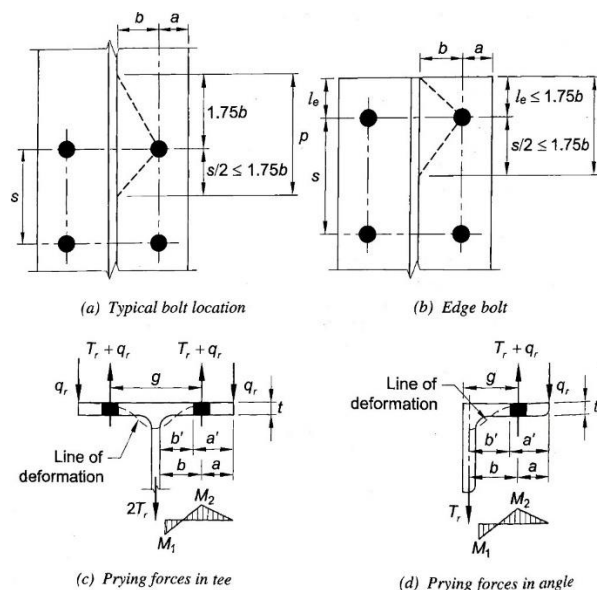
### AISC *Steel Construction Manual* and AISC 360 *Specification for Structural Steel Buildings*

AISC publishes the AISC *Steel Construction Manual*, or *Manual*, along with the AISC *Specification for Structural Steel Buildings*, or *Specification*. The most recent versions at the time of writing are the 16<sup>th</sup> edition of the *Manual* and AISC 360-22 for the *Specification*. For purposes of this literature review, the most recent versions will be referenced.

Part 9 of the *Manual* provides design guidance and equations to account for prying action, based on Thornton (1992) and Swanson (2002). Figure 4 illustrates the variables, forces, and dimensions, associated with prying action on tee and angle connections.

**Figure 4**

*Illustration of Variables in Prying Action Calculations*



*Note.* Adapted from *Steel Construction Manual (16th ed.)* by American Institute of Steel Construction, 2022a, p. 9-12.

There are two inequalities that govern the prying action design and analysis methods presented in the *Manual*, for the connecting element:

$$\frac{T_r}{T_c} \leq (1 + \delta\alpha) \left( \frac{t}{t_c} \right)^2, \quad (1)$$

and for the bolts:

$$\frac{T_r}{T_c} \leq \frac{1 + \delta\alpha}{1 + \delta\alpha(1 + \rho)}. \quad (2)$$

There are three solution methods given. The first method is to “design assuming no prying forces, which results in the smallest bolt diameter” (American Institute of Steel Construction, 2022a); this method will result in a thicker plate, with a thickness,  $t_{np}$ , that effectively eliminates prying action. Equations (3) and (4) calculate the required plate thickness for no prying action to occur for load and resistance factor design (LRFD) and allowable stress design (ASD), respectively:

$$t_{np} \geq \sqrt{\frac{4T_c b'}{\phi_b p F_u}} \sqrt{\frac{T_r}{T_c}} = \sqrt{\frac{4T_r b'}{\phi_b p F_u}}, \quad (3)$$

$$t_{np} \geq \sqrt{\frac{4\Omega_b T_c b'}{p F_u}} \sqrt{\frac{T_r}{T_c}} = \sqrt{\frac{4\Omega_b T_r b'}{p F_u}}. \quad (4)$$

The second method is the method that “results in the smallest required connecting element thickness” (American Institute of Steel Construction, 2022a). This method accounts for the effects of prying action and gives equations for the minimum required plate thickness,  $t_{min}$ , given by Equations (5) and Equation (6) for LRFD and ASD, respectively. Thus,



$$t_{min} = \sqrt{\frac{4T_r b'}{\phi_b p F_u (1 + \delta \alpha')}} \quad (5)$$

$$t_{min} = \sqrt{\frac{4\Omega_b T_r b'}{p F_u (1 + \delta \alpha')}} \quad (6)$$

The  $\alpha'$  factor for this method is determined by first calculating  $\beta$  using:

$$\beta = \frac{1}{\rho} \left( \frac{T_c}{T_r} - 1 \right). \quad (7)$$

If  $\beta$  is greater than or equal to one ( $\beta \geq 1$ ) then:

$$\alpha' = 1. \quad (8)$$

Otherwise, if  $\beta$  is less than 1 ( $\beta < 1$ ) then:

$$\alpha' = \frac{1}{\delta} \left( \frac{\beta}{1 - \beta} \right) \leq 1. \quad (9)$$

The third and final method involves an “analysis to find the tensile strength of the connecting element” (American Institute of Steel Construction, 2022a). A bolt factor,  $Q$ , is determined to adjust the bolt available strength. There are three steps to this method. The first step is to calculate the  $\alpha'$  factor for this method using the following equation:

$$\alpha' = \frac{1}{\delta(1+\rho)} \left[ \left( \frac{t_c}{t} \right)^2 - 1 \right]. \quad (10)$$

Depending on the value of  $\alpha'$ , there are three possible values for the bolt factor,  $Q$ . The first value for  $Q$  applies when  $\alpha'$  is less than or equal to zero ( $\alpha' \leq 0$ ), which occurs when the connecting element of thickness  $t$  has sufficient strength to develop the tensile strength of the bolt, and is given as

$$Q = 1. \quad (11)$$

The second value for  $Q$  applies when  $\alpha'$  falls between zero and one or is equal to one ( $0 < \alpha' \leq 1$ ), which occurs when the connecting element and bolt both control the design, and is given as

$$Q = \left(\frac{t_c}{t}\right)^2 - (1 + \delta\alpha'). \quad (12)$$

The third value for  $Q$  applies when  $\alpha'$  is greater than one ( $\alpha' > 1$ ), which occurs when the connecting element controls the design and is given as

$$Q = \left(\frac{t_c}{t}\right)^2 - (1 + \delta). \quad (13)$$

Finally, the adjusted bolt available tensile strength can be calculated using the calculated bolt force from Equation (11), (12), or (13) as

$$T_{c,adj} = QT_c. \quad (14)$$

Chapter J of the *Specification* contains information and equations on connecting elements, connectors, and the affected elements of connected members (American Institute of Steel Construction, 2022b). Section J2 addresses welds; the question provided to calculate the strength of fillet welds is

$$R_n = F_{nw}A_{we}k_{ds}, \quad (15)$$

where  $k_{ds}$  is the directional strength increase factor. If strain compatibility of the various weld elements is considered it can be calculated by:

$$k_{ds} = (1.0 + 0.50\sin^{1.5}\theta), \quad (16)$$

where if an increase of strength of up to one and a half times can be taken if the line of action of the force is perpendicular to the weld longitudinal axis. However, for fillet weld to the ends of rectangular HSS loaded in tension  $k_{ds}$  must be limited to 1.0:

$$k_{ds} = 1.0.$$

Section J3 addresses bolts; Section J3.7 addresses the tensile and shear strength of bolts and threaded parts and provides an equation based on the limit states of tension rupture and shear rupture:

$$R_n = F_n A_b. \quad (17)$$

It is mentioned that the required tensile strength should include tension resulting from prying action. A user note addresses the consideration of whether the bolt in shear is sheared through its shank or threads, as it will affect its available strength; if a bolt is loaded through the threads there will be less effective area,  $A_b$ , and thus lower strength. Another user note mentions the limit states of bolt bearing and bolt tearout in strength that are present in a snug-tightened or pre-tensioned high-strength bolt or threaded part; these additional limit states are addressed in Section J3.11, and the effective strength of an individual fastener may be taken as the lesser of the strength per Section J3.7 or J3.11. Section J3.7 addresses tension and shear acting alone, but special consideration must be taken when both tension and shear act together, this is address in Section J3.8, with the following equation:

$$R_n = F_{nt}' A_b, \quad (18)$$

where the combined effects are addressed by the modified tensile stress,  $F_{nt}'$ , given by Equation (19) and Equation (20) for LFRD and ASD, respectively:

$$F_{nt}' = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \leq F_{nt}, \quad (19)$$

$$F_{nt}' = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \leq F_{nt}. \quad (20)$$

Section J3 provides a table for the minimum edge distance from center of a standard hole to the edge of the connected part, see Table 1.

**Table 1**

*Bolt Hole Edge Distance*

Bolt Diameter, in.	Minimum Edge Distance
$\frac{1}{2}$	$\frac{3}{4}$
$\frac{5}{8}$	$\frac{7}{8}$
$\frac{3}{4}$	1
$\frac{7}{8}$	$1\frac{1}{8}$
1	$1\frac{1}{4}$
$1\frac{1}{8}$	$1\frac{1}{2}$
$1\frac{1}{4}$	$1\frac{5}{8}$
Over $1\frac{1}{4}$	$1\frac{1}{4} \times d$

*Note.* Adapted from ANSI/AISC 360-22: *Specification for Structural Steel Buildings* by American Institute of Steel Construction, (2022b), p.16.1-139.

Chapter K of the *Specification* addresses additional requirements for connections to HSS members. A user note states that that “connection strength is often governed by the size of HSS members..., and this must be considered in the initial design” (American Institute of Steel Construction, 2022b).

## **AISC Steel Design Guide 1: Base Connection Design, Fabrication, and Erection for Steel Structures**

A significant amount of research was conducted and changes to related codes and standards were made following the publication by AISC of the second edition of *Steel Design Guide 1* (Fisher & Kloiber, 2006), leading to the third edition. Kanvinde, Maamouri, and Buckholt (2024) authored the third edition of *Steel Design Guide 1*; it offers guidance in the design, fabrication, and erection of steel base connections (Kanvinde et al., 2024). Chapter 4 covers the design of exposed column base connections for axial, bending, and shear forces, alone and in combination, considering the various components of the connection, i.e., the anchor rods, steel base plate, and welds.

*Steel Design Guide 1* breaks out the design of base connections into six steps. The first step is to select the length and width of the base plate (“base plate footprint”). The second step is to determine how internal forces are distributed. The third step is to select the base plate thickness. The fourth step is anchoring design. The fifth step is footing design considerations. The sixth, and final, step is the design and detailing of the welds. The research of this paper will involve all steps, except the portion of the fifth step involving concrete design. These are the general steps involved with designing base connections; there are special considerations for the different loadings on the connection within the various steps that need to be accounted for.

*Steel Design Guide 1* Section 4.3.2 offers guidance for design for axial tension. Section 4.3.3 covers design for shear and Section 4.3.6 covers design for bending. There is no section in *Steel Design Guide 1* that addresses only the combination of bending and shear forces, but Sections 4.3.9 and 4.3.10 cover design for combined axial forces, bending, and shear. These

sections reference prior sections and offer additional commentary on any additional stresses that could arise from the combination of forces.

*Steel Design Guide 1* also covers material and type specification for the plates, anchors, and welds. The design guide provides tables for recommended base plate materials and anchor rod materials, see Tables 2 and 3. It is stated that “base plates should be designed using ASTM A572/A572M Grade 50 material unless the availability of an alternative grade is confirmed prior to specification”. It is also mentioned that a typical minimum thickness of ½ in for posts and light HSS columns, and ¾ in for other structural columns (Kanvinde et al., 2024).

**Table 2**

*Recommended Base Plate Materials*

Thickness ( $t_p$ )	Plate Availability
$t_p \leq 4$ in.	ASTM A36/A36M ASTM A572/A572M Gr 42 or 50 <sup>[a]</sup> ASTM A588/A588M Gr 50
4 in. < $t_p \leq 5$ in.	ASTM A36/A36M <sup>[a]</sup> ASTM A572/A572M Gr 42 ASTM A588/A588M Gr 46
5 in. < $t_p \leq 6$ in.	ASTM A36/A36M <sup>[a]</sup> ASTM A572/A572M Gr 42 ASTM A588/A588M Gr 42
6 in. < $t_p \leq 8$ in.	ASTM A36/A36M <sup>[a]</sup> ASTM A588/A588M Gr 42
$t_p > 8$ in.	ASTM A36/A36M <sup>[a]</sup>
<sup>[a]</sup> Preferred Material Specification	

*Note.* Adapted from *Steel Design Guide 1: Base Connection Design, Fabrication, and Erection for Steel Structures* (Third ed.) by A. M. Kanvinde et al., 2024, American Institute of Steel Construction, p. 8.

**Table 3***Recommended Anchor Rod Materials*

Material ASTM		Tensile Strength $F_u$ , ksi	Nominal Tensile Stress <sup>[a]</sup> $F_{nt} = 0.75F_u$ , ksi	Nominal Shear Stress (N type) <sup>[a, c]</sup> $F_{nv} = 0.450F_u$ , ksi	Nominal Shear Stress (X type) <sup>[a, b]</sup> $F_{nv} = 0.563F_u$ , ksi	Maximum Diameter, in.
F1554	Gr 36 <sup>[d]</sup>	58	43.5	26.1	32.7	4
	Gr 55 <sup>[d]</sup>	75	56.3	33.8	42.2	4
	Gr 105	125	93.8	56.3	70.4	3
A449		120	90.0	54.0	67.6	1
		105	78.8	47.3	59.1	1-1/2
		90	67.5	40.5	50.7	3
A36/A36M		58	43.5	26.1	32.7	15
A354 Gr BD		150	113	67.5	84.5	4
<sup>[a]</sup> Nominal stress on unthreaded body area of threaded part (gross area) <sup>[b]</sup> Threads excluded from shear plane <sup>[c]</sup> Threads included in the shear plane <sup>[d]</sup> Preferred material specification						

*Note.* Adapted from *Steel Design Guide 1: Base Connection Design, Fabrication, and Erection for Steel Structures* (Third ed.) by A. M. Kanvinde et al., 2024, American Institute of Steel Construction, p. 8.

The design guide mentions that load path from the column to the anchor rod should be considered when designing the weld between the column and the base plate. If the base plate is rigid, then the entirety of the weld could be considered effective. However, this is not often the case and, in most cases, only a portion of the weld could be considered effective in transferring the forces. An example that is given of such a case is when anchor rods are located at the corners of HSS columns; to account for the stress concentrations in this case, the design guide points to methods in *AISC Design Guide 10: Erection Bracing of Low-Rise Structural Steel Buildings* (West & Fisher, 2020) and the *AISC Hollow Structural Connections Manual* (American Institute of Steel Construction, 1997). It also mentions testing done by Christensen (Christensen, 2010) and Wilsmann (Wilsmann, 2012) which evaluated the methods and found them to be “generally conservative for the tested cases” (Kanvinde et al., 2024, p.36). Example 4.3 illustrates a second case in which “only welding adjacent to the anchor rod is considered effective” (Kanvinde et al., 2024). Ultimately it is noted that “a consistent model of load path from the column, through the

effective portions of the welding, through the effective portions of the base plate in bending, through the anchor rods, and into the concrete should be used” (Kanvinde et al., 2024, p. 36).



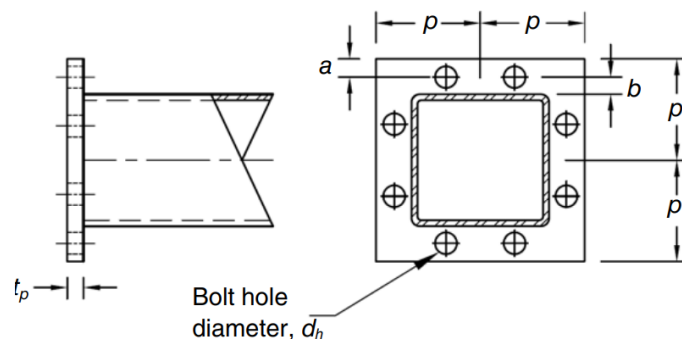
## AISC Steel Design Guide 24: Hollow Structural Steel Connections

Packer, Sherman, and Lecce (2010) authored *AISC Steel Design Guide 24* (Packer et al., 2010). It offers design methods and examples for various HSS connections. Chapter 5 covers HSS tension and compression connections, and Section 5.6 covers the design of an end-plate on rectangular HSS with bolts on four sides under axial tension. Chapter 4 covers HSS moment connections, with Section 4.1 covering W-beams to HSS columns and mentions “contemporary information concerning moment connections to HSS columns is limited in literature” (Packer et al., 2010, p. 29).

The design method for end-plate on rectangular HSS with bolts on four sides under axial tension, illustrated in Figure 4, in Section 5.6 outlines three limit states: yielding of the end-plate, tensile strength of the bolts (including prying action), and strength of the weld between the HSS and the end-plate.

**Figure 5**

*AISC Steel Design Guide 24 - Rectangular End-Plate with Bolts on Four Sides*



*Note.* Adapted from *Steel Design Guide 24: Hollow Structural Section Connections*, by J.A. Packer et al., 2010, American Institute of Steel Construction, p.53.

The design guide notes the difficulty in directly determining the capacity of this connection and load configuration based on AISC Specification due to the complexities of prying action and position of yield lines in the plate. The design guide does however present equations to assist in designing the thickness of the plate,  $t_p$ , the number of bolts,  $n$ , and the weld size,  $w$  in Section 5.6. Equation (21) and Equation (22) are presented in the design guide to calculate the minimum thickness required for no prying action to occur for LFRD and ASD, respectively. These equations are simplified from AISC *Manual* Part 9 (Packer et al., 2010). Thus,

$$t_p \geq t_{min} = \sqrt{\frac{4.44(P_u/n)b'}{pF_{yp}}}, \quad (21)$$

$$t_p \geq t_{min} = \sqrt{\frac{6.66(P_a/n)b'}{pF_{yp}}}. \quad (22)$$

The design guide also provides equations for determining a lesser thickness that accounts for the effects of prying action, Equations (23) and Equation (24) for LFRD and ASD, respectively.

Thus,

$$t_p \geq t_{min} = \sqrt{\frac{4.44(P_u/n)b'}{pF_{yp}(1+\delta\alpha')}} \quad (23)$$

$$t_p \geq t_{min} = \sqrt{\frac{6.66(P_a/n)b'}{pF_{yp}(1+\delta\alpha')}}. \quad (24)$$

These equations are also adapted from AISC *Manual* Part 9. The prying action equations in this section uses  $F_{yp}$  for the plate material strength, as experimentally validated by Willibald et al. (2003), upon which the procedure is based, rather than higher capacity of  $F_{up}$  as in the AISC *Manual*. Section 5.6 also provides equations for designing the weld based on the limit state of shear rupture from AISC *Specification* J2.4, Equations (25) and (26) for LFRD and ASD, respectively. Thus,

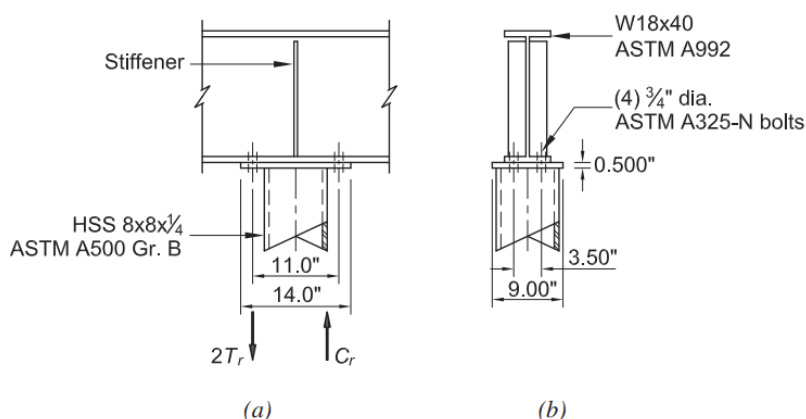
$$w \geq \frac{P_u \sqrt{2}}{F_{wc} L_w}, \quad (25)$$

$$w \geq \frac{P_a \sqrt{2}}{F_{wc} L_w}. \quad (26)$$

Limit states for HSS moment connections are covered in the examples section of chapter 4, Section 4.5. Example 4.1 involves a W-beam running over a HSS column connected by an end-plate moment connection, shown in Figure 5, and outlines the limit states that must be considered. These limit states include the effects of prying action on the W-shape flange and cap plate, tensile strength of the bolts, beam web local yielding, beam web local crippling, HSS wall strength, HSS wall local yielding, HSS wall crippling, and strength of the weld between the HSS and the end-plate.

## Figure 6

*AISC Steel Design Guide 24 - W-Beam Over HSS Column Connection (Moment Connection)*



*Note.* Adapted from *Steel Design Guide 24: Hollow Structural Section Connections*, by J.A. Packer et al., 2010, American Institute of Steel Construction, p. 31.

A simplified check for prying action based on “no prying action” equation in Part 9 of the AISC *Manual* is performed in Example 4.1 using Equations (27) and (28) for LFRD and ASD, respectively. Prying action equations in this section use  $F_u$ , based on the prying action equation in Part 9 of the *Manual*. Thus,

$$t_p \geq t_{min} = \sqrt{\frac{4.44T_u b'}{pF_u}}, \quad (27)$$

$$t_p \geq t_{min} = \sqrt{\frac{6.66T_u b'}{pF_u}}. \quad (28)$$

Example 4.1 also includes a method for designing the weld in which the effective length is determined assuming a 45° maximum load dispersion angle. Because a moment force will put one side of the HSS into compression, the HSS wall strength, wall local yielding, and wall local crippling are also checked in this example.

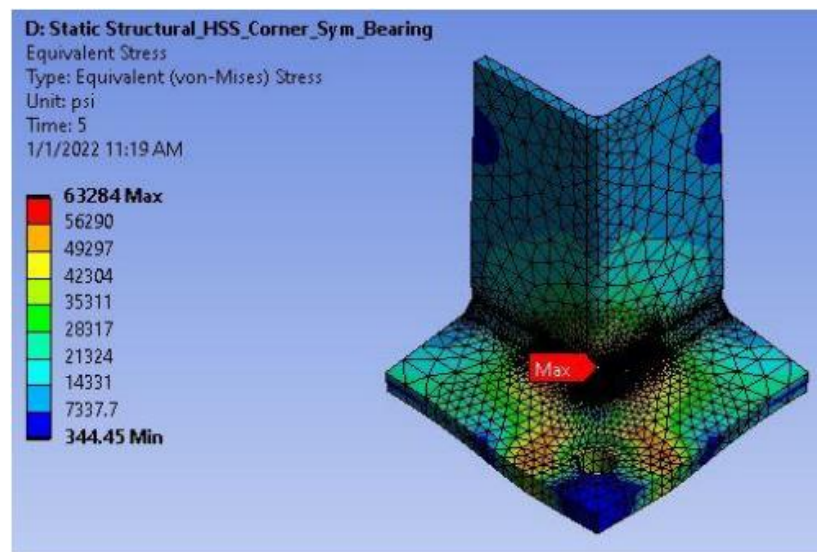
## Axial

### Evaluation of the Axial Capacity of Steel HSS End-Plate Connections using Nonlinear Finite Element Simulations and Analytical Methods

Zietlow (2022) used analytical methods and non-linear Finite Element Analysis (FEA) through ANSYS® Workbench R2 (2020) software to investigate the axial tensile capacity of HSS end-plate connections with all-around weld patterns and both side and corner bolt configurations. Figures 7 and 8 show the two FEA models investigated. Symmetry was used to simplify the model to one quarter of the specimen.

#### Figure 7

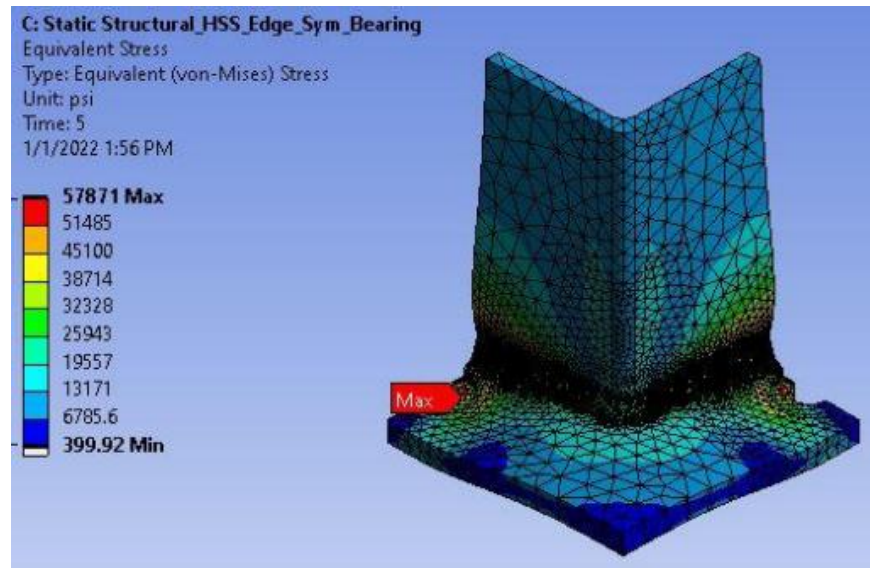
##### *Axial Corner Bolt FEA Model - von Mises Stress Distribution*



*Note.* Adapted from *Evaluation Of The Axial Capacity of Steel HSS End-Plate Connections Using Nonlinear Finite Element Simulations And Analytical Methods* [Unpublished master's thesis], by J. A. Zietlow, 2022, Milwaukee School of Engineering, p. 59.

**Figure 8**

*Axial Side Bolt FEA Model - von Mises Stress Distribution*



*Note. Adapted from Evaluation Of The Axial Capacity of Steel HSS End-Plate Connections Using Nonlinear Finite Element Simulations And Analytical Methods [Unpublished master's thesis], by J. A. Zietlow, 2022, Milwaukee School of Engineering, p. 60.*

For the corner bolt specimen, the maximum stress occurred in the weld at the corner of the HSS (see Figure 7). For the side bolt specimen, the maximum stress occurred at the bolt hole (see Figure 8). There was noticeable difference in the total deflection between the models, with the maximum deflection of the corner bolt specimen at an applied load of 20 kips being more than twice the maximum deflection of the side bolt specimen at an applied load of 25 kips. There was also a larger prying force in corner bolt specimen, with a maximum bolt force increase of 192% for the corner bolt specimen, while the maximum increase of 152% is seen in the side bolt specimen. The expected design strength of the connection calculated using *AISC Design Guide 24* was 119 kips, from the limit state of bolt strength (see Appendix A for calculations). However, it was found that significant plate and HSS yielding occurred at an applied load of 90

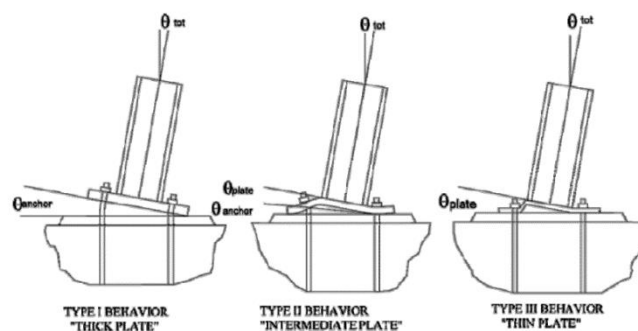
kip that would suggest a non-conservative solution from *AISC Steel Design Guide 24*, but it was also mentioned that “these results cannot be appropriately confirmed without additional data from experimental testing” (Zietlow, 2022, p. 73).

## Determining the Validity of Design Provisions for HSS to Base Plate Connections with Corner Anchor Rods Subjected to Axial Tension

Christensen (2010) performed several tests on HSS end-plate connections with varying plate and HSS thicknesses and sizes that were subjected to axial tension to verify the procedures in the *HSS Connections Manual* (1997). The manual suggested a design procedure with an equation that considered effective plate width, yield strength, base plate thickness, and the distance from anchor rod to HSS corner to determine the strength of the connection; however, there was no physical testing to validate it at the time. In the design process, base plate behavior was described as thick, intermediate, or thin (see Figure 9). The research focused on the “plate stress-based equation” from the manual, but also investigated weld strengths and strains. Linear strain gages and strain rosettes were used to measure strain in the base plates, HSS tubes, and welds; the strain rosettes were used to determine principal stress. Plate deflection was also measured using LVDTs (Christensen, 2010).

**Figure 9**

*Thick, Intermediate, and Thin Plate Behavior*



*Note.* Adapted from *Determining the Validity of Design Provisions for HSS to Base Plate Connections With Corner Anchor Rods Subjected to Axial Tension* [Unpublished master's thesis], H. Christensen, 2010, Milwaukee School of Engineering, p. 34.



It was found that the procedures based on plate thickness produced conservative results. The most important factor in determining plate behavior was found to be plate thickness, but HSS thickness was also found to impact stress results and was not considered in the manual (Christensen, 2010). Through the elimination of a factor from an error in the derivation of the equation and an additional factor for the HSS thickness,  $R_{hss}$ , calculations were comparable to experimental results (Christensen, 2010). The adjusted equation given for one corner of the connection was:

$$\phi P_n = \phi \frac{R_{hss} b_e t_p F_y}{L} = \phi 2 R_{hss} t_p^2 F_y \leq \phi \frac{4.75 R_{hss} t_p^2 F_y}{L}. \quad (29)$$

The results from weld data were found to be inconclusive as they did not match calculated design strength produced by the procedures in the manual; further investigation on weld strength was recommended.

### **Yield Line Approaches for Design of End-Plate Tension Connections for Square and Rectangular HSS Members Using End-Plate Tensile Strength**

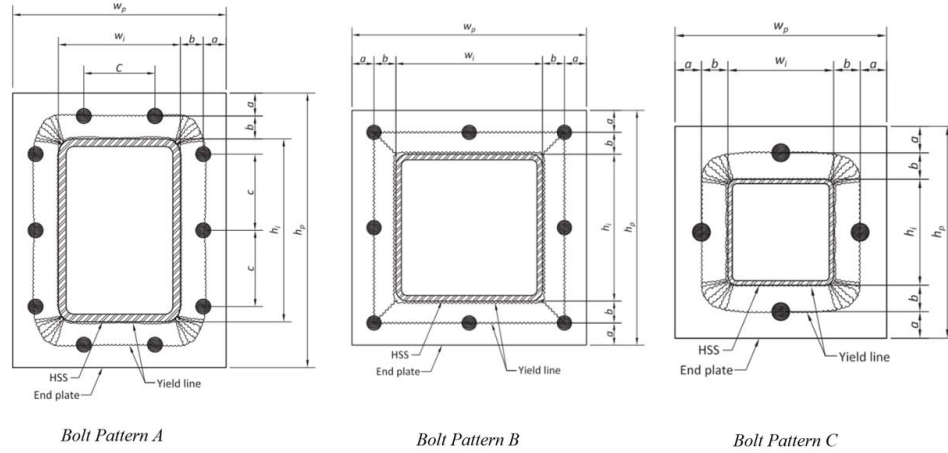
Thornton (2017) used experimental data for HSS end-plate connection subjected to axial tension produced by Willibald, Packer, and Puthli (2002, 2003); Kato and Mukai (1985); and Caravaggio (1988) to validate a design method based on the tensile strength  $F_u$ . The three bolt patterns considered in this paper, A, B, and C, are shown in Figure 10. The yield lines considered for each pattern are also shown in Figure 10.

There are many possible yield line families available for each of the three bolt patterns. For instance, circular yield lines at the HSS corners with radial fans are a possible family, as are straight line yield families. The author has reviewed a number of possibilities and determined by “trial and error” that the families chosen for this paper give the best correlation to the test data. (Thornton W. A., 2017, p. 142)

There were 55 physical tests available: 26 for pattern A, 2 for pattern B, and 27 for pattern C (Thornton W. A., 2017).

**Figure 10**

*Bolt Patterns A, B, and C and associated Yield Lines*



*Note.* Adapted From “Yield Line Approaches for Design of End-Plate Tension Connections for Square and Rectangular HSS Members Using End-Plate Tensile Strengths”, W. A. Thornton, 2017, *Engineering Journal*, 54, p. 142-143.

An equation for the tributary bolt length,  $p$ , is given for each bolt pattern: Equation (30) for pattern A, Equation (31) for pattern B, and Equation (32) for pattern C (Thornton W. A., 2017):

$$p_a = \frac{2(w_i + h_i + \pi b)}{n}, \quad (30)$$

$$p_b = \frac{2(w_i + h_i + 4b)}{n}, \quad (31)$$

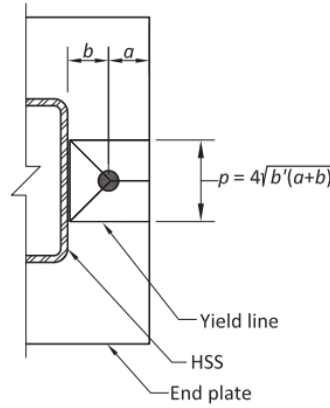
$$p_c = \frac{2(w_i + h_i + \pi b)}{n}. \quad (32)$$

It is assumed that the yield patterns shown in Figure 10 will develop as shown, but if the bolt spacing is too large, yield lines with less capacity than calculated by the above equations can develop (Thornton W. A., 2017). Dowswell (2011) showed that if the tributary length greater

than that shown in Figure 11, independent yield line with less capacity can develop at each bolt (Thornton W. A., 2017).

**Figure 11**

*Local Yield Line Pattern*



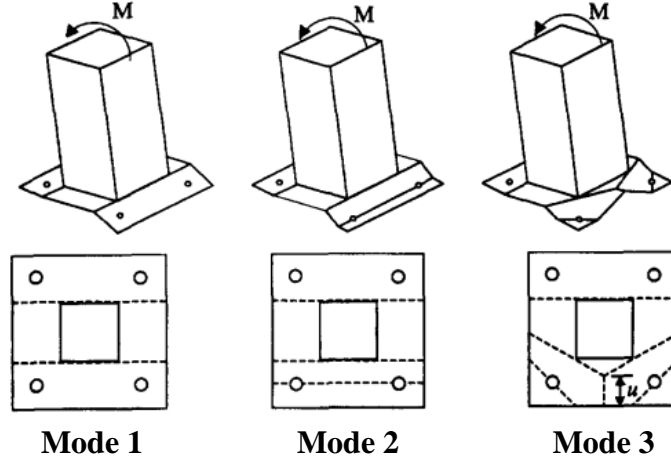
*Note.* Adapted From “Yield Line Approaches for Design of End-Plate Tension Connections for Square and Rectangular HSS Members Using End-Plate Tensile Strengths”, W. A. Thornton, 2017, *Engineering Journal*, 54, p. 144.

Therefore, the tributary lengths  $p_a$ ,  $p_b$ , and  $p_c$  are limited to:

$$p_{i,max} = 4\sqrt{b'(a + b)}. \quad (33)$$

This prevents the localized pattern from developing and maintains the validity of the patterns assumed in Figure 10 (Thornton W. A., 2017).



**Figure 13***Three Yield Line Modes of Failure*

*Note.* Adapted from “Design Model for Bolted Moment End-Plate Connections Using Rectangular Hollow Sections”, by A. T. Wheeler et al., 1998, *Journal of Structural Engineering*, 124(2), p. 167, (doi:[https://doi.org/10.1061/\(ASCE\)0733-9445\(1998\)124:2\(164\)](https://doi.org/10.1061/(ASCE)0733-9445(1998)124:2(164))).

Virtual work principles were used to obtain the analytical expressions for the yield moment for each mechanism, or mode. The three plate behaviors of thin, intermediate, and thick are considered and equations for the bolt force, including prying forces when applicable, and the moment capacity are derived. Ultimately equations are adapted for the capacity of the connection when limited by bolt failure and when limited by end-plate failure, Equations (34) and (35), respectively:

$$\phi_b M_{cb} = \phi_b \left( \frac{4n \left( B_{u1} a_p + \frac{\pi d_b^3 f_{yb}}{32} \right) d' + w_{eq} (d' + 2(s_0' + a_p)) t_p^2 f_p}{4(a_p + s_0') d'} \right) (d - t_s), \quad (34)$$

$$\phi_p M_{cp} = \phi_p \left( \frac{t_p^2 f_p (w_{eq} (d' + 2s_0')) + (w_{eq} - n d_f) + n \frac{\pi d_b^3 f_{yb}}{8} d'}{4d' s_0'} \right) (d - t_s). \quad (35)$$

The model presented is “simple and accurate” and is limited to square and rectangular HSS with two bolt rows, one above the top flange and one below the bottom flange (Wheeler, Clarke, Hancock, & Murray, 1998). The model was verified by experimentally tested connections with only two bolts per row; however, it is stated:

...the addition of extra bolts in the tensile (and compressive) bolt rows does not invalidate the model. The reason for this is that the use of additional bolts in the tensile row tends to enforce a mode 2 end-plate failure to which the model presented in this paper is well suited. (Wheeler et al., 1998, p.173)

It is recommended that connections be designed to act with intermediate behavior with the connection strength being limited by bolt failure. Equations are also presented for serviceability based on bolt and end-plate yielding, respectively:

$$\phi_b M_{cbs} = \phi_b \left( \frac{(d' + 2(s'_0 + a_e))w_{eq}t_p^2 f_y}{4(s'_0 + a_e)d'} + \frac{nB_{yl}a_e}{s'_0 + a_e} \right) (d - t_s), \quad (36)$$

$$\phi_p M_{cps} = \phi_p \left( \frac{((d' + s'_0)w_{eq} - nd_f d')t_p^2 f_y}{2s'_0 d'} \right) (d - t_s). \quad (37)$$

## Welds

### Weld Behavior in a Rectangular HSS Base Plate Connection with Corner Anchor Rods Subjected to an Axial Tensile Force

Following the research conducted by Christensen (2010) on axially loaded HSS end-plate connections, further research was conducted by Wilsmann (2012) with a focus on weld strength and validating the procedures for designing weld strength in the *HSS Connections Manual* (1997). Physical testing was done on specimens with varying base plate geometry and HSS size. Linear and rosette strain gages were placed on the weld to measure strain along its length, and LVDTs were used to measure displacements of the base plate and HSS column.

The experimental results were compared to two equations:

$$\phi R_n = \phi_w F_w W_t (4), \quad (38)$$

$$\phi R_n = \phi_p (50) W_t t_p^{1.5}, \quad (39)$$

where (4) and (50) are values for the effective length and grade of steel. The design capacity from Equation (38) is based on electrode strength and weld size. The design capacity from Equation (39) is based on weld size and plate thickness. Equation (38) was found to over calculate the weld strength because the four-inch effective length was not achieved in any of the tests. It was recommended that Equation (38) be modified to account for the effects from connection geometry on effective weld length, including plate size, anchor location, and HSS size. Equation (39) was determined to be overly conservative in determining weld strength and the origin could not be determined through literature review; because of these issues, it was recommended that Equation (39) be disregarded.

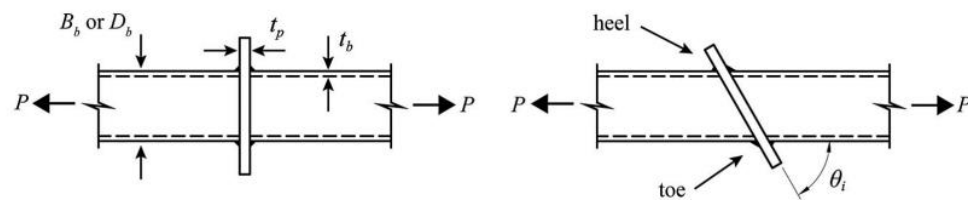


## Numerical Investigation of Fillet Welds in HSS-to-Rigid End-Plate Connections

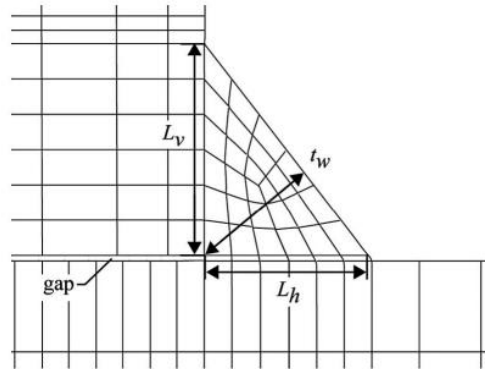
Tousignant and Packer (2017) performed a finite-element (FE) investigation on the behavior of fillet-welded HSS rigid end-plate connections. The validity of the FE models were confirmed by comparison to 33 experimental tests and then a parametric study was performed on 73 FE models. The FE models consisted of both rectangular hollow sections (RHS) and circular hollow sections (CHS) rigid end-plate connections that were tested to failure under axial tension loading. The study investigated “...the effect of the ratio of the weld throat dimension to the branch wall thickness ( $t_w/t_b$ ), the branch wall slenderness ( $D_b/t_b$  and  $B_b/t_b$ ), and the branch inclination angle ( $\theta_i$ ) on the weld strength” (Tousignant & Packer, 2017, p. 14). See Figures 14 and 15.

**Figure 14**

*HSS-To-Rigid Welded End-Plate Connection*



*Note.* Adapted from “Numerical Investigation of Fillet Welds in HSS-to-Rigid End-Plate Connections”, K. Tousignant and J. Packer, 2017, *Journal of Structural Engineering*, 143(12), p. 2 (doi:10.1061/(ASCE)ST.1943-541X.0001889).

**Figure 15***Effective Fillet Weld Dimensions*

*Note.* Adapted from “Numerical Investigation of Fillet Welds in HSS-to-Rigid End-Plate Connections”, K. Tousignant and J. Packer, 2017, *Journal of Structural Engineering*, 143(12), p. 3 (doi:10.1061/(ASCE)ST.1943-541X.0001889).

It was found that:

As  $t_w/t_b$  increases, the average stress on the weld throat area at failure significantly decreases. As  $D_b/t_b$  and  $B_b/t_b$  increase, the average stress on the weld throat area at failure slightly decreases. The branch inclination angle  $\theta_i$  has a negligible effect on the weld strength per unit throat area; however, the longer weld length that results from a reduction in branch angle increases the absolute strength of the weld. (Tousignant & Packer, 2017, p. 14)

Simplified equations for the required weld strength for both RHS and CHS connections were developed, Equations (40) and (41), respectively. Thus,

$$R_n = \left[ 0.90 - 0.25 \left( \frac{P_r}{P_y} \right) \right] A_w X_u, \quad (40)$$

$$R_n = \left[ 1.00 - 0.25 \left( \frac{P_r}{P_y} \right) \right] A_w X_u. \quad (41)$$

Based on Equations (40) and (41), the required weld size,  $t_w$ , can be determined as a function of the branch wall thickness,  $t_b$ . Equations to calculate  $t_w$  for both RHS and CHS are given as Equations (42) and (43), respectively:

$$t_w = \frac{1}{0.65} \left( \frac{F_y}{X_u} \right) t_b, \quad (42)$$

$$t_w = \frac{1}{0.75} \left( \frac{F_y}{X_u} \right) t_b. \quad (43)$$





It was ultimately recommended that “for all HSS connections, including HSS-to-HSS connections where the effective length concept is used, and even HSS connections in which the welds are “fully effective”, the current AISC and Canadian Standards Association (CSA) provisions (ANSI/AISC 360-16 and CSA S16-14, respectively) for the design of fillet welds be used without the directional strength increase factor, or the presented alternative method be used with North American resistance factors (Tousignant & Packer, 2017).

## Methods

To test the validity of *AISC Design Guide 24* design procedures and the variables affecting the capacity of HSS end-plate connections, the initial test matrix in Table 4 was developed, with Phase 1 consisting of twelve HSS specimens subjected to axial loading (axial tests) and Phase 2 consisting of twelve HSS specimens subjected to lateral loading (flexural tests). The steel for this testing was donated by Zalk Josephs fabricators, and due to their availability of steel, HSS7x7x5/16 was the selected shape for axial tests and HSS7x4x5/16 was selected for flexural tests.

**Table 4**

*Test Matrix*

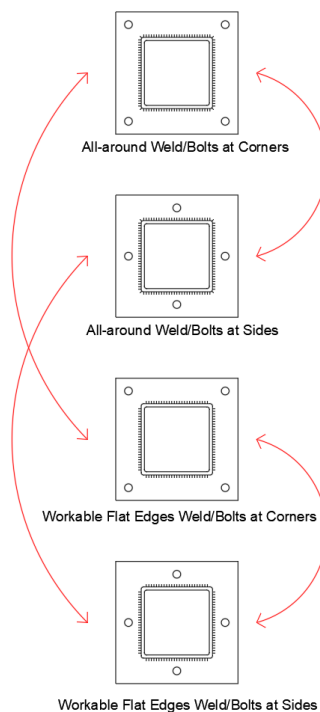
Weld Type		Experimental Tests and Phases				Instrumentation		
		PHASE 1 Axial Test  HSS 7x7x5/16		PHASE 2 Flexural Test  HSS 7x4x5/16		Number of Linear Strain Gauges	Number of Rosette Strain Gauges	Number of LVDT or String Pots.
						4 per specimen	1 per axial specimen 2 per flex Specimen	for all specimens
	All-around	Bolts at Corners	3	Strong Axis Bending Bolts at Corners	3	48	18	4
		Bolts at Sides	3	Weak Axis Bending Bolts at Corners	3			
	Workable Flat Edges	Bolts at Corners	3	Strong Axis Bending Bolts at Corners	3	48	18	
		Bolts at Sides	3	Weak Axis Bending Bolts at Corners	3			

## Phase 1: Axial

For the axial tests, two variable bolt patterns, side and corner located bolts, were used. Two variable weld patterns, all-around and workable-flat edges, were also used. Initially, as Table 4 suggests, the twelve specimens were divided into four categories of three specimens based on the differing bolt and weld patterns: three all-around weld corner bolted (AACB), three all-around weld side bolted (AASB), three workable-flat welds corner bolted (WFCB), and three workable-flat welds side bolted (WFSB). However, there were only two WFSB specimens and four AASB specimens tested. Figure 16 shows the different bolt and weld patterns of the specimens and the comparisons to be made between them. Figure 17 provides a closer look at the different weld patterns.

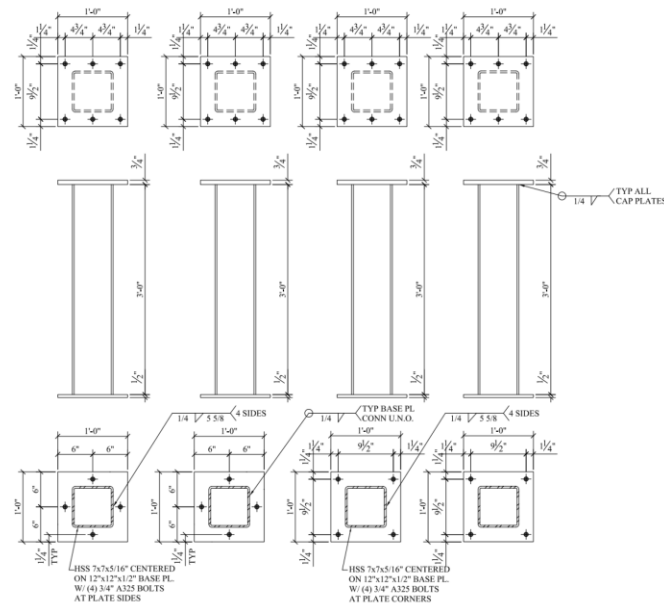
**Figure 16**

*Axial Specimens Variables and Comparisons*

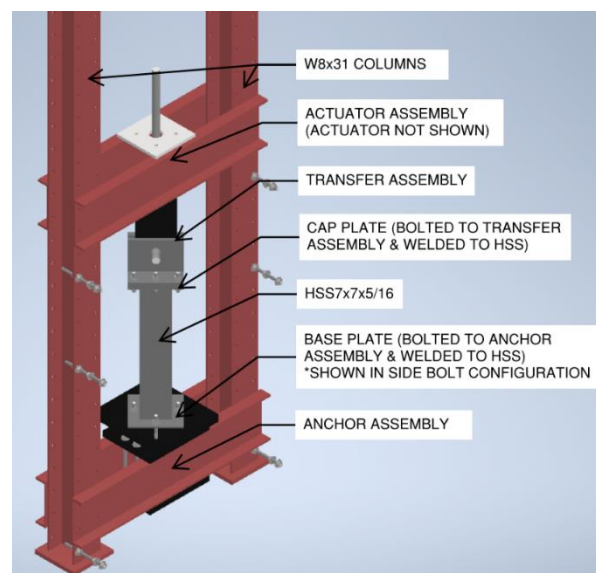


**Figure 17***Weld Patterns*

Initial calculations to size the axially loaded HSS end-plate connections were done by Zietlow (2022). See Appendix A for all calculations. Figure 18 shows the configurations and dimensions for the axial specimens. As previously mentioned HSS7x7x5/16 was used for all axial specimens. The HSS was centered on the base plate. The base plate was twelve inches by twelve inches (12 in. x 12 in.). A distance of one and one quarter inch ( $1\frac{1}{4}$  in.) was used for both edge distances for the corner bolts and the one edge for the side bolts. The side bolts were centered on the plate, or six inches (6 in.) from either edge. Rearranging Equation (25) and using a one-quarter inch ( $\frac{1}{4}$  in.) fillet weld and an effective length of twenty-eight inches (28 in.), the weld strength, from the limit state of weld rupture, was calculated to be 155.9 kips. The available strength from the limit states of HSS yield, HSS rupture, and bolt strength were 341.5 kips, 352.9 kips, and 119.2 kips, respectively. Therefore, the limiting available strength was 119.2 kips, from the available bolt strength; this strength was used in Equations (21) and (23), resulting in a minimum base plate thickness based on prying action to be calculated as 0.439 inch; therefore, base plates with thicknesses of one-half inch (0.500 in.) were used. Because the base plate was the focus of this research, the cap plate thickness was increased by one and a half times the base plate thickness. See Appendix D for full structural drawings and Appendix E for shop drawings of the specimens.

**Figure 18***Axial Specimen Structural Drawings*

An existing test frame was used to anchor the specimens for loading. Figure 19 shows a diagram of the test setup.

**Figure 19***Axial Specimen Test Frame and Setup*

The test frame consists of two W8x31 columns with C12x20.7 cross beams for the actuator and anchor assemblies. The actuator assembly is located above test specimen and consists of an actuator anchored between two plates to the two C12 cross beams. The load is transferred through the transfer assembly, consisting of a threaded rods and built-up plate connections, to the cap plate of the HSS. See Figure 20 for a photo of the upper portion of the test setup.

**Figure 20**

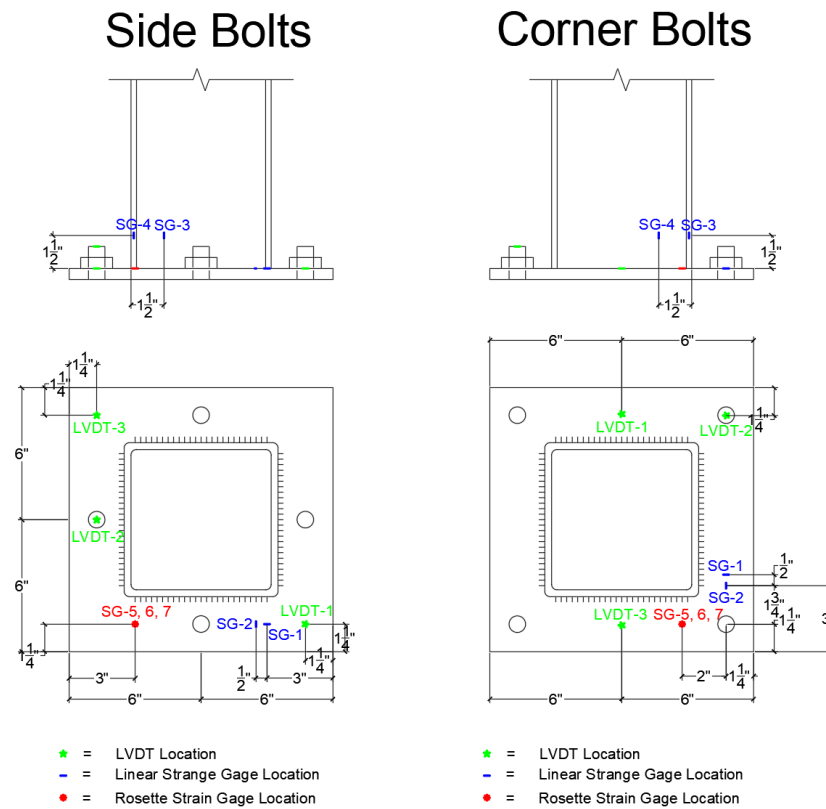
*Axial Test Setup Actuator and Transfer Assemblies*



The force is transferred through the cap plate into the HSS, then through the HSS and into the base plate. The base plate is held in place by the anchor assembly. The base plate was the focus of this research; thus, every other part of the test setup was sized to have greater capacity.

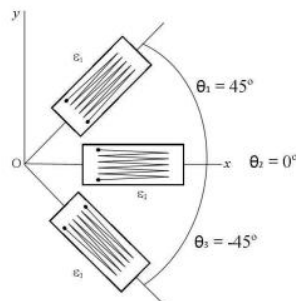
The actuator used to apply the axial tensile load to the specimen was an Enerpac Model RRH-606; the load was measured with a Sensotec Model 41-A530-01-03 load cell. The maximum load limit was 100,000 lbs (100 kips). Instrumentation, strain gages and LVDTs, were used to measure the strains and displacements, respectively. Figures 21, 22, and 23 show the positioning of instrumentation. For full instrumentation plans see Appendix F.



**Figure 21***Axial Tests Instrumentation Plans***Figure 22***Axial Corner Bolt Specimen Strain Gages*

**Figure 23***Axial Side Bolt Specimen Strain Gages*

Four linear strain gages (SG-1, SG-2, SG-3, SG-4) and one rosette strain gage (SG-5, SG-6, SG-7) were applied to each specimen. See Figure 24 for the rosette strain gage layout. Two LVDTs (LVDT-1 and LVDT-3) were used for all tests; for the first two tests an additional LVDT (LVDT-2) was placed on top of a bolt, but after bolt rupture on the second test (WFCB01) it was not used.

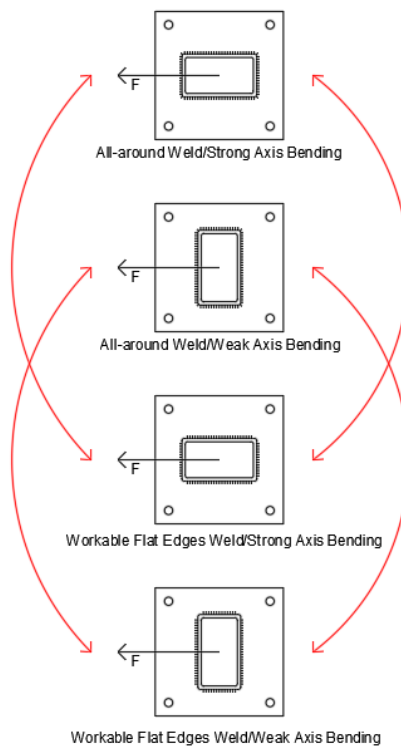
**Figure 24***Rosette Strain Gage Layout*

## Phase 2: Flexural

For the flexural tests, two variable HSS orientations, strong axis and weak axis bending, were used. The two variable weld patterns, all-around and workable-flat, were also used. As Table 4 suggests, the twelve specimens were divided into four categories of three specimens each based on the differing bolt and weld patterns: three all-around weld strong axis bending (AAST), three all-around weld strong axis bending (AAST), three workable-flat welds weak axis bending (WFWK), and three workable-flat welds strong axis bending (WFST). Figure 25 shows the different bolt and weld patterns of the specimens and the comparisons to be made between them.

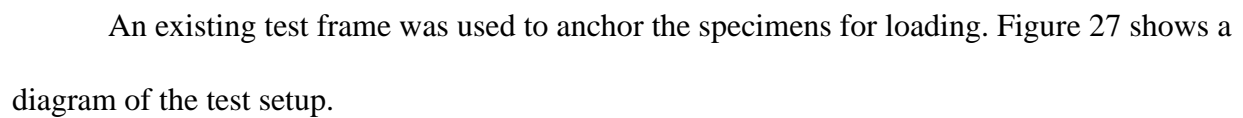
**Figure 25**

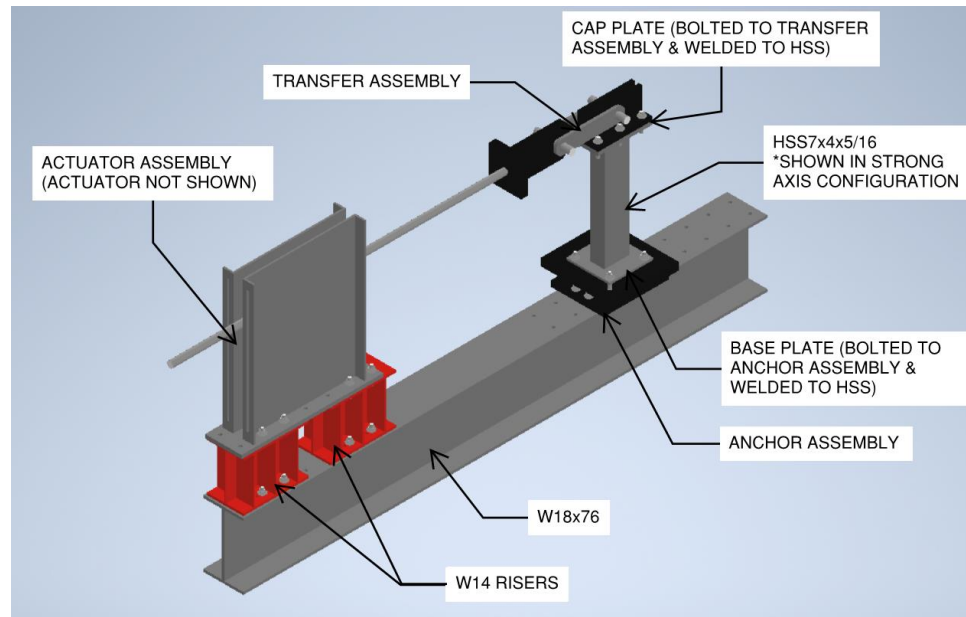
*Axial Specimens Variables and Comparisons*



Initial calculations to size the laterally loaded HSS end-plate connections were also done by Zietlow (2022). See Appendix A for all calculations. Figure 26 shows the configurations and dimensions for the axial specimens. As previously mentioned HSS7x4x5/16 was used for all flexural specimens. The HSS was centered on the base plate. The base plate was twelve inches by twelve inches (12 in. x 12 in.). A distance of one and one quarter inch ( $1 \frac{1}{4}$  in.) was used for edge distances for the bolts. Rearranging Equation (25) and using a one-quarter inch ( $\frac{1}{4}$  in.) fillet weld and an effective length of twenty-two inches (22 in.), the weld strength, from the limit state of weld rupture, was calculated to be 122.5 kips. The available strength from the limit states of HSS yield, HSS rupture, and bolt strength were 263.2 kips, 272.0 kips, and 119.2 kips, respectively. Therefore, the limiting available strength was 119.2 kips, from the available bolt strength; this strength was used in Equations (21) and (23), resulting in a minimum base plate thickness based on prying action to be calculated as 0.599 inch; therefore, base plates with thicknesses of five-eighths inch (0.625 in.) were used. Because the base plate was the focus of this research, the cap plate thickness was increased by one and a half times the base plate thickness. See Appendix D for full structural drawings and Appendix E for shop drawings of the specimens.

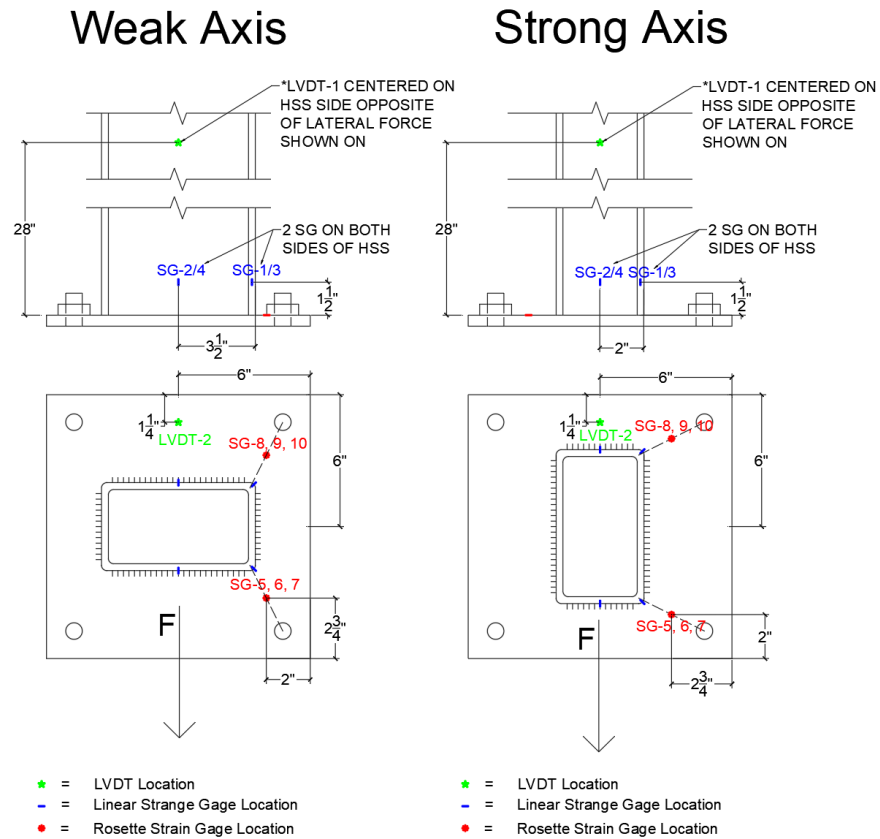
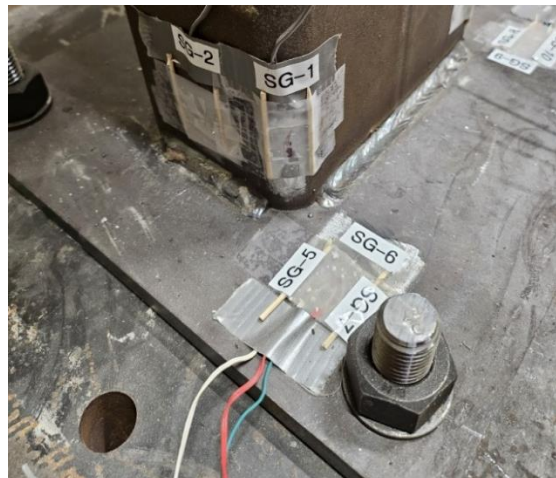
### *Axial Specimen Structural Drawings*



**Figure 27***Flexural Specimen Test Frame and Setup*

The test frame consists of a W18x76 beam base. The actuator assembly consists of an actuator connected to a plate assembly that is then connected through two W14 risers to the base. The force from the actuator is transferred through the transfer assembly, consisting of threaded rods and built-up plate connections, to the HSS cap plate, then through the cap plate into the HSS, and finally through the HSS and into the base plate. The base plate is held in place by the anchor assembly, which is attached to the base. The base plate was the focus of this research; thus, every other part of the test setup was sized to have greater capacity.

The actuator used to apply the lateral load to the specimen was an Enerpac Model RRH-606; the load was measured with a Sensotec Model 41-A530-01-03 load cell. The maximum load limit was 100,000 lbs (100 kips). Instrumentation, strain gages and LVDTs, were used to measure the strains and displacements, respectively. Figures 28 to 31 show the positioning of instrumentation. For instrumentation plans see Appendix F.

**Figure 28***Axial Tests Instrumentation Plans***Figure 29***Flexural Specimen Compression Side Strain Gages*



**Figure 30**

*Flexural Specimen Tension Side Strain Gages*

**Figure 31**

*Flexural Specimen LVDT 1*





Four linear strain gages (SG-1, SG-2, SG-3, SG-4) and two rosette strain gage (one consisting of SG-5, SG-6, SG-7 and the other consisting of SG-8, SG-9, SG-10) were applied to each specimen. See Figure 24 for the rosette strain gage layout. Two LVDTs (LVDT-1 and LVDT-2) were used for all tests. LVDT-1 was placed twenty-eight inches up from the end-plate on the side of the HSS to measure lateral displacement. LVDT-2 was placed on the end-plate between the bolts on the tension side to measure end-plate deformation.

## Material Strengths

Threaded rods were used in various parts of the test setup. Four three-quarter inch ( $\frac{3}{4}$  in.) diameter ASTM A193, Grade B7, were used to anchor the base plate to the anchor assembly for all axial and flexural tests. ASTM A193, Grade B7, rods were selected because of availability and having similar material properties to ASTM A325 bolts and ASTM F1852 threaded rods, which were used in the initial calculations performed by Zietlow (2022). See Tables 5 and 6 for ASTM A193 and ASTM A325/ASTM F1852 material properties, respectively. Because the base plate was the focus of the research, all threaded rods elsewhere, used for the transfer, anchor, and cap plate assemblies, were high strength ASTM A354, Grade B, threaded rods.

**Table 5**

### *ASTM A193 Mechanical Properties*

**A193 Mechanical Properties**

Grade	Size	Tensile ksi, min	Yield, ksi, min	Elong, %, min	RA % min
B7	Up to 2-1/2	125	105	16	50
	2-5/8 - 4	115	95	16	50
	4-1/8 - 7	100	75	18	50

*Note.* Adapted from *ASTM A193*, by Portland Bolt, retrieved November, 2023,

<https://www.portlandbolt.com/technical/specifications/astm-a193/>

**Table 6**

### *ASTM A325/F1852 Mechanical Properties*

**F3125 Mechanical Properties**

Grade	Tensile, ksi	Yield, ksi min	Elongation, % min	RA, % min
120ksi (A325/F1852)	120 min	92	14	35
150ksi (A490/F2280)	150-173	130	14	40

*Note.* Adapted from *ASTM F3125*, by Portland Bolt, retrieved November, 2023,

<https://www.portlandbolt.com/technical/specifications/astm-f3125/>

The plate used for the specimens was certified to be in conformance with both ASTM A36, “Standard Specification for Carbon Structural Steel,” and ASTM A572, Grade 50, “Standard Specification for High-Strength Low-Alloy Columbium-Vanadium Structural Steels.” The HSS used for the specimens was certified to be in conformance with ASTM A500, Grade B, “Standard Specification for Cold-Formed Welded and Seamless Carbon Steel Structural Tubing in Rounds and Shapes,” and ASTM A1085, “Standard Specification for Cold-Formed Welded Carbon Steel Hollow Structural Sections (HSS).” See Table 7 for results of a tensile test report from Metallurgical Associates, Inc. See Appendix G for the full report. For the plate, the average tensile strength was 82.9 ksi, and the average yield strength was 57.1 ksi. For the HSS, the average tensile strength was 66.8 ksi, and the average yield strength was 53.2 ksi. The HSS likely also conforms to ASTM A500 Grade C, but it was not formally confirmed by Metallurgical Associates.

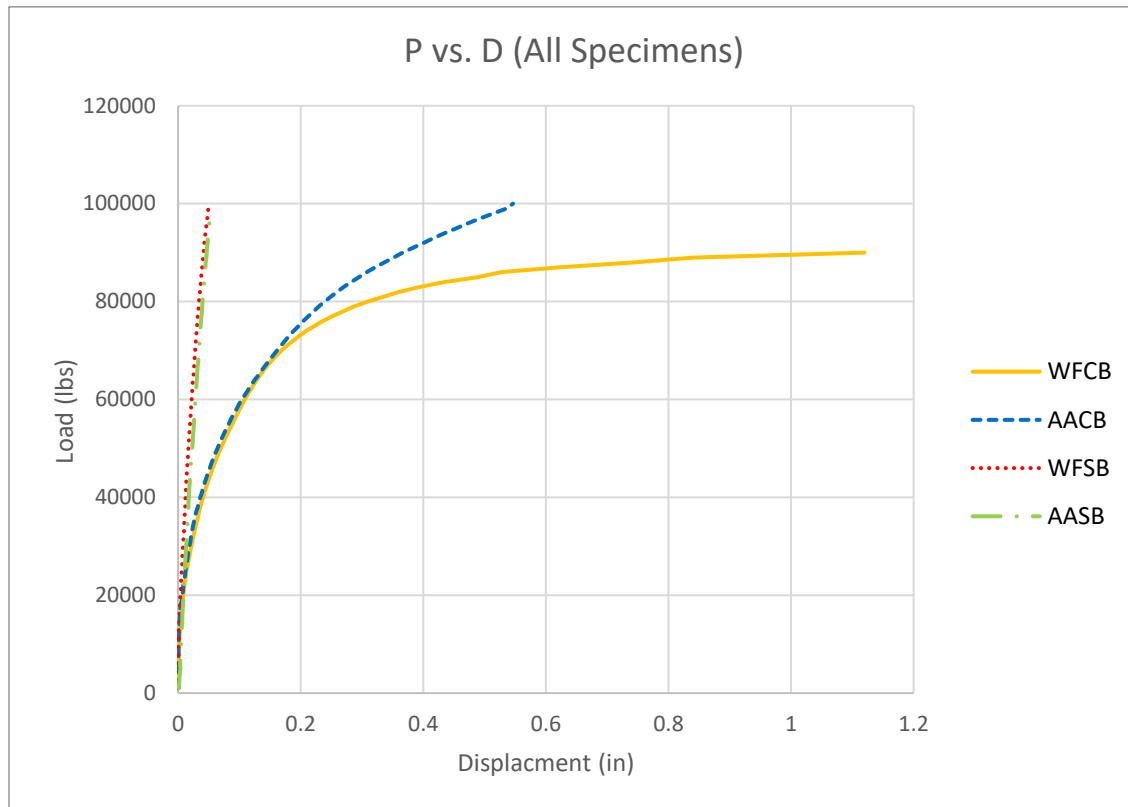
**Table 7***Tensile Testing Report*

Property	Plate 1	Plate 2	Plate 3	ASTM A36	ASTM A572, Grade 50
Test Bar Dimensions, Diameter, inch	0.496	0.495	0.496	D	D
Gage Length, inches	2.0	2.0	2.0	4D	4D
Tensile Strength, psi	82,900	82,900	82,900	58,000 - 80,000	65,000 min.
Yield Strength, psi (1)	57,300	56,900	57,200	36,000 min.	50,000 min.
Elongation, %	30	30	29	23 min.	21 min.
Elongation, %	67	66	67	Not Specified	Not Specified

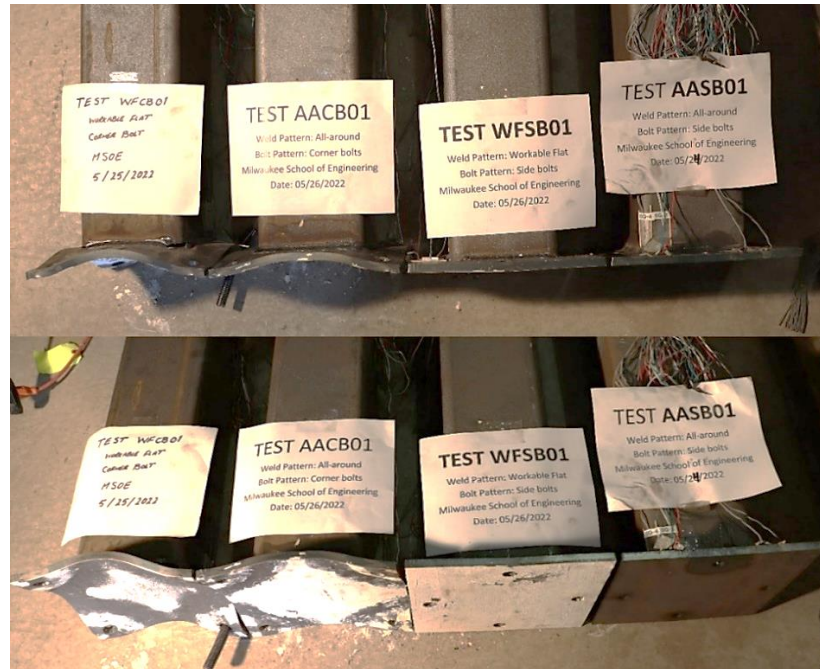
Property	Tube 1	Tube 2	Tube 3	ASTM A500, Grade B	ASTM A1085
Test Bar Dimensions Width, inch	0.493	0.489	0.493	0.50	0.50
Thickness, inch	0.295	0.293	0.293	Material Thickness	Material Thickness
Gage Length, inches	2.0	2.0	2.0	2.0	2.0
Tensile Strength, psi	65,500	67,200	67,700	58,000 min.	65,000 min.
Yield Strength, psi (1)	53,100	53,300	53,300	46,000 min.	50,000 - 70,000
Elongation, %	35	32	32	23 min.	21 min.

(1): at 0.2% offset



**Figure 32***Axial Tests Average Tension Loads versus Average Displacements*

The specimens with corner bolt patterns had significant yielding and high displacements compared to the specimens with side bolt patterns with almost no yielding and very small displacements, as seen in Figures 33 and 34. The strain for the specimens with corner bolts showed higher and more concentrated stress closer to the hole, perpendicular to the HSS, and up into the corner of the HSS whereas the strain for the specimens with side bolts show lower and better distributed stress with higher strain parallel to the HSS and in the HSS side wall.

**Figure 33***Axial First Trial Specimen Comparison***Figure 34***Axial Second Trial Specimen Comparison*

**All-Around Weld with Corner Bolts (AACB)**

This configuration was tested to the 100-kip capacity of the load cell. The base plate exhibited significant yielding with an average maximum displacement of 0.673 in. The highest strain values of 6000 to 10000  $\mu\text{in/in}$  occur at the strain gages closest to the holes (SG-2 and SG-5). SG-2 was perpendicular to the HSS, and SG-5 was at a 135-degree angle in line with the corner of the HSS (for all strain gage locations and orientations see Appendices B and F). SG-3, located on the corner of the HSS, had higher strain than SG-4 which was located one- and one-half inches away from the corner. There was slight bending and permanent deformation visible in the rods indicating they have yielded, but very little to no rupture visually apparent in the welds.

**Workable-Flat Welds with Corner Bolts (WFCB)**

This configuration was tested to its ultimate capacity, with two of the three specimens being loaded to rupture of the rods with an average maximum load of just under 90 kips. The specimens had significant deformation and yielding with an average maximum plate displacement of 1.308 in. The strain values were similar to that of the all-around weld specimens with the highest strain of 6000 to 8000  $\mu\text{in/in}$  occurring at the strain gages closest to the holes (SG-2 and SG-5). SG-3 again had higher strains than SG-4. Rupture was observable at the ends of the welds and significant bending and permanent deformation was observable in the rods. Some of these specimens also exhibited rupture in the rods, after the specimens were loaded past their ultimate strength, as shown in Figures 35 and 36.

**Figure 35***WFCB01 Bolt Rupture***Figure 36***WFCB02 Bolt Rupture*



This rupture was likely caused by prying action as the plate exhibited significant yielding. In the initial calculations the thickness of the plate was determined by using the controlling limit state of the bolt tensile strength for A325 bolts to be under one-half inch and it was verified for the change to A193 rods to still be under one-half inch; see calculations in Appendix A. However, the equations used from *AISC Design Guide 24* are for a side bolt configuration and may not be directly applicable to the corner bolt configuration.

### **All-Around Weld with Side Bolts (AASB)**

This configuration was tested to the 100-kip capacity of the load cell used in this testing, and the specimen did not undergo any significant yielding with a small max displacement of 0.089 in. The highest strains of 1000 to 2000  $\mu\text{in/in}$  occurred at SG-1 and SG-5. SG-1 was parallel to the HSS, and SG-5 was at a 135-degree angle. SG-3, located on the HSS one- and one-half inch from the corner, had higher strain than SG-4, located on the HSS corner, for all specimens. There was no observable bending or permanent deformation in the rods and no observable rupture in the welds.

### **Workable-Flat Welds with Side Bolts (WFSB)**

This configuration was tested to the 100-kip capacity of the load cell used in this testing, and the specimen did not undergo any significant yielding with a small max displacement of 0.096 in. Similar to the all-around weld with side bolts specimens, the highest strains of 1000 to 2000  $\mu\text{in/in}$  occurred at SG-1 and SG-5. SG-3 also had higher strain than SG-4 in all specimens. There was minor bending/yielding in the rods and no rupture in the welds.

## Discussion

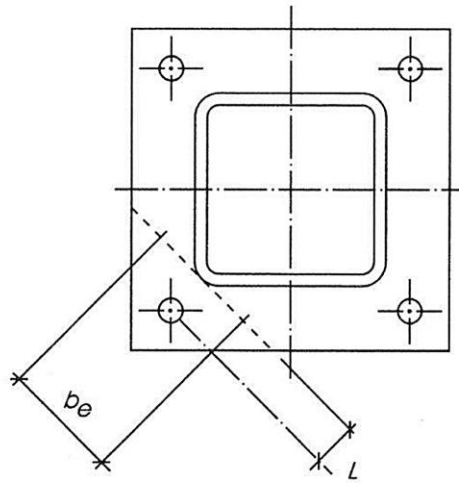
For the corner bolt configuration, using Equation (29), the capacity of the connection was calculated to be 114.2 kips using no  $R_{hss}$  factor, 68.52 kips using a  $R_{hss}$  factor of 0.6 for a  $t_{hss}$  of one-quarter inch, and 79.94 kips using a  $R_{hss}$  factor of 0.7 for a  $t_{hss}$  of three-eighths inch. The bolt tributary width,  $p$ , used in Equations (21) and (23) for the initial calculations performed by Zietlow (2022) was taken as the greater of the plate width and length, or twelve inches; using this, and a load of 119.2 kips from the limiting state of bolt strength (using A325N bolts), a minimum plate thickness of 0.439 inch was calculated. Recalculating the minimum plate thickness using Equation (23) with the same bolt tributary width,  $p$ , of twelve inches but considering the higher capacity of the A193 rods used, 139.2 kips, and the average plate yield strength from MAI material testing, 57.1 ksi, resulted in a thickness of 0.444 inch. This thickness is still under the end-plate thickness used of one-half inch; however, these design procedures and equations apply directly to the side bolt configuration.

Plate thicknesses using more recent procedures from Thornton (2017) and the *Manual* (AISC, 2022) were calculated; the bolt tributary width,  $p$ , was calculated as 5.916 inches from the minimum of Equations (32) and (33). Equations from the *Manual* were then used to calculate the required plate thicknesses. From Equation (3), for no prying action the required thickness of the plate was 0.533 inch. This is over the one-half inch plate thickness of the end-plates used; the actual thickness of one-half inch was used and Equation (3) was rearranged to calculate a tension force of 113.6 kips. However, the end-plates of the corner bolt specimens underwent significant deformation well before 113.6 kips, and this procedure still applies to the side bolt configuration. One way the procedure could be modified to possibly account for the corner bolt configuration is to set the tributary width equal to the effective width,  $b_e$  (see Figure 37); another way may be to

increase the a and b dimensions. Calculations considering these adjustments can be found in Appendix A.

**Figure 37**

*Axial Corner Bolt Effective Width,  $b_e$*



*Note.* Adapted from *Hollow Structural Sections Connections Manual* by American Institute of Steel Construction, 1997.

The specimens with all-around welds showed to have greater capacity over the specimens with workable-flat welds for the corner bolt configuration; the all-around specimens reached the 100-kip capacity of the load cell without ultimate failure, whereas the workable-flat welds reached ultimate failure at an average load of just under 90 kips. Although both weld patterns for the side bolt configurations reached the 100-kip capacity of the load cell, the specimens with welds on the workable flats exhibited slightly higher displacement and more visible permanent deformation compared to the all-around weld specimens.

## Phase 2: Flexural

Twelve physical flexural specimens were tested. Table 9 provides a summary of the tests and the maximum measured lateral forces and displacements. The moments were also calculated using a distance of 35.5 inches from the base of the end-plate to the middle of the transfer rod.

**Table 9**

*Flexural Tests Maximum Forces and Displacements*

Weld Pattern	Bending Axis	Specimen	Max Lateral Force (lbs)	Max Moment at Base (lb-ft)	HSS Displacement at Max Load (in)	Plate Displacement at Max Load (in)
All-Around Welds	Weak	AAWK01	13621	40296	5.362	0.639
		AAWK02	13175	38977	4.668	0.552
		AAWK03	12692	37548	4.391	0.520
	Strong	AAST01	16161	47809	4.566	1.275
		AAST02	17573	51986	4.965	1.357
		AAST03	16393	48496	5.056	1.355
Workable-Flat Welds	Weak	WFWK01	11772	34827	5.157	0.560
		WFWK02	10986	32501	2.914	0.477
		WFWK03	10516	31109	2.348	0.459
	Strong	WFST01	12834	37968	2.854	0.764
		WFST02	11827	34989	3.227	-
		WFST03	12686	37530	2.257	0.661

Weld Pattern	Bending Axis	Average Max Lateral Force (lb)	Average Max Moment at Base (lb-ft)	Average Max HSS Displacement (in)	Average Max Plate Displacement (in)
All-Around Welds	Weak	13163	38940	4.807	0.571
	Strong	16709	49431	4.862	1.329
Workable-Flat Welds	Weak	11092	32813	3.473	0.499
	Strong	12449	36829	2.780	0.713

The specimens with welds at the workable flats had relatively less capacity than the specimens with welds all-around when considering the same bending axis, due to weld rupture occurring in most of the workable-flat weld specimens. In all specimens when weld rupture did not occur, the ultimate load was not reached before the actuator reached maximum stroke. However, there was significant deformation and yielding of the end-plate; the all-around weld specimens exhibited more deformation and higher displacements in both the HSS (LVDT-1) and the end-plate (LVDT-2). Specimen comparison photos can be seen in Figures 38 through 44; the plate failure for all end-plate was observed to be Mode 3 failure as shown in Wheeler et al. (1998), see Figure 13. Load versus displacement plots for all specimens and LVDTs and load versus strain plots for all specimens and gages can be found in Appendix C.

**Figure 38**

*Comparison Photo of AAWK Specimens*



**Figure 39**

*Comparison Photo of AAST Specimens*

**Figure 40**

*Comparison Photo of WFWK Specimens*





**Figure 41**

*Comparison Photo of WFST Specimens*

**Figure 42**

*Comparison Photo of First Trial Specimens*



**Figure 43**

*Comparison Photo of Second Trial Specimens*

**Figure 44**

*Comparison Photo of Third Trial Specimens*





### All-Around Weld Strong Axis

This configuration reached an average maximum lateral load of 16.7 kips, resulting in an average maximum moment at the base of 49.4 kip-ft, before the actuator reached its maximum stroke at an average displacement of 4.807 inches at LVDT 1; the actuator reached full stroke before the ultimate load was reached for all three specimens. There was a small fracture in the weld on the corners of the tension side of the HSS, see Figure 45. There was significant deformation in the base plate with an average maximum displacement of 1.329 inches at LVDT 2, see Figure 46.

#### Figure 45

*Corner Weld Cracking in All-Around Weld Strong Axis Specimen (AAST03)*

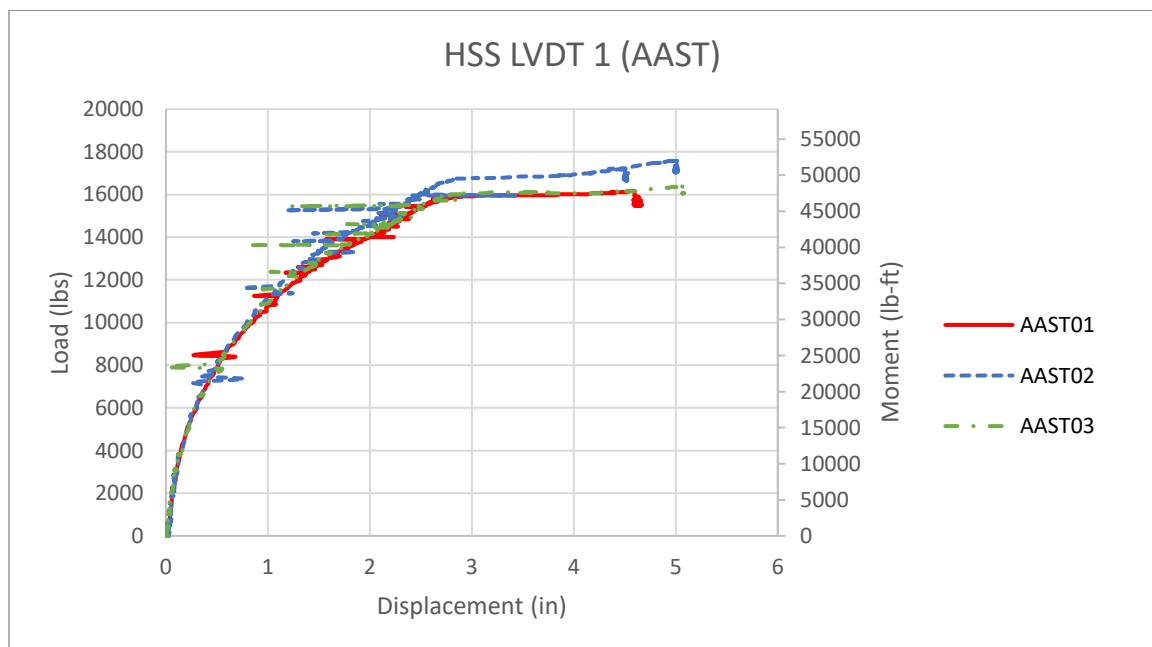


**Figure 46**

*End-Plate Deformation in All-Around Weld Strong Axis Specimen (AAST03)*

**Figure 47**

*AAST Specimens Load versus Displacement for LVDT 1 (End-plate)*



**Figure 48**

*AAST Specimens Load versus Displacement for LVDT 2 (HSS)*

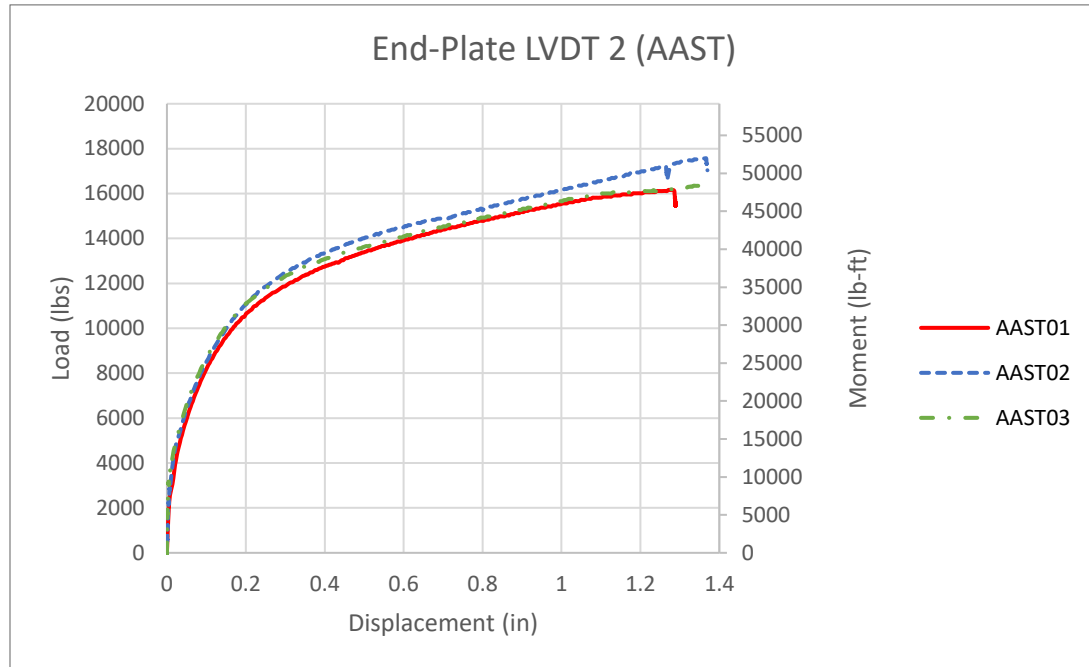


Figure 47 shows a load versus displacement plot for LVDT 1, located at the top of the HSS, and Figure 48 shows a load versus displacement plot for LVDT 2, located on the tension side of the plate between bolts.

### Workable-Flat Welds Strong Axis

This configuration reached an average maximum lateral load of 12.4 kips, resulting in a maximum moment at the base of 36.8 kip-ft, before weld rupture. Complete weld rupture occurred in all three specimens; therefore, all loads were ultimate loads. The HSS reached an average maximum displacement of 2.780 inches before weld rupture, see Figure 49. There was also significant deformation in the base plate before the weld rupture, with an average maximum displacement of 0.527 inches at LVDT 2, see Figure 50.

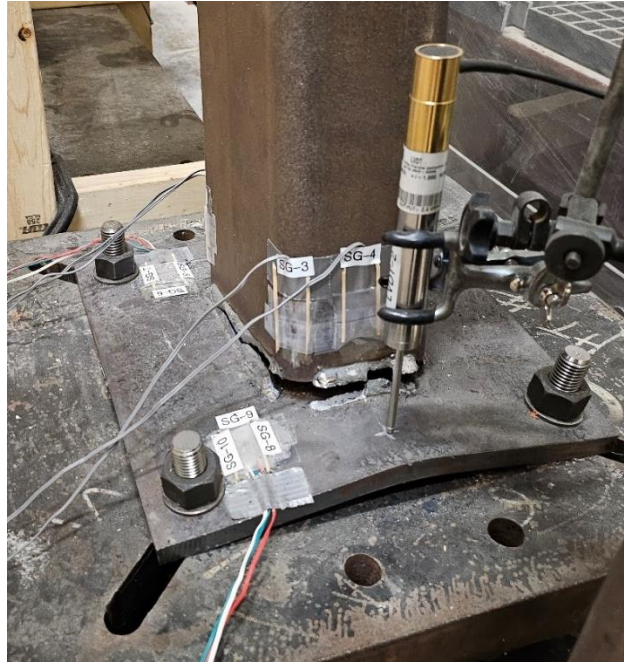
**Figure 49**

*Weld Rupture in Workable-Flat Welds Strong Axis Specimen (WFST01)*

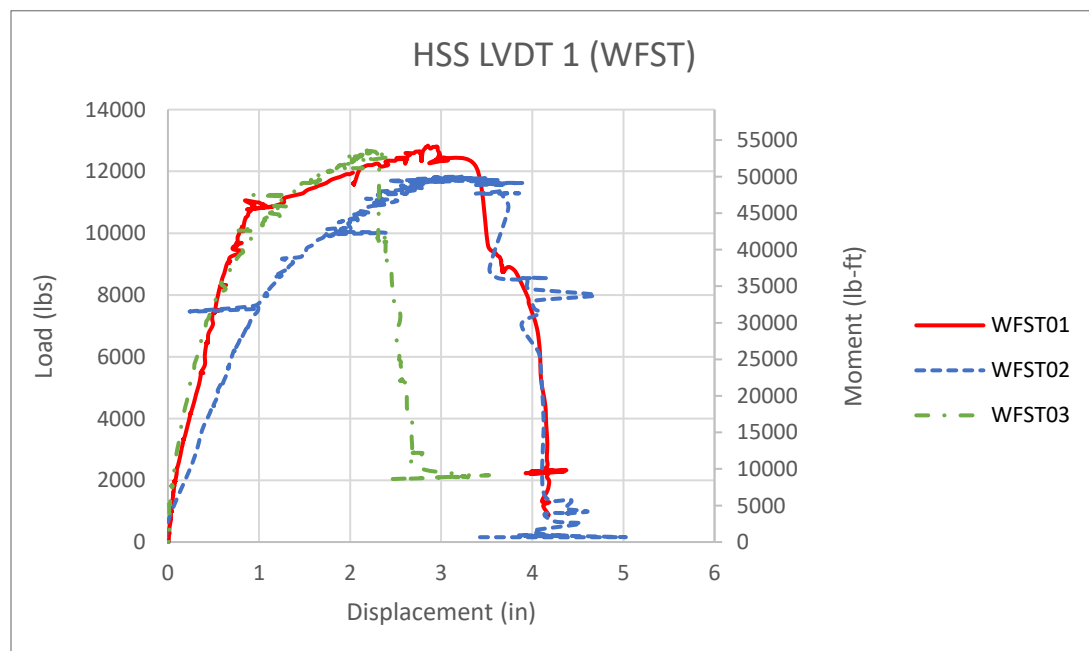


**Figure 50**

*End-Plate Deformation in Workable-Flat Welds Strong Axis Specimen (WFST01)*

**Figure 51**

*WFST Specimens Load versus Displacement for LVDT 1 (End-plate)*



**Figure 52**

*WFST Specimens Load versus Displacement for LVDT 2 (HSS)*

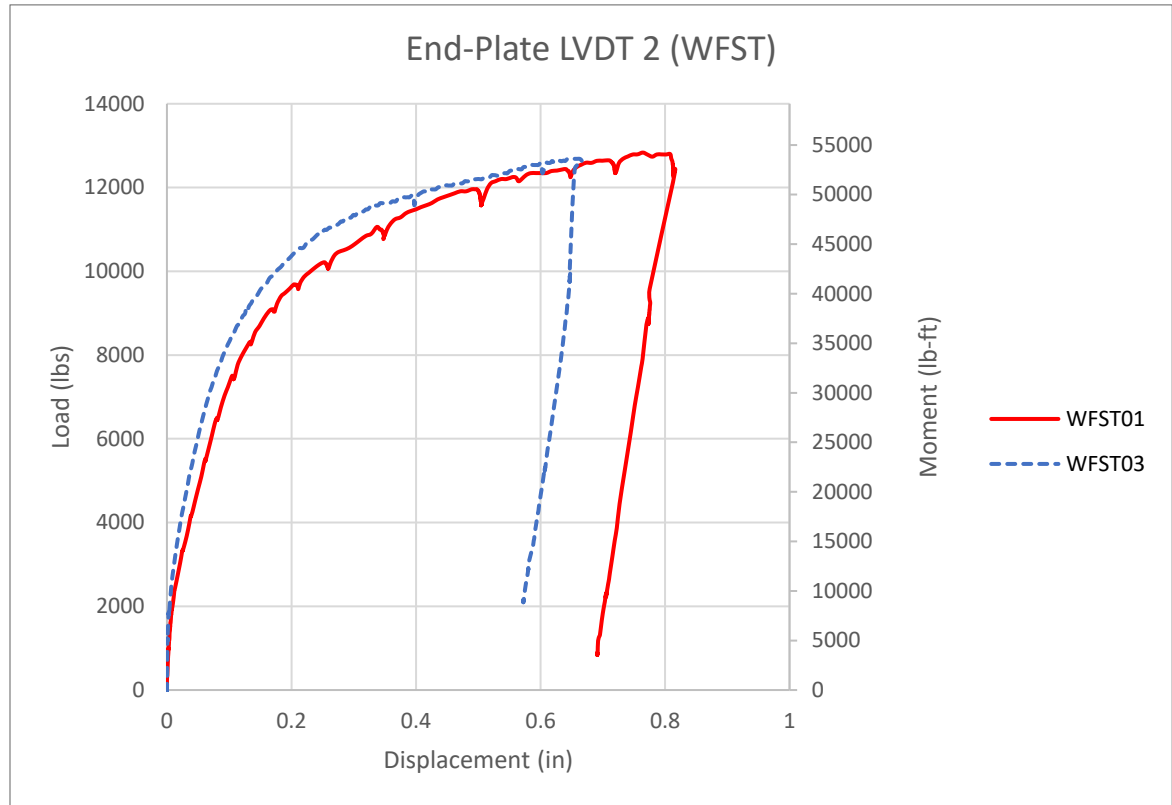


Figure 51 shows a load versus displacement plot for LVDT 1, located at the top of the HSS, and Figure 52 shows a load versus displacement plot for LVDT 2, located on the tension side of the plate between the bolts.



**All-Around Weld Weak Axis**

This configuration reached an average maximum lateral load of 13.2 kips resulting in an average maximum moment at the base of 38.9 kip-ft, before the actuator reached its maximum stroke at an average displacement of 4.862 inches at LVDT 1; the actuator reached full stroke before the ultimate load was reached for all three specimens. There was no observable cracking in the weld on the corners of the tension side of the HSS, see Figure 53. There was significant deformation in the base plate with an average maximum displacement of 0.713 inches at LVDT 2, see Figure 54.

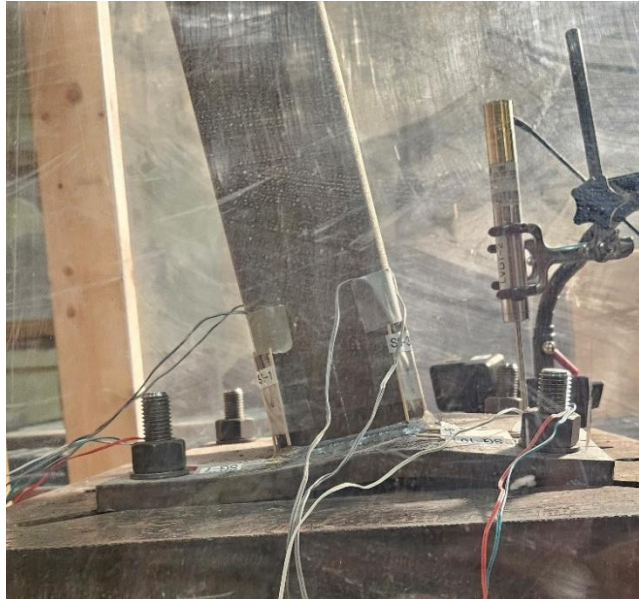
**Figure 53**

*Corner Weld in All-Around Weld Weak Axis Specimen (AAWK03)*

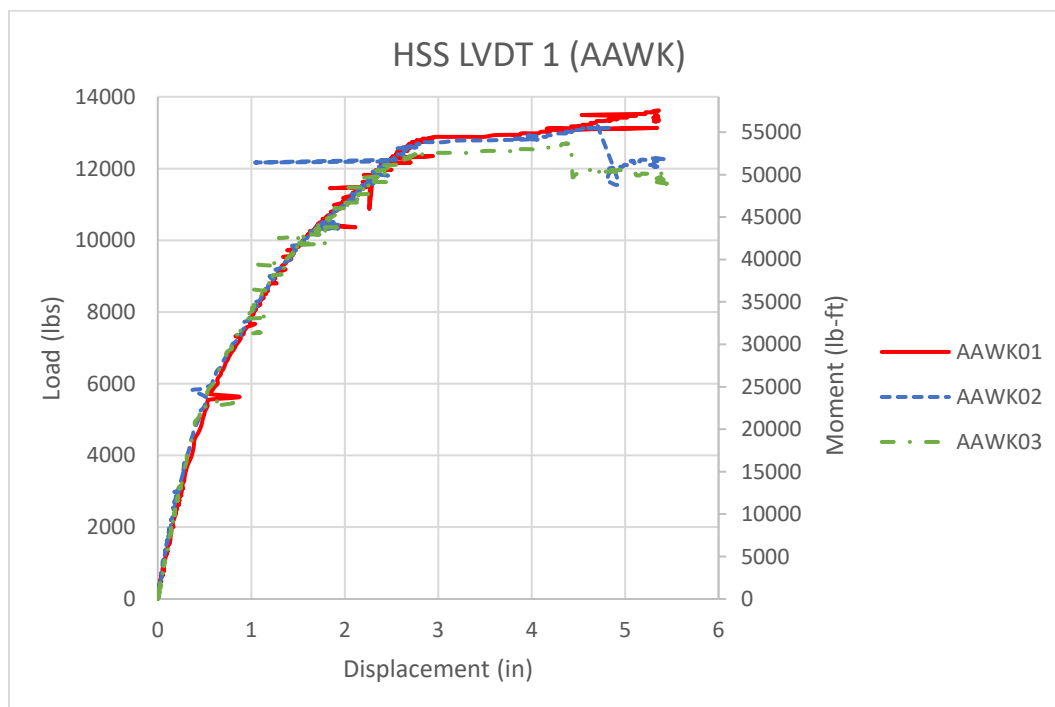


**Figure 54**

*End-Plate Deformation in All-Around Weld Strong Axis Specimen (AAWK02)*

**Figure 55**

*AAWK Specimens Load versus Displacement for LVDT 1 (End-plate)*





**Figure 56**

*AAWK Specimens Load versus Displacement for LVDT 2 (HSS)*

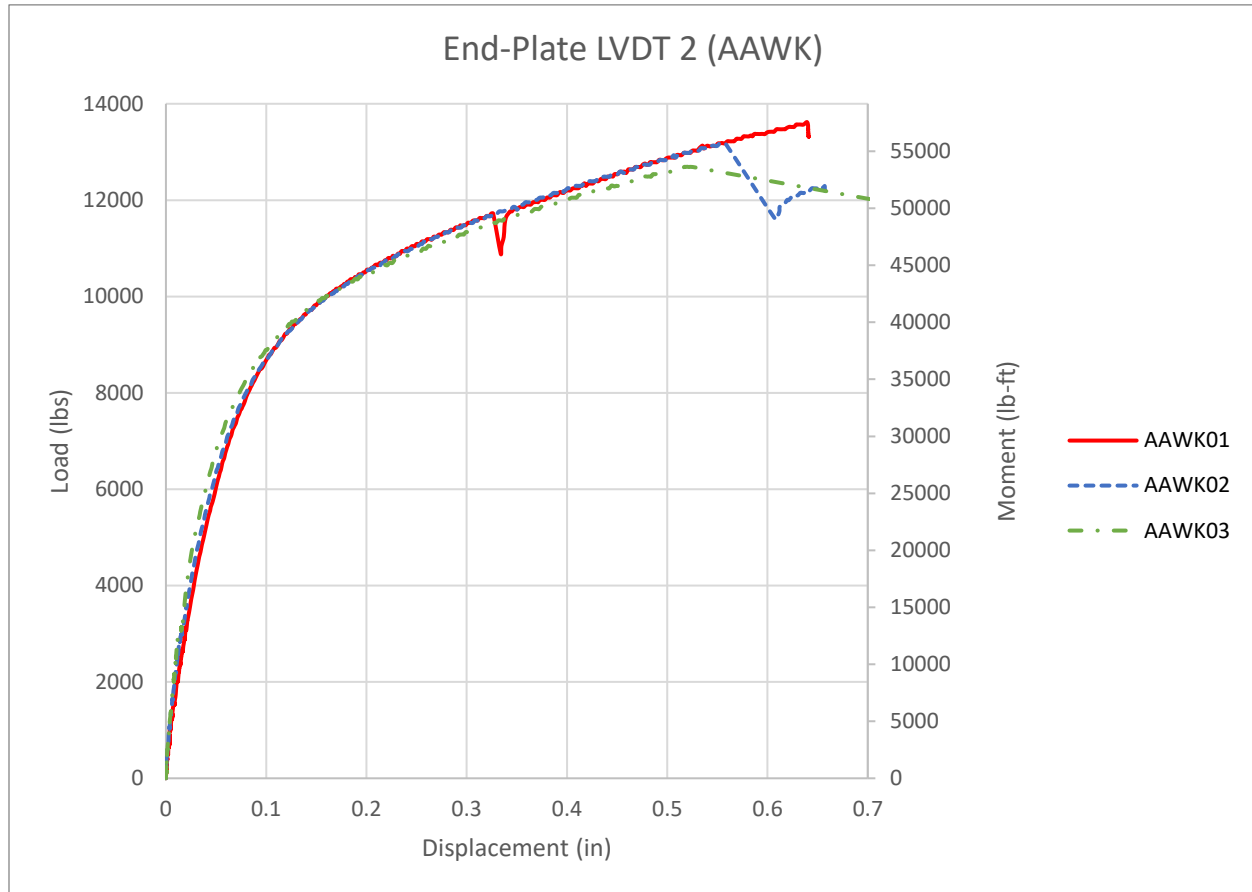


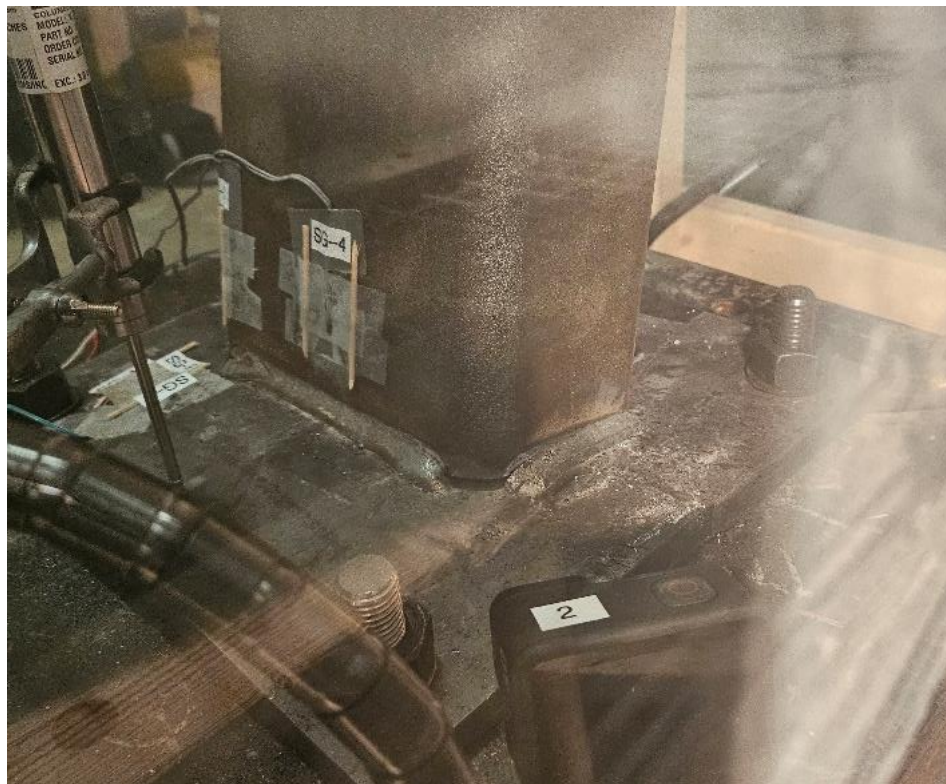
Figure 55 shows a load versus displacement plot for LVDT 1, located at the top of the HSS, and Figure 56 shows a load versus displacement plot for LVDT 2, located on the tension side of the plate between the bolts.

### Workable-Flat Welds Weak Axis

This configuration reached an average maximum lateral load of 11.1 kips resulting in an average maximum moment at the base of 32.8 kip-ft. For the first specimen, WFK01, the actuator reached full stroke before the ultimate load was reached; there was observable cracking in the welds, see Figure 57. The second specimen, WFWK02, reached an ultimate load at a HSS displacement of 2.914 inches, measured by LVDT 1; there was observable rupture in the welds, see Figure 58. The third specimen, WFWK03, reached an ultimate load at a HSS displacement of 2.348 inches, measured by LVDT 1; there was complete rupture in the welds, see Figure 59. The average maximum HSS displacement at LVDT 1 for these specimens was 3.473 inches; the average maximum end-plate displacement at LVDT 2 was 0.499 inches.

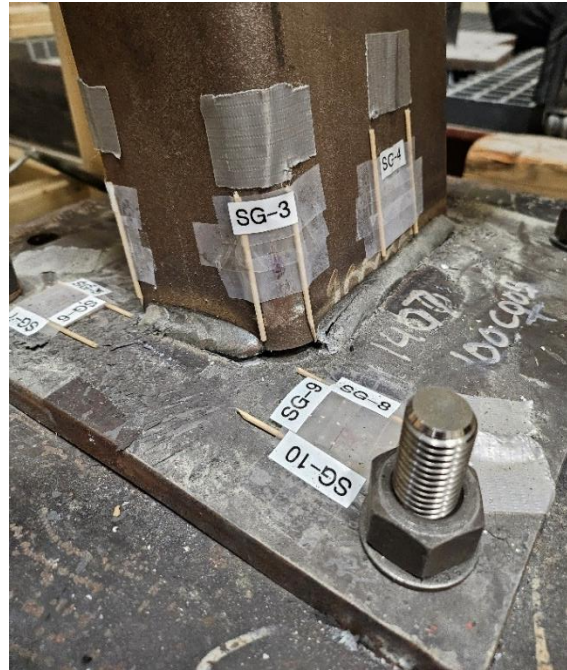
**Figure 57**

*Corner Weld Cracking in All-Around Weld Weak Axis Specimen (WFWK01)*



**Figure 58**

*Weld Rupture in All-Around Weld Weak Axis Specimen (WFWK02)*

**Figure 59**

*Weld Rupture in All-Around Weld Weak Axis Specimen (WFWK03)*



Figure 60

AAWK Specimens Load versus Displacement for LVDT 1 (End-plate)

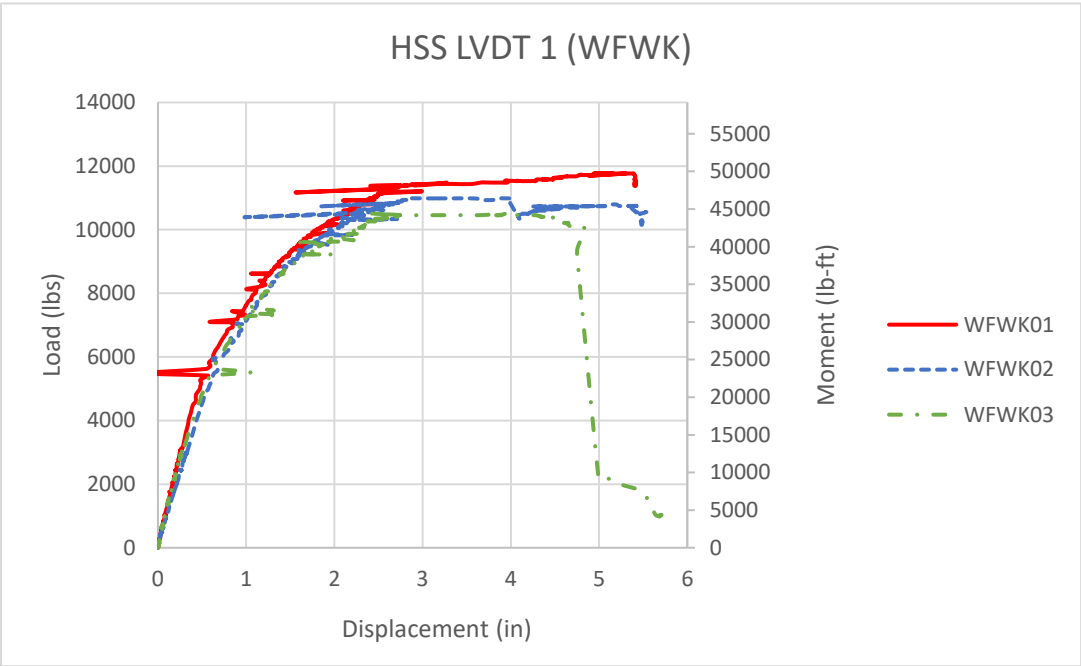


Figure 61

AAWK Specimens Load versus Displacement for LVDT 2 (HSS)

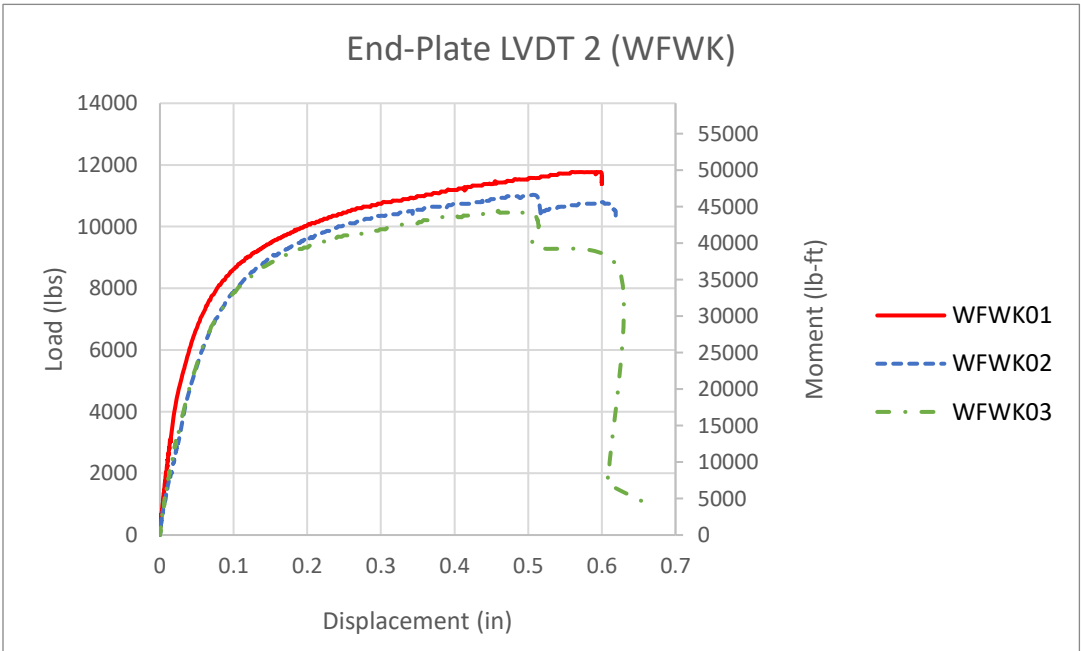


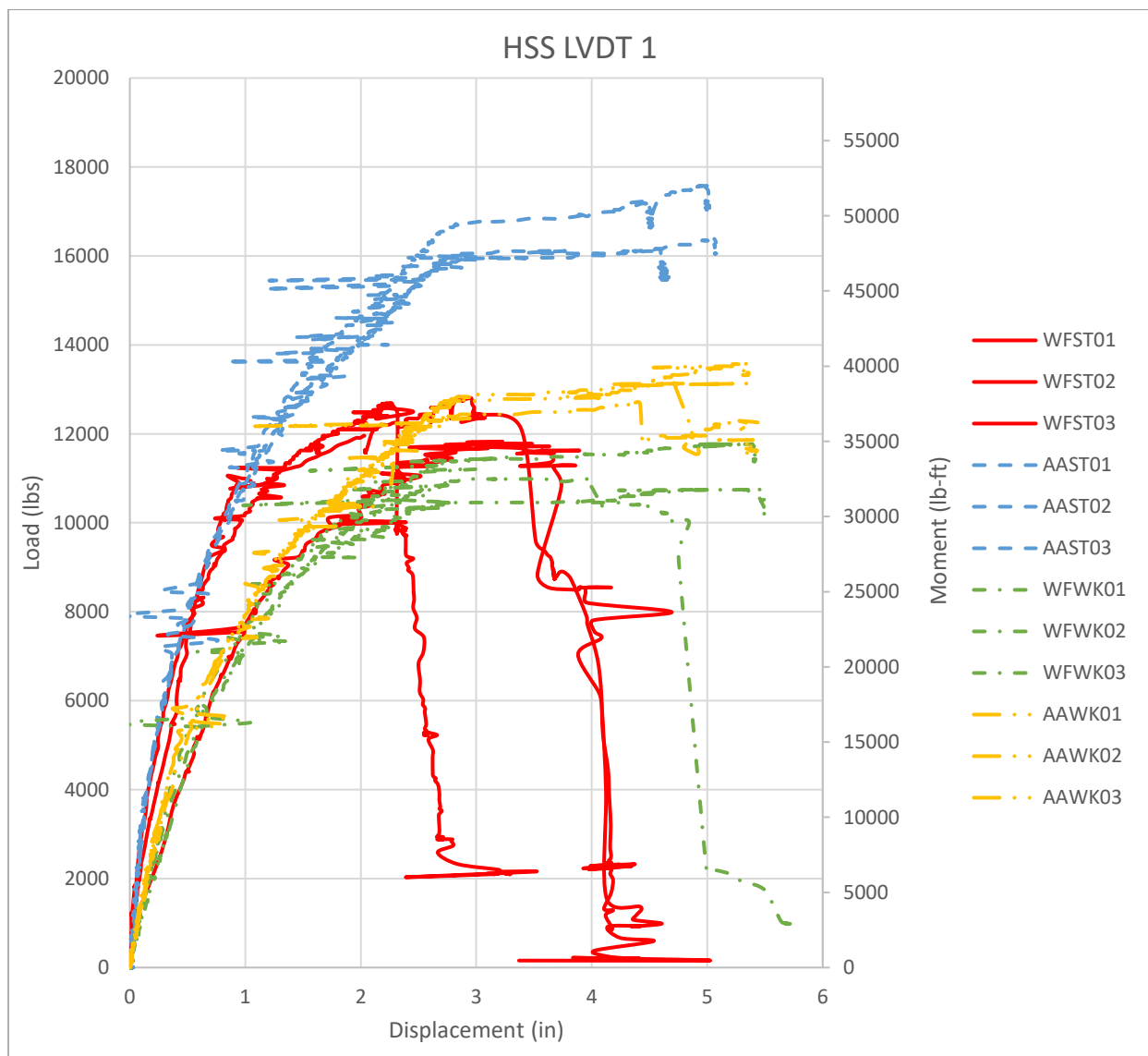
Figure 60 shows a load versus displacement plot for LVDT 1, located at the top of the HSS, and Figure 61 shows a load versus displacement plot for LVDT 2, located on the tension side of the plate between the bolts.

## Discussion

As expected, the strong axis specimens were stronger than the weak axis specimens and the all-around weld specimens were stronger than the workable-flat welds specimens. The difference in stiffnesses can also be seen from the load versus displacement graphs for both LVDTs for all specimens, see Figures 62 and 63.

**Figure 62**

*All Specimens Load versus Displacement for LVDT 1 (End-plate)*



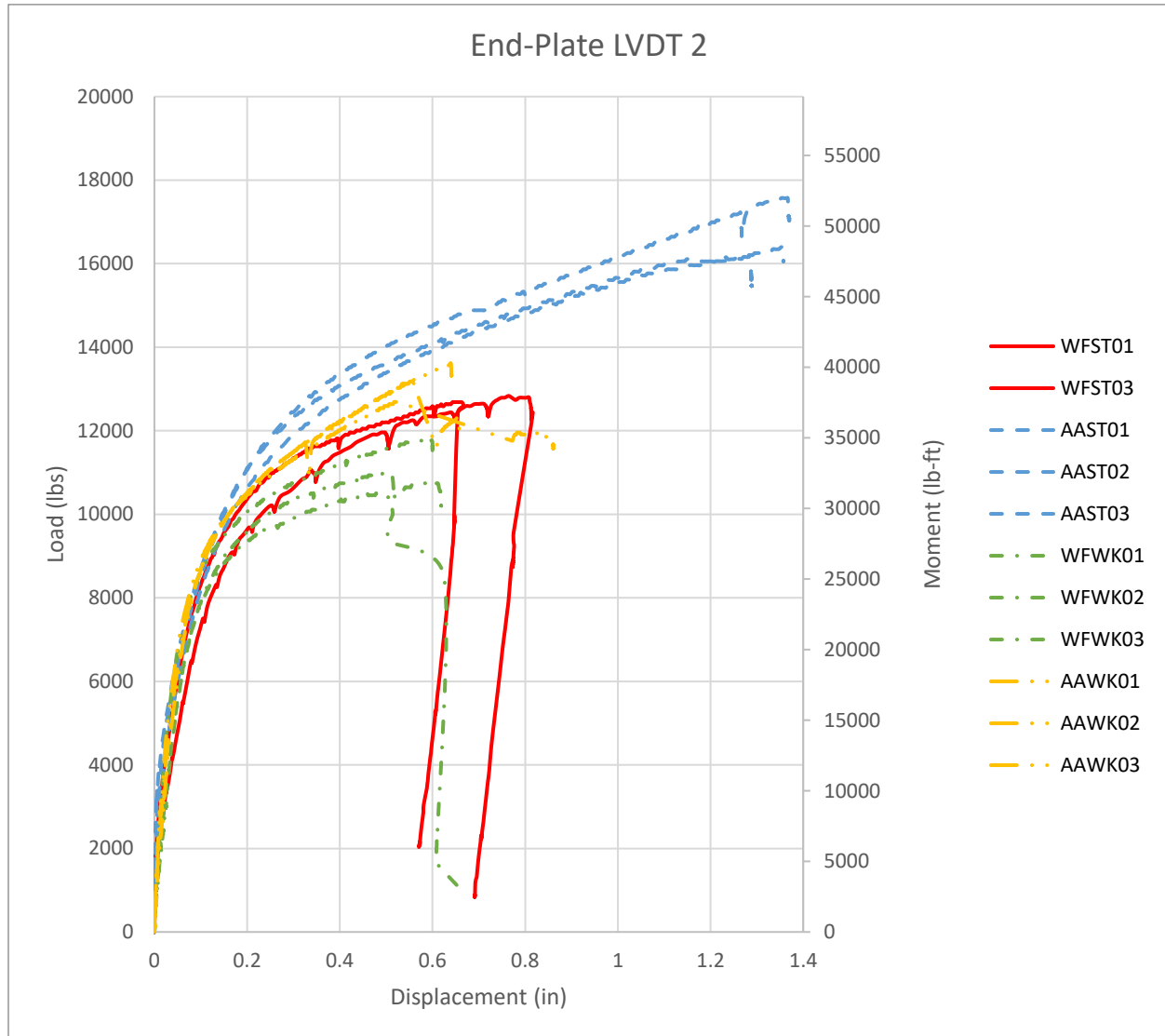
**Figure 63***All Specimens Load versus Displacement for LVDT 2 (HSS)*

Figure 62 shows that the all-around weld specimens had higher overall stiffnesses over the workable-flat welds specimens for both the weak and strong orientations. Figure 63 shows that after yield the plate stiffnesses of the all-around weld specimens are higher than the workable-flat welds specimens. The stiffnesses are also similar between the same weld patterns and different HSS orientations.



Strength capacities were calculated using the design procedure presented in Wheeler et al. (1998) and AISC Design Guide 24 (2010). Serviceability loads were also calculated using the design procedure from Wheeler et al. (1998). All capacities are tabulated in Table 10; the tabulated strength loads do not include any safety factors. See Appendix A for all calculations. The maximum and estimated yield loads from experimental testing are tabulated in Table 11.

**Table 10***Flexural Calculated Capacities*

Axis	Force	Strength				Serviceability	
		Wheeler et al.		AISC 24		Wheeler et al.	
		Bolt	Plate	Weld	Prying	Bolt	Plate
Weak	Moment (k-ft)	28.4	<b>27.8</b>	35.3	<b>14.6</b>	22.1	<b>19.5</b>
	Lateral (k)	9.60	<b>9.39</b>	11.9	<b>4.93</b>	7.46	<b>6.58</b>
Strong	Moment (k-ft)	48.1	<b>37.3</b>	61.7	<b>39.6</b>	37.7	<b>20.5</b>
	Lateral (k)	16.3	<b>12.6</b>	20.9	<b>13.4</b>	12.8	<b>6.93</b>

	$\phi$	0.8	0.9	0.75	0.9
Weak	Moment (k-ft)	22.7	<b>25.0</b>	26.4	<b>13.1</b>
	Lateral (k)	7.68	<b>8.45</b>	8.94	<b>4.44</b>
Strong	Moment (k-ft)	38.5	<b>33.6</b>	46.3	<b>35.6</b>
	Lateral (k)	13.0	<b>11.4</b>	15.6	<b>12.0</b>

**Table 11***Flexural Experimental Capacities*

		Average Max Lateral Force (k)	Average Max Moment at Base (k-ft)
Weak	AAWK	13.2	38.9
	WFWK	11.1	32.8
Strong	AAST	16.7	49.4
	WFST	12.4	36.8



The strengths calculated from the Wheeler et al. (1998) design procedures were controlled by plate failure and were conservative when compared to the average maximum forces for all specimen configurations except for the WFST specimens. The average maximum lateral force for the WFST was 12.4 kips, which was 0.2 kips less than the calculated 12.6 kips. However, if a safety factor ( $\Phi=0.9$ ) is applied this capacity would become conservative (11.4 kips). The serviceability loads calculated from the Wheeler et al. (1998) design procedures were controlled by plate yielding and were close to the experimental yield loads seen in the plate from LVDT 1, see Figure 63.

The strengths calculated from the design procedure from Example 4.1 in *AISC Design Guide 24* were governed by prying action and were conservative when compared to the average maximum forces for all specimen configurations except for the WFST specimens. The average maximum lateral force for the WFST was 12.4 kips, which was 1.0 kips less than the calculated 13.4 kips. However, if a safety factor ( $\Phi=0.9$ ) was applied this capacity would become conservative (12.0 kips). The strengths from weld capacity calculated from the design procedure from Example 4.1 in *AISC Design Guide 24* were conservative for the AAWK specimens (with and without safety factor), conservative for the AAST specimens (with safety factor), conservative for the WFWK specimens (with safety factor), and unconservative for the WFST specimens. It should also be noted that the all-around weld specimens did not exhibit failure in the welds, rather the maximum load was reached through yielding of the system and maximum stroke of the actuator being reached. Therefore, the all-around welds had some additional capacity, making the weld calculations more conservative.

## Conclusions

Results from the axial tests indicate that the side bolt configurations provide more capacity than the corner bolt configurations and the all-around weld provides more capacity than the workable-flats weld. The *AISC Design Guide 24* procedure for designing HSS end-plate connections under axial tension with the side bolt configuration cannot be fully validated because the specimens reached the 100-kip capacity of the actuator. However, the strength of the side bolt connections based on the design procedures exceeded 100 kips and the small displacement and yielding seen in the connection suggest that the higher capacity could be reached. The results for the corner bolt specimens suggest that the design procedure is not directly applicable to this different configuration. Ultimate failure occurred in the WFCB specimens at a load of just under 90 kips, well under the calculated capacity from the design procedures. Although the AACB specimens reached the 100-kip capacity of the load cell, they all exhibited significant displacement and yielding starting well before the capacity was reached. Although an ultimate capacity was not reached, this significant deformation in the plate and rods could be considered failure and suggest that the higher strength based on rod strength would not have been reached before prying action would have caused rupture in the rods. The significant deformation in the corner bolt specimens also correlates with the results of the finite element analysis conducted by Zietlow (2022).

Results from the flexural tests indicate that the strong axis configurations provide more capacity than the weak axis configurations and the all-around weld provides more capacity than the workable-flats weld. *AISC Design Guide 24* does not have any apparent design procedure that is directly applicable to the lateral loading applied to this configuration of an HSS end-plate connection, and the design procedures for an axially loaded connection that were applied in the

initial design calculations appear to have overestimated the capacity of the laterally loaded connection. Capacities of the connections for strength calculated using equations from the design procedures presented in Wheeler et al. (1998) and Example 4.1 in *AISC Design Guide 24* were more comparable to the experimental capacities of the HSS end-plate connection subjected to lateral/flexural loading.

### Recommendations

Results from the axial tests suggest that the procedure in *AISC Design Guide 24* is not directly applicable to HSS end-plate connection with corner bolt configurations, and that the corner bolt configuration offers less capacity than the side bolt configuration. Therefore, a recommendation is to clearly state that the design procedure should not be applied to the design of corner bolt HSS end-plate configurations in axial tension and the side bolt configuration should be used, or that a separate design procedure be developed, potentially involving a reduced bolt tributary length,  $p$ , increased  $a$  and  $b$  dimensions; the latter option would require more testing and research for development and validation. All prying action calculations for the axial specimens conducted in this research used the yield strength ( $F_y$ ), as it is used in the axial prying action equations in *AISC Design Guide 24*. However, the most current *AISC Steel Construction Manual* uses the tensile strength ( $F_u$ ). Thornton (2017) notes this discrepancy and explains the reasons for  $F_u$  being initially adopted in 13<sup>th</sup> Edition *Manual* (2005). Thornton developed design procedures using  $F_u$  and showed “excellent correlation” between experimental and predicted capacities. Therefore, it recommended that the design procedure be considered for implementation in *AISC Design Guide 24* for the side bolt configuration. The results also suggest that the all-around weld pattern offers more capacity than the workable-flat welds pattern. Thus, another recommendation is to use all-around welds for HSS end-plates in axial tension.

Results from the flexural tests suggest the weld design procedures of Example 4.1 in *AISC Design Guide 24* are conservative for the all-around weld but not necessarily for the workable-flat welds. The results also suggest that the all-around weld pattern offers significantly more capacity than the workable-flat welds pattern. Therefore, it is also recommended to use all-around welds for HSS end-plates in flexure. The results showed good correlation with the design procedure presented by Wheeler et al. (1998) and calculated capacities were conservative for all-around weld specimens. Therefore, it is recommended that the design procedure be considered for implementation in *AISC Design Guide 24*. The results from the testing conducted by Wheeler et al. (1998) show that specimens with bolts in line with the HSS walls ( $c=0$ ) had higher capacities compared to otherwise similar specimens with bolts located beyond the HSS walls ( $c>0$ ). Therefore, if the bending is uniaxial, it may be best to locate the bolts in line with the HSS walls or inside as in Example 4.1 from *AISC Design Guide 24*. If bending is biaxial, it may be best to place bolts all around the HSS, that is to place additional bolts at the sides of the HSS walls.

For future research and testing it is recommended to focus on one variable of the connection. For example, it may be worth focusing on the welds with gages focused on them or the effects of prying action with instrumentation to measure bolt force. Instrumentation could have been better implemented; strain gages could have been focused better on anticipated yield lines and a LVDT should have been located off the HSS to measure overall displacement in the axial tests. It is also recommended that future research and testing be focused on one type of loading and to design specimens with a capacity that does not exceed the capacity of the actuators and load cells; another alternative would be to use actuators and load cells with higher capacities.

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## **Appendix A – Calculations**

## Zietlow (2022) Calculations

Calculation of HSS 7x4x5/16 strength in strong axis bending:

Given:

- Grade A500C Steel

Yield strength, specified	$F_{yh} := 50 \frac{\text{kip}}{\text{in}^2}$
Ultimate strength, specified	$F_{uh} := 62 \frac{\text{kip}}{\text{in}^2}$
Ultimate strength, experimental	$F_{u\_exp} := \max \left( F_{yh} \cdot 1.25 \cdot \frac{\text{in}^2}{\text{kip}}, F_{uh} \cdot \frac{\text{in}^2}{\text{kip}} \right) \cdot \frac{\text{kip}}{\text{in}^2} \rightarrow \frac{62.5 \cdot \text{kip}}{\text{in}^2}$

Bending Limit States to check:

- Yield
- Flange Local Buckling
- Web Local Buckling
- Lateral Torsional Buckling

Limit State: Yield

Plastic section modulus	$Z_x := 13.1 \text{ in}^3$
Nominal Moment Capacity	$M_{n\_y} := F_{yh} \cdot Z_x \text{ float}, 3 \rightarrow 655.0 \cdot \text{in} \cdot \text{kip}$

Limit State: Flange Local Buckling

Since HSS7x4x5/16 is compact per AISC Tab. 1-12a, the limit state of flange local buckling does not apply.

Limit State: Web Local Buckling

Since HSS7x4x5/16 is compact, the limit state of web local buckling does not apply.

Limit State: Lateral Torsional Buckling

Modulus of Elasticity	$E := 29000 \frac{\text{kip}}{\text{in}^2}$
Radius of Gyration	$r_y := 1.61 \text{ in}$
Polar Moment of Inertia	$J := 35.4 \text{ in}^4$
Gross Area of Steel	$A_g := 5.85 \text{ in}^2$
Nominal Plastic Moment Capacity	$M_p := M_{n_y} \rightarrow 655.0 \cdot \text{in} \cdot \text{kip}$
Unbraced Length	$L_b := 30 \text{ in}$
Inelastic Bending Limit	$L_p := 0.13 \cdot E \cdot r_y \cdot \sqrt{\frac{(J \cdot A_g)}{\text{in}^6}} \cdot \text{in}^3 \text{ float}, 3 \rightarrow 133.0 \cdot \text{in}$
	$\text{if} \left( \frac{L_b}{\text{in}} < \frac{L_p}{\text{in}}, \text{"LTB does not apply"} , \text{"LTB applies"} \right) \rightarrow \text{"LTB does not apply"}$

**Verification of W18x76 Support Beam:**

Nominal Moment Strength	$M_n := 1.25 M_{n_y} \text{ float}, 3 \rightarrow 819.0 \cdot \text{in} \cdot \text{kip}$
Actuator Moment Arm (see CAD file)	$d_{\text{act}} := L_b \rightarrow 30 \cdot \text{in}$
Required Actuator Force	$F_{\text{act}} := \frac{M_n}{d_{\text{act}}} \text{ float}, 3 \rightarrow 27.3 \cdot \text{kip}$
Thickness of base plate (assumed)	$t_{\text{bp}_a} := 1 \text{ in}$
Allowable cap eccentricity (assumed)	$e_{\text{cp}} := 6 \text{ in}$
Beam depth	$d_{\text{W18}} := 18.2 \text{ in}$
Total moment arm	$d_{\text{tot}} := t_{\text{bp}_a} + e_{\text{cp}} + \frac{d_{\text{W18}}}{2} + d_{\text{act}} \text{ float}, 3 \rightarrow 46.1 \cdot \text{in}$
Total moment in W18x76 beam	$M_u := F_{\text{act}} \cdot d_{\text{tot}} \text{ float}, 3 \rightarrow 1258.0 \cdot \text{in} \cdot \text{kip}$

Limit states for W18x76 to be checked:

- Flexural capacity
- Web shear
- Flange bending
- Web buckling

Flexural Capacity:

Yield strength, W18 beam	$F_{y\_W18} := 50 \frac{\text{kip}}{\text{in}^2}$
Plastic moment capacity	$\phi M_{p\_w18} := 611 \text{ kip}\cdot\text{ft}$
Provided moment capacity	$\phi M_{n\_w18} := \phi M_{p\_w18} \rightarrow 611 \cdot \text{ft}\cdot\text{kip}$

$$\text{if} \left( \frac{\phi M_{n\_w18}}{\text{kip}\cdot\text{in}} \cdot \frac{12\text{in}}{\text{ft}} > \frac{M_u}{\text{kip}\cdot\text{in}}, "Flexure OK", "Flexure NG" \right) \rightarrow "Flexure OK"$$

Web Shear: [G2]

Bolt spacing provided	$S_{\text{bolt}} := 15\text{in}$
Total number of bolts provided	$n_{W18} := 4$
Shear force per bolt	$F_{\text{bolt}} := \frac{M_n}{S_{\text{bolt}} \cdot \frac{n_{W18}}{2}} \text{ float}, 3 \rightarrow 27.3 \cdot \text{kip}$
Web shear strength reduction factor	$\phi_{ws} := 1.0$
Web thickness	$t_w := 0.425\text{in}$
Area of beam web	$A_w := t_w \cdot d_{W18} \rightarrow 7.735 \cdot \text{in}^2$
Shear constant	$c_{v1} := 1.0$
Available shear capacity	$\phi V_n := 0.6 \cdot \phi_{ws} \cdot F_{y\_W18} \cdot A_w \cdot c_{v1} \text{ float}, 3 \rightarrow 232.0 \cdot \text{kip}$
Ultimate shear force	$V_u := 2 \cdot F_{\text{bolt}} \rightarrow 54.6 \cdot \text{kip}$

$$\text{if} \left( \frac{\phi V_n}{\text{kip}} > \frac{V_u}{\text{kip}}, "web shear OK", "web shear NG" \right) \rightarrow "web shear OK"$$

Flange Local Bending: [J10]

Flange local bending reduction factor  $\phi_{FLB} := 0.9$

Flange thickness  $t_f := 0.68\text{in}$

Available capacity  $\phi R_{n\_FLB} := 6.25 \cdot \phi_{FLB} \cdot F_{y\_W18} \cdot t_f^2 \text{ float}, 3 \rightarrow 130.0 \cdot \text{kip}$

$$\text{if} \left( \frac{\phi R_{n\_FLB}}{\text{kip}} > \frac{F_{\text{bolt}}}{\text{kip}}, "FLB OK", "FLB NG" \right) \rightarrow "FLB OK"$$

Web Local Yielding: [J10]

Web local yielding reduction factor  $\phi_{WLY} := 1.0$

Web yielding factor  $k_1 := 1.0625\text{in}$

Flange width  $b_f := 11\text{in}$

$$k := \frac{b_f}{2} - k_1 \text{ float}, 3 \rightarrow 4.44 \cdot \text{in}$$

Web yielding factor  $l_b := k \rightarrow 4.44 \cdot \text{in}$

Available capacity  $\phi R_{n\_WLY} := \phi_{WLY} \cdot F_{y\_W18} \cdot t_w \cdot (5k + l_b) \text{ float}, 3 \rightarrow 566.0 \cdot \text{kip}$

$$\text{if} \left( \frac{\phi R_{n\_WLY}}{\text{kip}} > \frac{2 \cdot F_{\text{bolt}}}{\text{kip}}, "WLY OK", "WLY NG" \right) \rightarrow "WLY OK"$$

Web Local Crippling: [J10]

WLC strength reduction factor  $\phi_{WLC} := 0.75$

WLC constant  $Q_f := 1.0$

Provided capacity

$$\phi R_{n\_WLC} := 0.8 \cdot \phi_{WLC} \cdot t_w^2 \cdot \left[ 1 + 3 \cdot \left( \frac{l_b}{d_{W18}} \right) \cdot \left( \frac{t_w}{t_f} \right)^{1.5} \right] \cdot \sqrt{\frac{E \cdot F_{y\_W18} \cdot t_f}{t_w} \cdot \frac{\text{in}^4}{\text{kip}^2} \cdot \frac{\text{kip}}{\text{in}^2}} \cdot Q_f$$

$\phi R_{n\_WLC} \text{ float}, 3 \rightarrow 225.0 \cdot \text{kip}$

$$\text{if} \left( \frac{\phi R_{n\_WLC}}{\text{kip}} > \frac{2 \cdot F_{\text{bolt}}}{\text{kip}}, "WLC OK", "WLC NG" \right) \rightarrow "WLC OK"$$

Web Compression Buckling: [J10]

Web compression buckling reduction factor  $\phi_{\text{WCB}} := 0.9$

Web height  $h := d_{\text{W18}} - 2 \cdot k_1 \text{ float}, 3 \rightarrow 16.1 \cdot \text{in}$

Available capacity  $\phi R_{\text{n\_WCB}} := \phi_{\text{WCB}} \cdot \left( \frac{24 \cdot t_{\text{w}}^3 \cdot \sqrt{\frac{E \cdot F_{\text{y\_W18}} \cdot \text{in}^4}{\text{kip}^2}}}{h} \right) \cdot Q_{\text{f}} \cdot \frac{\text{kip}}{\text{in}^2}$

$\phi R_{\text{n\_WCB}} \text{ float}, 3 \rightarrow 124.0 \cdot \text{kip}$

$\text{if} \left( \frac{\phi R_{\text{n\_WCB}}}{\text{kip}} > \frac{2 \cdot F_{\text{bolt}}}{\text{kip}}, " \text{WCB OK} ", " \text{WCB NG} " \right) \rightarrow " \text{WCB OK} "$

**Base/Cap Plate Design (HSS7x7x5/16)**Given:

Strength reduction factor, weld	$\phi_w := 0.75$
Strength reduction factor, tensile yield	$\phi_y := 0.9$
Strength reduction factor, tensile rupture	$\phi_r := 0.75$
Yield strength, HSS	$F_{yh} := 50 \frac{\text{kip}}{\text{in}^2}$
Rupture Strength, HSS	$F_{uh} := 62 \frac{\text{kip}}{\text{in}^2}$
Length, HSS	$l_h := 7\text{in}$
Width, HSS	$b_h := 7\text{in}$
Thickness, HSS	$t := 0.3125\text{in}$
Length workable flat	$f_l := 5.625\text{in}$
Width workable flat	$f_b := 5.625\text{in}$
Yield strength, base plate	$F_{yp} := 50 \frac{\text{kip}}{\text{in}^2}$
Length, plate	$l_p := 12\text{in}$
Width, plate	$b_p := 12\text{in}$
Electrode classification number	$F_{EXX} := 70 \frac{\text{kip}}{\text{in}^2}$
Weld size	$w := 0.25\text{in}$
Bolt diameter	$d_b := 0.75\text{in}$
Hole diameter	$d_h := \frac{13\text{in}}{16} \text{ float}, 4 \rightarrow 0.8125\text{in}$
Bolt available tensile strength (AISC Tab. 7-2)	$\phi_{rn} := 29.8\text{kip}$
Gross area, HSS	$A_g := 7.59\text{in}^2$
Net area, HSS	$A_n := 7.59\text{in}^2$
Shear lag factor	$U := 1$
Number of bolts	$n := 4$

Design:

60% weld electrode	$F_w := F_{EXX} \cdot 0.6 \rightarrow \frac{42.0 \cdot \text{kip}}{\text{in}^2}$
Design weld strength	$F_{wc} := F_w \cdot \phi_w \rightarrow \frac{31.5 \cdot \text{kip}}{\text{in}^2}$
Weld length	$L_w := 2(l_h + b_h) \rightarrow 28 \cdot \text{in}$
Effective Area	$A_e := A_n \cdot U \rightarrow 7.59 \cdot \text{in}^2$
Available strength, weld rupture (AISC Design Guide 24, Equation 5-25a)	$P_{u1} := \frac{w \cdot F_{wc} \cdot L_w}{\sqrt{2}} \text{ float}, 4 \rightarrow 155.9 \cdot \text{kip}$
Available strength, HSS yield (AISC Manual Equation D2-1)	$P_{u2} := \phi_y \cdot F_{yh} \cdot A_g \text{ float}, 4 \rightarrow 341.5 \cdot \text{kip}$
Available strength, HSS rupture (AISC Manual Equation D2-2)	$P_{u3} := \phi_r \cdot F_{uh} \cdot A_e \text{ float}, 4 \rightarrow 352.9 \cdot \text{kip}$
Available strength, bolts (AISC Manual Table 7-2)	$P_{u4} := n \cdot \phi_{rm} \text{ float}, 4 \rightarrow 119.2 \cdot \text{kip}$
Limiting available strength	$P_u := \min\left(\frac{P_{u1}}{\text{kip}}, \frac{P_{u2}}{\text{kip}}, \frac{P_{u3}}{\text{kip}}, \frac{P_{u4}}{\text{kip}}\right) \cdot \text{kip} \rightarrow 119.2 \cdot \text{kip}$

**NOTE: Bolt strength governs design**

HSS edge to bolt edge	$b' := \frac{(\min(l_p, b_p) - \min(l_h, b_h))}{4} - \frac{d_b}{2} \text{ float}, 3 \rightarrow 0.875 \cdot \text{in}$
Bolt end tributary length	$p := \max(l_p, b_p) \rightarrow 12 \cdot \text{in}$
Prying factor	$\delta := 1 - \frac{d_h}{p} \text{ float}, 3 \rightarrow 0.932$
Prying factor	$a_e := \frac{(b_p - b_h)}{4} \text{ float}, 3 \rightarrow 1.25 \cdot \text{in}$
Prying factor	$a' := a_e + \frac{d_b}{2} \text{ float}, 3 \rightarrow 1.62 \cdot \text{in}$
Prying factor	$\rho := \frac{b'}{a'} \text{ float}, 3 \rightarrow 0.54$
Prying factor (LRFD Design)	$\beta := \frac{\left(\phi_{rm} \cdot \frac{n}{P_u} - 1\right)}{\rho} \text{ float}, 3 \rightarrow -4.67\text{e-}29$
Prying factor	$\alpha' := \text{if } \left[ \beta < 1, \min\left[1, \frac{\left[\frac{\beta}{(1-\beta)}\right]}{\delta}\right], 1 \right] \text{ float}, 3 \rightarrow -5.01\text{e-}29$



Minimum base plate thickness  
(AISC Design Guide 24, Equation 5-23a)

$$t_{bp1} := \left[ \sqrt{\frac{4.44 \cdot P_u}{in^2 \cdot n} \cdot \frac{b'}{(p \cdot F_{yp})}} \right] \cdot in \text{ float}, 3 \rightarrow 0.439 \cdot in$$

$$t_{bp1\_eff} := \text{ceil}\left(t_{bp1} \cdot \frac{16}{in}\right) \rightarrow 8 \text{ [16th inch]}$$

Minimum base plate thickness  
(AISC Design Guide 24, Equation 5-26a)

$$t_{bp2} := \left[ \sqrt{\frac{4.44 \cdot P_u}{in^2 \cdot n} \cdot \frac{b'}{[p \cdot F_{yp} \cdot (1 + \delta \cdot \alpha')]} \right] \cdot in \text{ float}, 3 \rightarrow 0.439 \cdot in$$

$$t_{bp2\_eff} := \text{ceil}\left(t_{bp2} \cdot \frac{16}{in}\right) \rightarrow 8 \text{ [16th inch]}$$

Minimum base plate thickness

$$t_{bp} := \min(t_{bp1\_eff}, t_{bp2\_eff}) \rightarrow 8 \text{ [16th inch]}$$

Minimum cap plate thickness

$$t_{cp} := \text{ceil}\left(1.5 \cdot t_{bp}\right) \rightarrow 12 \text{ [16th inch]}$$

## Nelson Calculations

### Base/Cap Plate Design (HSS7x4x5/16)

Given:

Strength reduction factor, weld	$\phi_w := 0.75$
Strength reduction factor, tensile yield	$\phi_y := 0.9$
Strength reduction factor, tensile rupture	$\phi_r := 0.75$
Yield strength, HSS	$F_{yh} := 50 \frac{\text{kip}}{\text{in}^2}$
Rupture Strength, HSS	$F_{uh} := 62 \frac{\text{kip}}{\text{in}^2}$
Length, HSS	$l_h := 7\text{in}$
Width, HSS	$b_h := 4\text{in}$
Thickness, HSS	$t := 0.3125\text{in}$
Length workable flat	$f_l := 5.625\text{in}$
Width workable flat	$f_b := 2.625\text{in}$
Yield strength, base plate	$F_{yp} := 50 \frac{\text{kip}}{\text{in}^2}$
Length, plate	$l_p := 12\text{in}$
Width, plate	$b_p := 12\text{in}$
Weld electrode	$F_{EXX} := 70 \frac{\text{kip}}{\text{in}^2}$
Weld size	$w := 0.25\text{in}$
Bolt diameter	$d_b := 0.75\text{in}$
Hole diameter	$d_h := \frac{13\text{in}}{16} \text{ float}, 4 \rightarrow 0.8125\text{in}$
Bolt available tensile strength (AISC Tab. 7-2)	$\phi_m := 29.8\text{kip}$
Gross area, HSS	$A_g := 5.85\text{in}^2$
Net area, HSS	$A_n := 5.85\text{in}^2$
Shear lag factor	$U := 1$
Number of bolts	$n := 4$

Design:

60% weld electrode	$F_w := F_{EXX} \cdot 0.6 \rightarrow \frac{42.0 \cdot \text{kip}}{\text{in}^2}$
Design weld strength	$F_{wc} := F_w \cdot \phi_w \rightarrow \frac{31.5 \cdot \text{kip}}{\text{in}^2}$
Weld length	$L_w := 2(l_h + b_h) \rightarrow 22 \cdot \text{in}$
Effective Area	$A_e := A_n \cdot U \rightarrow 5.85 \cdot \text{in}^2$
Available strength, weld rupture (AISC Design Guide 24, Equation 5-25a)	$P_{u1} := \frac{w \cdot F_{wc} \cdot L_w}{\sqrt{2}} \text{ float}, 4 \rightarrow 122.5 \cdot \text{kip}$
Available strength, HSS yield (AISC Manual Equation D2-1)	$P_{u2} := \phi_y \cdot F_{yh} \cdot A_g \text{ float}, 4 \rightarrow 263.2 \cdot \text{kip}$
Available strength, HSS rupture (AISC Manual Equation D2-2)	$P_{u3} := \phi_r \cdot F_{uh} \cdot A_e \text{ float}, 4 \rightarrow 272.0 \cdot \text{kip}$
Available strength, bolts (AISC Manual Table 7-2)	$P_{u4} := n \cdot \phi_{rm} \text{ float}, 4 \rightarrow 119.2 \cdot \text{kip}$
Limiting available strength	$P_u := \min\left(\frac{P_{u1}}{\text{kip}}, \frac{P_{u2}}{\text{kip}}, \frac{P_{u3}}{\text{kip}}, \frac{P_{u4}}{\text{kip}}\right) \cdot \text{kip} \rightarrow 119.2 \cdot \text{kip}$

Note: Bolt strength governs design

HSS edge to bolt edge	$b' := \frac{\left(\min\left(\frac{l_p}{\text{in}}, \frac{b_p}{\text{in}}\right) - \min\left(\frac{l_h}{\text{in}}, \frac{b_h}{\text{in}}\right)\right)}{4} \cdot \text{in} - \frac{d_b}{2}$ $b' \text{ float}, 4 \rightarrow 1.625 \cdot \text{in}$
Bolt end tributary length	$p := \max(l_p, b_p) \rightarrow 12 \cdot \text{in}$
Prying factor	$\delta := 1 - \frac{d_h}{p} \text{ float}, 3 \rightarrow 0.932$
Prying factor	$a_e := \frac{(b_p - b_h)}{4} \text{ float}, 3 \rightarrow 2.0 \cdot \text{in}$
Prying factor	$a' := a_e + \frac{d_b}{2} \text{ float}, 3 \rightarrow 2.37 \cdot \text{in}$
Prying factor	$\rho := \frac{b'}{a'} \text{ float}, 3 \rightarrow 0.686$
Prying factor (LRFD Design)	$\beta := \frac{\left(\phi_{rm} \cdot \frac{n}{P_u} - 1\right)}{\rho} \text{ float}, 3 \rightarrow -3.68e-29$

Prying factor

$$\alpha' := \text{if} \left[ \beta < 1, \min \left[ 1, \frac{\left[ \frac{\beta}{(1-\beta)} \right]}{\delta} \right], 1 \right] \text{float}, 3 \rightarrow -3.95\text{e-}29$$

Minimum base plate thickness  
(AISC Design Guide 24, Equation 5-23a)

$$t_{bp1} := \sqrt{\frac{4.44 \cdot P_u}{\text{in}^2 \cdot n} \cdot \frac{b'}{(p \cdot F_{yp})}} \cdot \text{in} \text{float}, 3 \rightarrow 0.599 \cdot \text{in}$$

$$= \text{ceil} \left( t_{bp1} \cdot \frac{16}{\text{in}} \right) \rightarrow 10 \quad [16\text{th inch}]$$

Minimum base plate thickness  
(AISC Design Guide 24, Equation 5-26a)

$$t_{bp2} := \sqrt{\frac{4.44 \cdot P_u}{\text{in}^2 \cdot n} \cdot \frac{b'}{[p \cdot F_{yp} \cdot (1 + \delta \cdot \alpha')]}} \cdot \text{in} \text{float}, 3 \rightarrow 0.599 \cdot \text{in}$$

$$= \text{ceil} \left( t_{bp2} \cdot \frac{16}{\text{in}} \right) \rightarrow 10 \quad [16\text{th inch}]$$

Minimum base plate thickness

$$t_{bp} := \min(t_{bp1\_eff}, t_{bp2\_eff}) \rightarrow 10 \quad [16\text{th inch}]$$

Minimum cap plate thickness

$$t_{cp} := \text{ceil} \left( 1.5 \cdot t_{bp} \right) \rightarrow 15 \quad [16\text{th inch}]$$

## Axial Calculations

### A193 Bolt Strength

$$f_{yb} := 105 \text{ ksi}$$

$$d_b := \frac{3}{4} \text{ in}$$

$$A_b := \pi \cdot \left( \frac{d_b}{2} \right)^2 = 0.442 \text{ in}^2$$

$$r_n := A_b \cdot f_{yb} = 46.388 \text{ kip}$$

$$\phi := 0.75$$

$$\phi r_n := \phi \cdot r_n = 34.791 \text{ kip}$$

$$n := 4$$

$$R_n := n \cdot r_n = 185.55 \text{ kip}$$

$$\phi R_n := n \cdot \phi r_n = 139.163 \text{ kip}$$

$$T_r := \frac{\phi R_n}{n} = 34.791 \text{ kip}$$

### AISC Design Guide 24 (2010) Prying Action

$$f_{yp} := 57.1 \text{ ksi} \quad f_{up} := 82.9 \text{ ksi}$$

$$d_h := \frac{13}{16} \text{ in}$$

$$b := 1.25 \text{ in}$$

$$p := 12 \text{ in}$$

$$a := 1.25 \text{ in}$$

$$a_e := 1.25 \text{ in}$$

$$a' := a_e + \frac{d_b}{2} = 1.625 \text{ in}$$

$$b' := b - \frac{d_b}{2} = 0.875 \text{ in}$$

$$t_{min} := \sqrt{\frac{4.44 \cdot (T_r) b'}{p \cdot f_{yp}}} = 0.444 \text{ in}$$

Calculate Force Using Actual Plate Thickness of 1/2"

$$t := \frac{1}{2} \text{ in}$$

$$P_n := \frac{t^2 \cdot p \cdot f_{yp}}{4.44 \cdot b'} \cdot n = 176.371 \text{ kip}$$

### Corner Specimen Capacity - HSS Connections Manual (1997) and Christensen (2010)

$$L := \sqrt{(1.25 \text{ in})^2 + (1.25 \text{ in})^2} = 1.768 \text{ in}$$

$$b_e := 2 \cdot L = 3.536 \text{ in}$$

$$R_n := \frac{b_e \cdot t^2 \cdot f_{yp}}{L} = 28.55 \text{ kip} \quad \text{Equation (29)}$$

$$P_n := 4 \cdot R_n = 114.2 \text{ kip}$$

$$t_{HSS} := \frac{1}{4} \text{ in}$$

$$R_{HSS} := 0.6$$

$$P_n := R_{HSS} \cdot 4 \cdot R_n = 68.52 \text{ kip}$$

$$t_{HSS} := \frac{3}{8} \text{ in}$$

$$R_{HSS} := 0.7$$

$$P_n := R_{HSS} \cdot 4 \cdot R_n = 79.94 \text{ kip}$$

**Prying Actions for Side Bolts with Adjusted Tributary Width - Thornton (2017) and AISC *Manual*, 16th Ed., Equations**

$$P_u := 100 \text{ kip} \quad (100\text{-kip Capacity of Load Cell})$$

$$w_i := 7 \text{ in}$$

$$h_i := 7 \text{ in}$$

$$p_c := \frac{2 \cdot (w_i + h_i + \pi \cdot b)}{n} = 8.963 \text{ in} \quad \text{Equation (32)}$$

$$p_{imax} := 4 \cdot \sqrt{b' \cdot (a + b)} = 5.916 \text{ in} \quad \text{Equation (33)}$$

$$\phi_b := 0.9$$

$$t_{min} := \sqrt{\frac{4.44 \cdot (T_r) \cdot b'}{\phi_b \cdot p_{imax} \cdot f_{up}}} = 0.553 \text{ in} \quad \text{No Prying Action - Equation (3)}$$

$$t := \frac{1}{2} \text{ in}$$

$$P_n := \frac{t^2 \cdot \phi_b \cdot p_{imax} \cdot f_{up}}{4.44 \cdot b'} \cdot n = 113.616 \text{ kip}$$

$$\delta := 1 - \frac{d_h}{p_{imax}} = 0.863$$

$$\rho := \frac{b'}{a'} = 0.538$$

$$\beta := \left[ \frac{\phi r_n}{\left( \frac{P_u}{n} \right)} - 1 \right] \cdot \frac{1}{\rho} = [0.727]$$

$$\alpha' := \min \left( 1.0, \frac{1}{\delta} \cdot \left( \frac{\beta}{1 - \beta} \right) \right) = 1$$

$$t_{min} := \sqrt{\frac{4.44 \cdot (T_r) \cdot b'}{\phi_b \cdot p_{imax} \cdot f_{up} \cdot (1 + \delta \cdot \alpha')}} = 0.405 \text{ in} \quad \text{Prying Action - Equation (5)}$$

$$P_n := \frac{t^2 \cdot \phi_b \cdot p_{imax} \cdot f_{up} \cdot (1 + \delta \cdot \alpha')}{4.44 \cdot b'} \cdot n = 211.629 \text{ kip}$$

### Prying Action for Corner Bolts with p=be

$$p := b_e = 3.536 \text{ in}$$

$$a' := a + \frac{d_b}{2} = 1.625 \text{ in}$$

$$b' := b - \frac{d_b}{2} = 0.875 \text{ in}$$

$$t_{min} := \sqrt{\frac{4.44 \cdot (T_r) \cdot b'}{\phi_b \cdot p \cdot f_{up}}} = 0.716 \text{ in}$$

No Prying Action - Equation (3)

$$t := \frac{1}{2} \text{ in}$$

$$P_n := \frac{t^2 \cdot \phi_b \cdot p \cdot f_{up}}{4.44 \cdot b'} \cdot n = 67.899 \text{ kip}$$

$$\delta := 1 - \frac{d_h}{p} = 0.77$$

$$\rho := \frac{b'}{a'} = 0.538$$

$$\beta := \left[ \frac{\phi r_n}{\left( \frac{P_u}{n} \right)} - 1 \right] \cdot \frac{1}{\rho} = [0.727]$$

$$\alpha' := \min \left( 1.0, \frac{1}{\delta} \cdot \left( \frac{\beta}{1 - \beta} \right) \right) = 1$$



$$t_{min} := \sqrt{\frac{4.44 \cdot (T_r) \cdot b'}{\phi_b \cdot p \cdot f_{up} \cdot (1 + \delta \cdot \alpha')}} = 0.538 \text{ in} \quad \text{Prying Action - Equation (5)}$$

$$P_n := \frac{t^2 \cdot \phi_b \cdot p \cdot f_{up} \cdot (1 + \delta \cdot \alpha')}{4.44 \cdot b'} \cdot n = 120.193 \text{ kip}$$

### Prying Action for Corner Bolts with Adjusted a and b Dimensions

$$p := p_{imax} = 5.916 \text{ in}$$

$$b := \sqrt{(1.25 \text{ in})^2 + (1.25 \text{ in})^2} = 1.768 \text{ in}$$

$$a := \sqrt{(1.25 \text{ in})^2 + (1.25 \text{ in})^2} = 1.768 \text{ in}$$

$$a' := a + \frac{d_b}{2} = 2.143 \text{ in}$$

$$b' := b - \frac{d_b}{2} = 1.393 \text{ in}$$

$$t_{min} := \sqrt{\frac{4.44 \cdot (T_r) \cdot b'}{\phi_b \cdot p \cdot f_{up}}} = 0.698 \text{ in} \quad \text{No Prying Action - Equation (3)}$$

$$t := \frac{1}{2} \text{ in}$$

$$P_n := \frac{t^2 \cdot \phi_b \cdot p \cdot f_{up}}{4.44 \cdot b'} \cdot n = 71.379 \text{ kip}$$

$$\delta := 1 - \frac{d_h}{p} = 0.863$$

$$\rho := \frac{b'}{a'} = 0.65$$

$$\beta := \left[ \frac{\phi r_n}{\left( \frac{P_u}{n} \right)} - 1 \right] \cdot \frac{1}{\rho} = [0.603]$$

$$\alpha' := \min \left( 1.0, \frac{1}{\delta} \cdot \left( \frac{\beta}{1 - \beta} \right) \right) = 1$$

$$t_{min} := \sqrt{\frac{4.44 \cdot \langle T_r \rangle \cdot b'}{\phi_b \cdot p \cdot f_{up} \cdot (1 + \delta \cdot \alpha')}} = 0.512 \text{ in} \quad \text{Prying Action - Equation (5)}$$

$$P_n := \frac{t^2 \cdot \phi_b \cdot p \cdot f_{up} \cdot (1 + \delta \cdot \alpha')}{4.44 \cdot b'} \cdot n = 132.955 \text{ kip}$$

### Prying Action for Corner Bolts with Adjusted a and b Dimensions and p=be

$$p := b_e = 3.536 \text{ in}$$

$$b := \sqrt{(1.25 \text{ in})^2 + (1.25 \text{ in})^2} = 1.768 \text{ in}$$

$$a := \sqrt{(1.25 \text{ in})^2 + (1.25 \text{ in})^2} = 1.768 \text{ in}$$

$$a' := a + \frac{d_b}{2} = 2.143 \text{ in}$$

$$b' := b - \frac{d_b}{2} = 1.393 \text{ in}$$

$$t_{min} := \sqrt{\frac{4.44 \cdot \langle T_r \rangle \cdot b'}{\phi_b \cdot p \cdot f_{up}}} = 0.903 \text{ in} \quad \text{No Prying Action - Equation (3)}$$

$$P_n := \frac{t^2 \cdot \phi_b \cdot p \cdot f_{up}}{4.44 \cdot b'} \cdot n = 42.657 \text{ kip}$$

$$\delta := 1 - \frac{d_h}{p} = 0.77$$

$$\rho := \frac{b'}{a'} = 0.65$$

$$\beta := \left[ \frac{\phi r_n}{\left( \frac{P_u}{n} \right)} - 1 \right] \cdot \frac{1}{\rho} = [0.603]$$

$$\alpha' := \min \left( 1.0, \frac{1}{\delta} \cdot \left( \frac{\beta}{1 - \beta} \right) \right) = 1$$

$$t_{min} := \sqrt{\frac{4.44 \cdot \langle T_r \rangle \cdot b'}{\phi_b \cdot p \cdot f_{up} \cdot (1 + \delta \cdot \alpha')}} = 0.679 \text{ in}$$

Prying Action - Equation (5)

$$P_n := \frac{t^2 \cdot \phi_b \cdot p \cdot f_{up} \cdot (1 + \delta \cdot \alpha')}{4.44 \cdot b'} \cdot n = 75.511 \text{ kip}$$

**Strong Axis - AISC Design Guide 24 (LRFD)**

HSS7x4x5/16

$$F_{yHSS} := 53.2 \text{ ksi}$$

$$F_{uHSS} := 66.8 \text{ ksi}$$

$$S_x := 10.4 \text{ in}^3$$

$$M_u := F_{yHSS} \cdot S_x = 46.107 \text{ kip} \cdot \text{ft}$$

$$E := 29000 \text{ ksi}$$

Base Plate

$$F_{yp} := 57.1 \text{ ksi}$$

$$F_{up} := 82.9 \text{ ksi}$$

From AISC *Manual*/Table 1-12, the HSS geometric properties are as follows:

HSS7x4x5/16

$$H := 4.00 \text{ in}$$

$$B := 7.00 \text{ in}$$

$$t := \frac{5}{16} \text{ in} = 0.313 \text{ in}$$

Gage

$$g := 9.5 \text{ in}$$

Geometric properties of end-plate and bolts are as follows:

Base Plate

$$L := 12.0 \text{ in}$$

$$W := 12.0 \text{ in}$$

$$t_p := \frac{5}{8} \text{ in} = 0.625 \text{ in}$$

A193 B7 Rods

$$F_{nt} := 125 \text{ ksi}$$

$$d_b := \frac{3}{4} \text{ in} = 0.75 \text{ in}$$

Tensile load in bolt

$$M_u = 553.28 \text{ kip} \cdot \text{in}$$

$$T_u := \frac{M_u}{\frac{g}{2}} \cdot \frac{1}{2} = 58.24 \text{ kip}$$

Effect of prying action - end-plate minimum thickness

$$b := \frac{(g - B)}{2} = 1.25 \text{ in}$$

$$b' := b - \frac{d_b}{2} = 0.875 \text{ in}$$

$$p := \frac{W}{2} = 6 \text{ in}$$

$$t_{min} := \sqrt{\frac{4.44 \cdot T_u \cdot b'}{p \cdot F_{up}}} = 0.674 \text{ in}$$

Effect of prying action - capacity using actual 5/8" end-plate thickness

$$T_n := \frac{t_p^2 \cdot p \cdot F_{up}}{4.44 \cdot b'} = 50.012 \text{ kip}$$

$$M_n := T_n \cdot \frac{g}{2} \cdot 2 = 39.593 \text{ kip} \cdot \text{ft}$$

$$P_n := \frac{M_n}{35.5 \text{ in}} = 13.384 \text{ kip}$$

$$\phi_p := 0.9$$

$$\phi_p M_n := \phi_p \cdot M_n = 35.634 \text{ kip} \cdot \text{ft}$$

$$\phi_p P_n := \phi_p \cdot P_n = 12.045 \text{ kip}$$

Bolt available tensile strength

$$\phi r_n := 0.75 \cdot F_{nt} \cdot \pi \cdot \left( \frac{d_b}{2} \right)^2 = 41.417 \text{ kip}$$

Weld size between the HSS and end-plate

$$2\ g = 19\ \text{in} > B$$

$$2 \cdot (d_b + 2 \cdot b) = 6.5\ \text{in}$$

$$B = 7\ \text{in}$$

$$\frac{2 \cdot T_u}{B} = 16.64\ \frac{\text{kip}}{\text{in}}$$

$$D := \frac{\frac{2 \cdot T_u}{B}}{1.392\ \frac{\text{kip}}{\text{in}}} = 11.954$$

Force using actual 1/4" weld size (D=4)

$$D := 4$$

$$T_u := \frac{D \cdot 1.392\ \frac{\text{kip}}{\text{in}} \cdot (B)}{2} = 19.488\ \text{kip}$$

$$\phi M_u := (T_u \cdot 12) \cdot \frac{g}{2} \cdot \frac{1}{2} = 46.284\ \text{kip} \cdot \text{ft}$$

$$\phi := 0.75$$

$$M_u := \frac{\phi M_u}{\phi} = 61.712\ \text{kip} \cdot \text{ft}$$

$$P := \frac{M_u}{35.5\ \text{in}} = 20.86\ \text{kip}$$

**Weak Axis - AISC Design Guide 24 (LRFD)**

HSS7x4x5/16

$$F_{yHSS} := 53.2 \text{ ksi}$$

$$F_{uHSS} := 66.8 \text{ ksi}$$

$$S_y := 5.16 \text{ in}^3$$

$$M_u := F_{yHSS} \cdot S_y = 22.876 \text{ kip} \cdot \text{ft}$$

$$E := 29000 \text{ ksi}$$

Base Plate

$$F_{yp} := 57.1 \text{ ksi}$$

$$F_{up} := 82.9 \text{ ksi}$$

From AISC *Manual*/Table 1-12, the HSS geometric properties are as follows:

HSS7x4x5/16

$$H := 7.00 \text{ in}$$

$$B := 4.00 \text{ in}$$

$$t := \frac{5}{16} \text{ in} = 0.313 \text{ in}$$

Gage

$$g := 9.5 \text{ in}$$

Geometric properties of end-plate plate and bolts are as follows:

Base Plate

$$L := 12.0 \text{ in}$$

$$W := 12.0 \text{ in}$$

$$t_p := \frac{5}{8} \text{ in} = 0.625 \text{ in}$$

A193 B7 Rods

$$F_{nt} := 90 \text{ ksi}$$

$$d_b := \frac{3}{4} \text{ in} = 0.75 \text{ in}$$

Tensile load in bolt

$$M_u = 274.512 \text{ kip} \cdot \text{in}$$

$$T_u := \frac{M_u}{\frac{g}{2}} \cdot \frac{1}{2} = 28.896 \text{ kip}$$

Effect of prying action - end-plate minimum thickness

$$b := \frac{(g - B)}{2} = 2.75 \text{ in}$$

$$b' := b - \frac{d_b}{2} = 2.375 \text{ in}$$

$$p := \frac{W}{2} = 6 \text{ in}$$

$$t_{min} := \sqrt{\frac{4.44 \cdot T_u \cdot b'}{p \cdot F_{up}}} = 0.783 \text{ in}$$

Effect of prying action - capacity using actual 5/8" end-plate thickness

$$T_n := \frac{t_p^2 \cdot p \cdot F_{up}}{4.44 \cdot b'} = 18.425 \text{ kip}$$

$$M_n := T_n \cdot \frac{g}{2} \cdot 2 = 14.587 \text{ kip} \cdot \text{ft}$$

$$P_n := \frac{M_n}{35.5 \text{ in}} = 4.931 \text{ kip}$$

$$\phi_p := 0.9$$

$$\phi_p M_n := \phi_p \cdot M_n = 13.128 \text{ kip} \cdot \text{ft}$$

$$\phi_p P_n := \phi_p \cdot P_n = 4.438 \text{ kip}$$

Bolt available tensile strength

$$\phi r_n := 0.75 \cdot F_{nt} \cdot \pi \cdot \left( \frac{d_b}{2} \right)^2 = 29.821 \text{ kip}$$



Weld size between the HSS and end-plate

$$2g = 19 \text{ in} > B$$

$$2 \cdot (d_b + 2 \cdot b) = 12.5 \text{ in} > B$$

$$B = 4 \text{ in}$$

$$\frac{2 \cdot T_u}{B} = 14.448 \frac{\text{kip}}{\text{in}}$$

$$D := \frac{\frac{2 \cdot T_u}{B}}{1.392 \frac{\text{kip}}{\text{in}}} = 10.379$$

Force Using 1/4" Weld (D=4)

$$D := 4$$

$$T_u := \frac{D \cdot 1.392 \frac{\text{kip}}{\text{in}} \cdot (B)}{2} = 11.136 \text{ kip}$$

$$\phi M_u := (T_u \cdot 12) \cdot \frac{g}{2} \cdot \frac{1}{2} = 26.448 \text{ ft} \cdot \text{kip}$$

$$\phi := 0.75$$

$$M_u := \frac{\phi M_u}{\phi} = 35.264 \text{ ft} \cdot \text{kip}$$

$$P := \frac{M_u}{35.5 \text{ in}} = 11.92 \text{ kip}$$



$$d_{34} := \frac{a_e \cdot (2 \cdot q \cdot v - r) \cdot \left( \sqrt{\left(\frac{2}{r}\right)^2 + \left(\frac{1}{q \cdot v}\right)^2} \right)}{2} - d_f = 3.799 \text{ in}$$

$$m_p := \frac{1}{4} \cdot t_p^2 \cdot f_y = 5.576 \text{ kip}$$

Mode 1

$$M_{y1} := \left( \frac{(d' + 2 \cdot (s_0' + a_e)) \cdot w}{(s_0' + a_e) \cdot d'} \cdot m_p + \frac{n \cdot B_y \cdot a_e}{s_0' + a_e} \right) \cdot (d - t_s) = 54.3 \text{ kip} \cdot \text{ft}$$

Mode 2

$$M_{y2} := \left( \frac{2 \cdot (d' + s_0') \cdot w}{s_0' \cdot d'} - \frac{n \cdot d_f}{s_0'} \right) \cdot m_p \cdot (d - t_s) = 75.2 \text{ kip} \cdot \text{ft}$$

Mode 3

$$M_{y3} := \left( \frac{w}{d'} + \frac{\frac{v \cdot w}{b} \cdot (b^2 + 4 \cdot q^2) + (\sqrt{r^2 + 4 \cdot v^2 \cdot q^2}) \cdot d_{34} + 4 \cdot u \cdot q \cdot v}{(2 \cdot q \cdot c + s_0' \cdot b) \cdot d'} \right) \cdot d \cdot m_p = 34.3 \text{ kip} \cdot \text{ft}$$

Failure Mode

$$M_y := \min(M_{y1}, M_{y2}, M_{y3}) = 34.326 \text{ kip} \cdot \text{ft}$$

Design Model

$$w_{eq} := \left( \frac{4 \cdot M_{y3}}{d \cdot t_p^2 \cdot f_p} + \frac{n \cdot d_f}{s_0'} \right) \cdot \left( \frac{d' \cdot s_0'}{2 \cdot (d' + s_0')} \right) = 4.493 \text{ in}$$

$$\phi_b := 0.8$$

$$\phi_p := 0.9$$

## Bolt Failure

$$f_{yb} := \frac{B_y}{\pi \cdot \left(\frac{d_b}{2}\right)^2} = 0.724 \frac{kN}{mm^2}$$

$$M_{cb} := \left( \frac{4 \cdot n \cdot \left( B_u \cdot a_p + \frac{\pi \cdot d_b^3 \cdot f_{yb}}{32} \right) \cdot d' + w_{eq} \cdot (d' + 2 \cdot (s_0' + a_p)) \cdot t_p^2 \cdot f_p}{4 \cdot (a_p + s_0') \cdot d'} \right) \cdot (d - t_s) = 48.086 \text{ kip} \cdot ft$$

$$\phi_b \cdot M_{cb} = 38.469 \text{ kip} \cdot ft$$

$$P_{cb} := \frac{M_{cb}}{35.5 \text{ in}} = 16.254 \text{ kip}$$

$$t_{bu} := \sqrt{\frac{4 \cdot \left( \frac{M_{cb} \cdot (a_p + s_0')}{\phi_b \cdot (d - t_s)} - n \cdot \left( B_u \cdot a_p + \frac{\pi \cdot d_b^3 \cdot f_{yb}}{32} \right) \right) \cdot d'}{w_{eq} \cdot (d' + 2 \cdot (s_0' + a_p)) \cdot f_p}} = 0.869 \text{ in}$$

$$t_{pmax} := \sqrt{\frac{4 \cdot n \cdot B_u \cdot s_0'}{w_{eq} \cdot f_p}} = 1.192 \text{ in}$$

## Plate Failure

$$M_{cp} := \left( \frac{t_p^2 \cdot f_p \cdot (w_{eq} \cdot (d' + 2 \cdot s_0') + (w_{eq} - n \cdot d_f) \cdot d') + n \cdot \frac{\pi \cdot d_b^3 \cdot f_{yb}}{8} \cdot d'}{4 \cdot d' \cdot s_0'} \right) \cdot (d - t_s) = 37.31 \text{ kip} \cdot ft$$

$$\phi_p \cdot M_{cp} = 33.579 \text{ kip} \cdot ft$$

$$P_{cp} := \frac{M_{cp}}{35.5 \text{ in}} = 12.612 \text{ kip}$$

$$t_{pu} := 2 \cdot \sqrt{\frac{\left( \frac{M_{cb} \cdot s_0'}{\phi_p \cdot (d + t_s)} - n \cdot \frac{\pi \cdot d_b^3 \cdot f_{yb}}{32} \right) \cdot d'}{(w_{eq} \cdot (d' + 2 \cdot s_0') + (w_{eq} - n \cdot d_f) \cdot d') \cdot f_p}} = 0.727 \text{ in}$$

## Serviceability

### Bolt Yielding

$$M_{cbs} := \left( \frac{(d' + 2 \cdot (s_0' + a_e)) \cdot w_{eq} \cdot t_p^2 \cdot f_y}{4 \cdot (s_0' + a_e) \cdot d'} + \frac{n \cdot B_y \cdot a_e}{s_0' + a_e} \right) \cdot (d - t_s) = 37.718 \text{ kip} \cdot \text{ft}$$

$$\phi_b \cdot M_{cbs} = 30.175 \text{ kip} \cdot \text{ft}$$

$$P_{cbs} := \frac{M_{cbs}}{35.5 \text{ in}} = 12.75 \text{ kip}$$

### Plate Yielding

$$M_{cps} := \left( \frac{((d' + s_0') \cdot w_{eq} - n \cdot d_f \cdot d') \cdot t_p^2 \cdot f_y}{2 \cdot d' \cdot s_0'} \right) \cdot (d - t_s) = 20.497 \text{ kip} \cdot \text{ft}$$

$$\phi_p \cdot M_{cps} = 18.447 \text{ kip} \cdot \text{ft}$$

$$P_{cps} := \frac{M_{cps}}{35.5 \text{ in}} = 6.929 \text{ kip}$$

### Weak Axis - Wheeler et al. Method

$$f_y := 57.1 \text{ ksi}$$

$$f_u := 82.9 \text{ ksi}$$

$$f_p := \frac{f_y + 2 \cdot f_u}{3} = 74.3 \text{ ksi}$$

$$W_p := 12 \text{ in}$$

$$b := 7 \text{ in}$$

$$d_f := \frac{13}{16} \text{ in}$$

$$D_p := 12 \text{ in}$$

$d := 4$  *in*

$$d_b := \frac{3}{4} \text{ in}$$

$$a_e := 1.25 \text{ in}$$

$$t_s := \frac{5}{16} \text{ in}$$

$$B_y := 105 \text{ ksi} \cdot \pi \cdot \left(\frac{d_b}{2}\right)^2 = 46.388 \text{ kip}$$

$$c := 1.25 \text{ in}$$

$$t_p := \frac{5}{8} \text{ in}$$

$$B_u := 125 \text{ ksi} \cdot \pi \cdot \left( \frac{d_b}{2} \right)^2 = 55.223 \text{ kip}$$

$s_0 := 2.75$  *in*

$s := 0.25$  *in*

$$n := 2$$

$$a_p := \min(a_e, (2 \cdot t_p)) = 1.25 \text{ in}$$

$$s_0' := s_0 - \frac{s}{\sqrt{2}} = 2.573 \text{ in}$$

$$d' := d + \frac{s}{\sqrt{2}} = 4.177 \text{ in}$$

$$w := b + 2 \cdot a_e + 2 \cdot c = 12 \text{ in}$$

$$v := d' + s_0' = 6.75 \text{ in}$$

$$u_{max} := s_0' + a_e = 3.823 \text{ in}$$

$$0 \leq u < u_{max}$$

$$u := 0 \text{ in}$$

$$q := a_e + s_0' - u = 3.823 \text{ in}$$

$$r := 2 \cdot q \cdot c - b \cdot d' = -19.679 \text{ in}^2$$

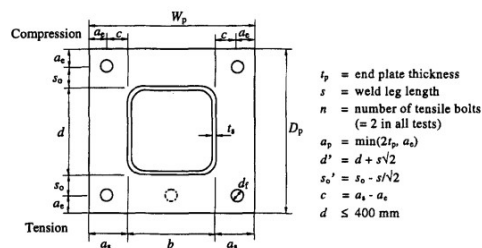


FIG. 3. End Plate Layout and Model Parameters

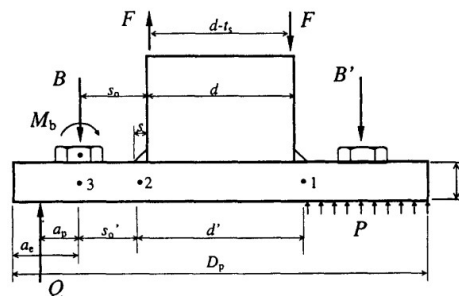
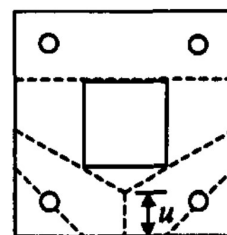


FIG. 6. Analytical Model Used in Modified Stub-Tee Analysis



$$d_{34} := \frac{a_e \cdot (2 \cdot q \cdot v - r) \cdot \left( \sqrt{\left( \frac{2}{r} \right)^2 + \left( \frac{1}{q \cdot v} \right)^2} \right)}{2} - d_f = 4.034 \text{ in}$$

$$m_p := \frac{1}{4} \cdot t_p^2 \cdot f_y = 5.576 \text{ kip}$$

Mode 1

$$M_{y1} := \left( \frac{(d' + 2 \cdot (s_0' + a_e)) \cdot w}{(s_0' + a_e) \cdot d'} \cdot m_p + \frac{n \cdot B_y \cdot a_e}{s_0' + a_e} \right) \cdot (d - t_s) = 24.5 \text{ kip} \cdot \text{ft}$$

Mode 2

$$M_{y2} := \left( \frac{2 \cdot (d' + s_0') \cdot w}{s_0' \cdot d'} - \frac{n \cdot d_f}{s_0'} \right) \cdot m_p \cdot (d - t_s) = 24.7 \text{ kip} \cdot \text{ft}$$

Mode 3

$$M_{y3} := \left( \frac{w}{d'} + \frac{\frac{v \cdot w}{b} \cdot (b^2 + 4 \cdot q^2) + (\sqrt{r^2 + 4 \cdot v^2 \cdot q^2}) \cdot d_{34} + 4 \cdot u \cdot q \cdot v}{(2 \cdot q \cdot c + s_0' \cdot b) \cdot d'} \right) \cdot d \cdot m_p = 29 \text{ kip} \cdot \text{ft}$$

Failure Mode

$$M_y := \min(M_{y1}, M_{y2}, M_{y3}) = 24.545 \text{ kip} \cdot \text{ft}$$

Design Model

$$w_{eq} := \left( \frac{4 \cdot M_{y3}}{d \cdot t_p^2 \cdot f_p} + \frac{n \cdot d_f}{s_0'} \right) \cdot \left( \frac{d' \cdot s_0'}{2 \cdot (d' + s_0')} \right) = 10.052 \text{ in}$$

$$\phi_b := 0.8$$

$$\phi_p := 0.9$$

## Bolt Failure

$$f_{yb} := \frac{B_y}{\pi \cdot \left(\frac{d_b}{2}\right)^2} = 0.724 \frac{kN}{mm^2}$$

$$M_{cb} := \left( \frac{4 \cdot n \cdot \left( B_u \cdot a_p + \frac{\pi \cdot d_b^3 \cdot f_{yb}}{32} \right) \cdot d' + w_{eq} \cdot (d' + 2 \cdot (s_0' + a_p)) \cdot t_p^2 \cdot f_p}{4 \cdot (a_p + s_0') \cdot d'} \right) \cdot (d - t_s) = 28.389 \text{ kip} \cdot ft$$

$$\phi_b \cdot M_{cb} = 22.711 \text{ kip} \cdot ft$$

$$P_{cb} := \frac{M_{cb}}{35.5 \text{ in}} = 9.596 \text{ kip}$$

$$t_{bu} := \sqrt{\frac{4 \cdot \left( \frac{M_{cb} \cdot (a_p + s_0')}{\phi_b \cdot (d - t_s)} - n \cdot \left( B_u \cdot a_p + \frac{\pi \cdot d_b^3 \cdot f_{yb}}{32} \right) \right) \cdot d'}{w_{eq} \cdot (d' + 2 \cdot (s_0' + a_p)) \cdot f_p}} = 0.747 \text{ in}$$

$$t_{pmax} := \sqrt{\frac{4 \cdot n \cdot B_u \cdot s_0'}{w_{eq} \cdot f_p}} = 1.234 \text{ in}$$

## Plate Failure

$$M_{cp} := \left( \frac{t_p^2 \cdot f_p \cdot (w_{eq} \cdot (d' + 2 \cdot s_0') + (w_{eq} - n \cdot d_f) \cdot d') + n \cdot \frac{\pi \cdot d_b^3 \cdot f_{yb}}{8} \cdot d'}{4 \cdot d' \cdot s_0'} \right) \cdot (d - t_s) = 27.781 \text{ kip} \cdot ft$$

$$\phi_p \cdot M_{cp} = 25.003 \text{ kip} \cdot ft$$

$$P_{cp} := \frac{M_{cp}}{35.5 \text{ in}} = 9.391 \text{ kip}$$

$$t_{pu} := 2 \cdot \sqrt{\frac{\left( \frac{M_{cb} \cdot s_0'}{\phi_p \cdot (d + t_s)} - n \cdot \frac{\pi \cdot d_b^3 \cdot f_{yb}}{32} \right) \cdot d'}{(w_{eq} \cdot (d' + 2 \cdot s_0') + (w_{eq} - n \cdot d_f) \cdot d') \cdot f_p}} = 0.615 \text{ in}$$



## Serviceability

### Bolt Yielding

$$M_{cbs} := \left( \frac{(d' + 2 \cdot (s_0' + a_e)) \cdot w_{eq} \cdot t_p^2 \cdot f_y}{4 \cdot (s_0' + a_e) \cdot d'} + \frac{n \cdot B_y \cdot a_e}{s_0' + a_e} \right) \cdot (d - t_s) = 22.073 \text{ kip} \cdot \text{ft}$$

$$\phi_b \cdot M_{cbs} = 17.659 \text{ kip} \cdot \text{ft}$$

$$P_{cbs} := \frac{M_{cbs}}{35.5 \text{ in}} = 7.461 \text{ kip}$$

### Plate Yielding

$$M_{cps} := \left( \frac{((d' + s_0') \cdot w_{eq} - n \cdot d_f \cdot d') \cdot t_p^2 \cdot f_y}{2 \cdot d' \cdot s_0'} \right) \cdot (d - t_s) = 19.47 \text{ kip} \cdot \text{ft}$$

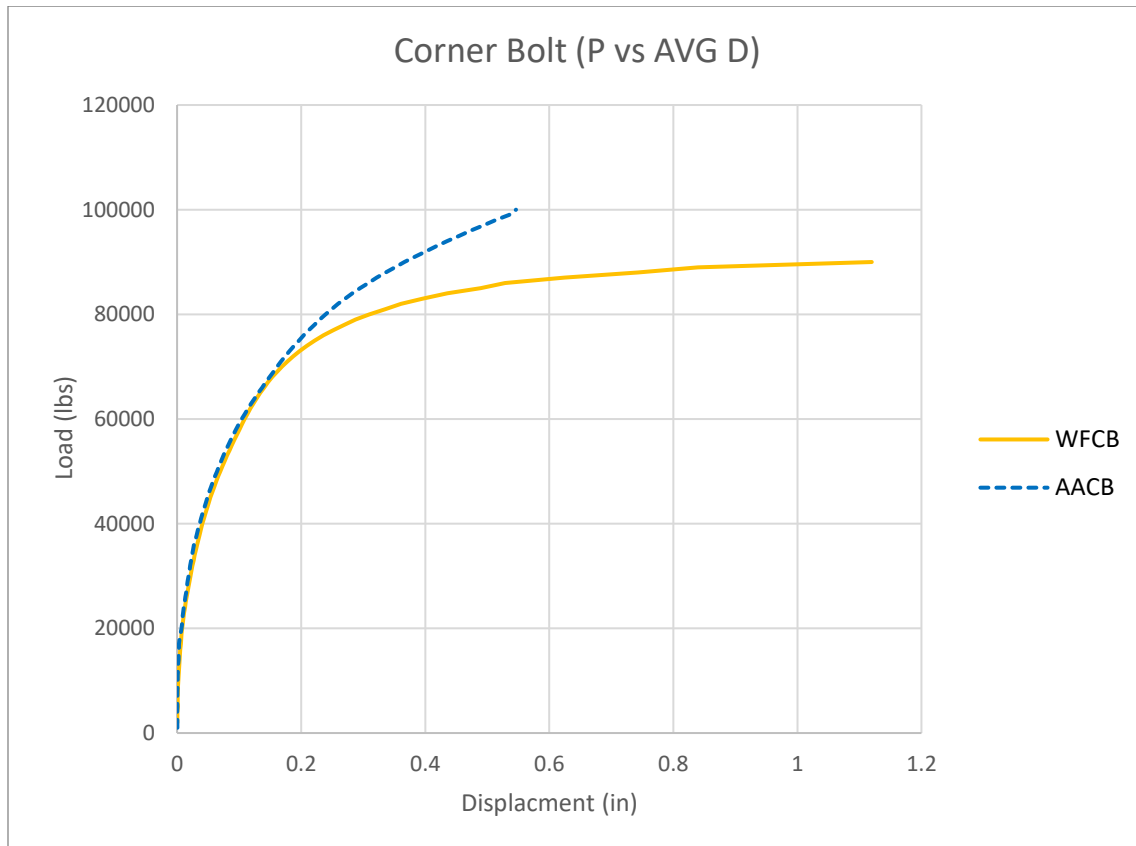
$$\phi_p \cdot M_{cps} = 17.523 \text{ kip} \cdot \text{ft}$$

$$P_{cps} := \frac{M_{cps}}{35.5 \text{ in}} = 6.581 \text{ kip}$$

## **Appendix B – Axial Tests Plots**

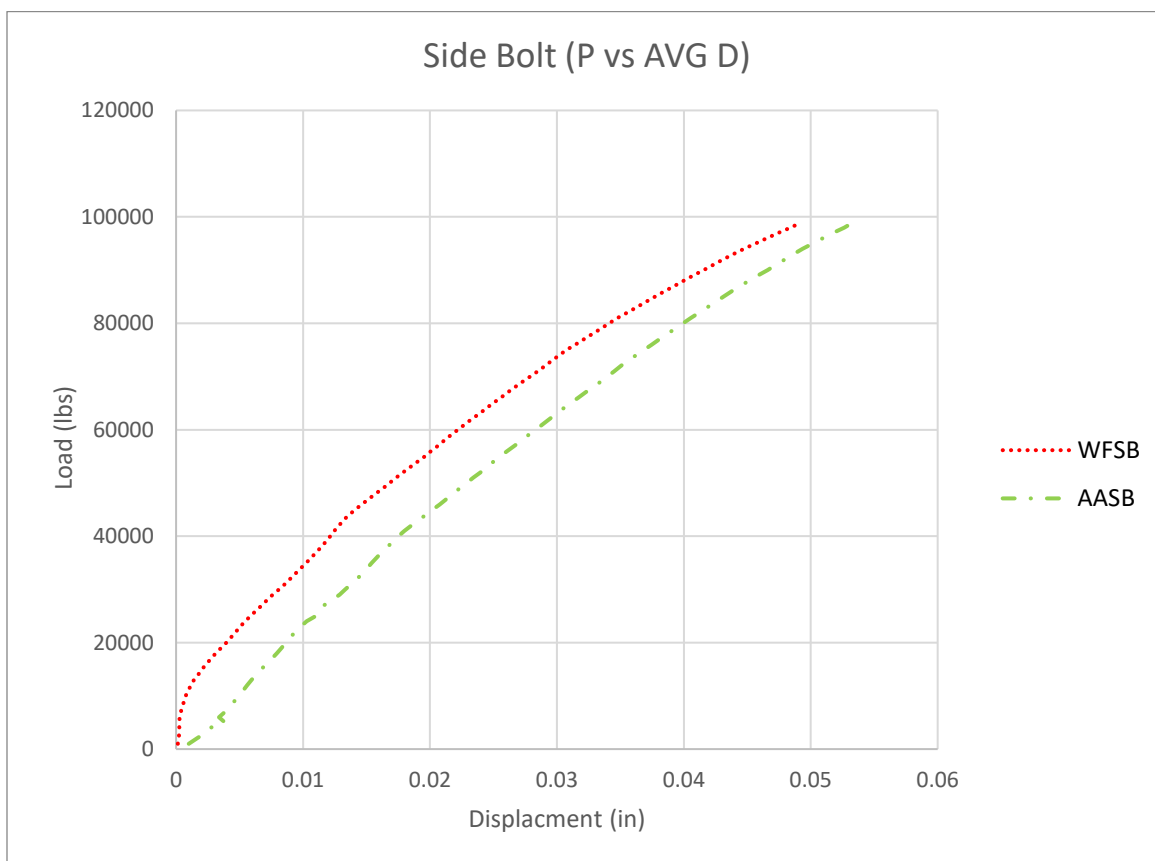
**Load versus Displacement Plots****Figure B1**

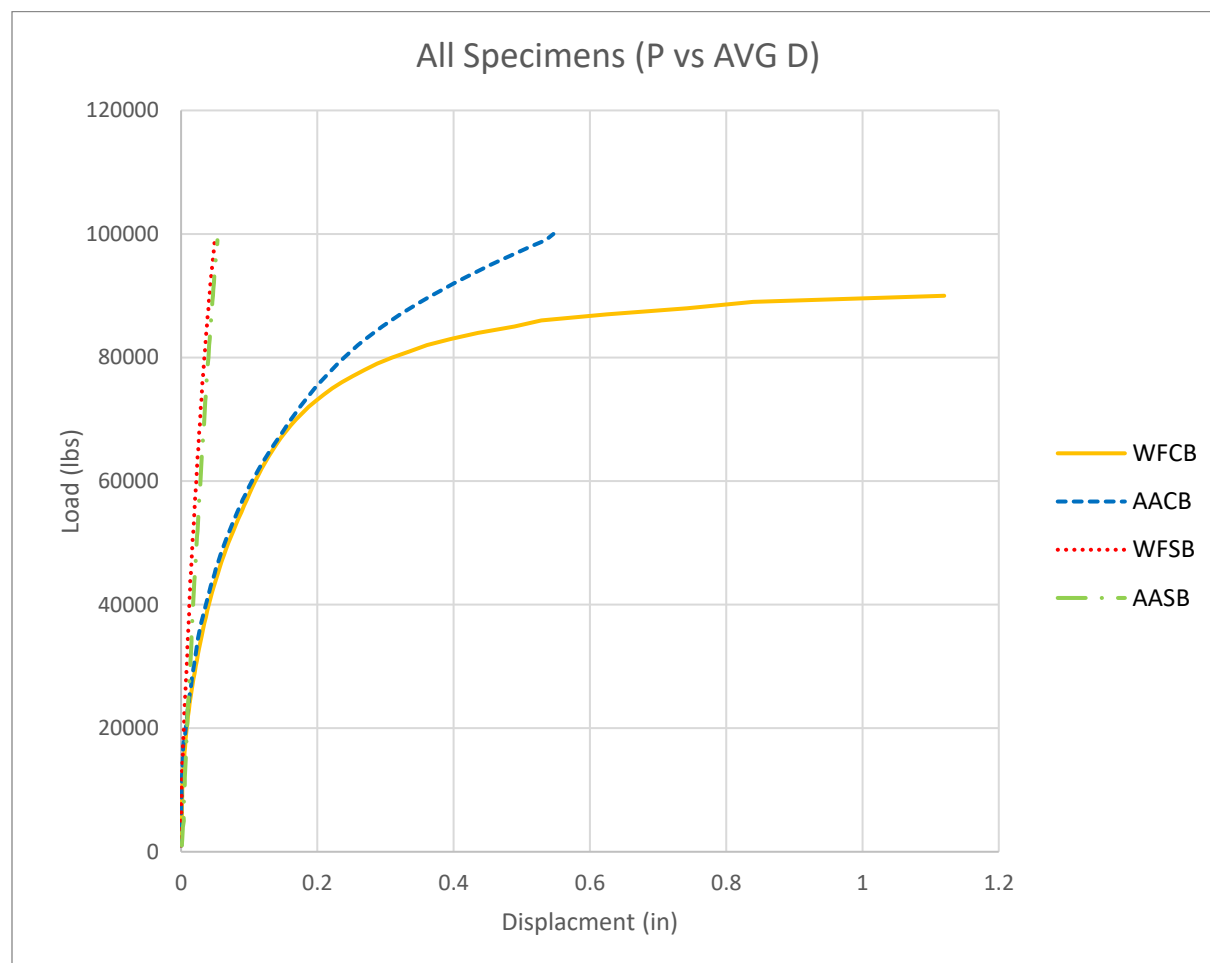
*Load versus Average Displacement for Corner Bolt Specimens*



**Figure B2**

*Load versus Average Displacement for Side Bolt Specimens*



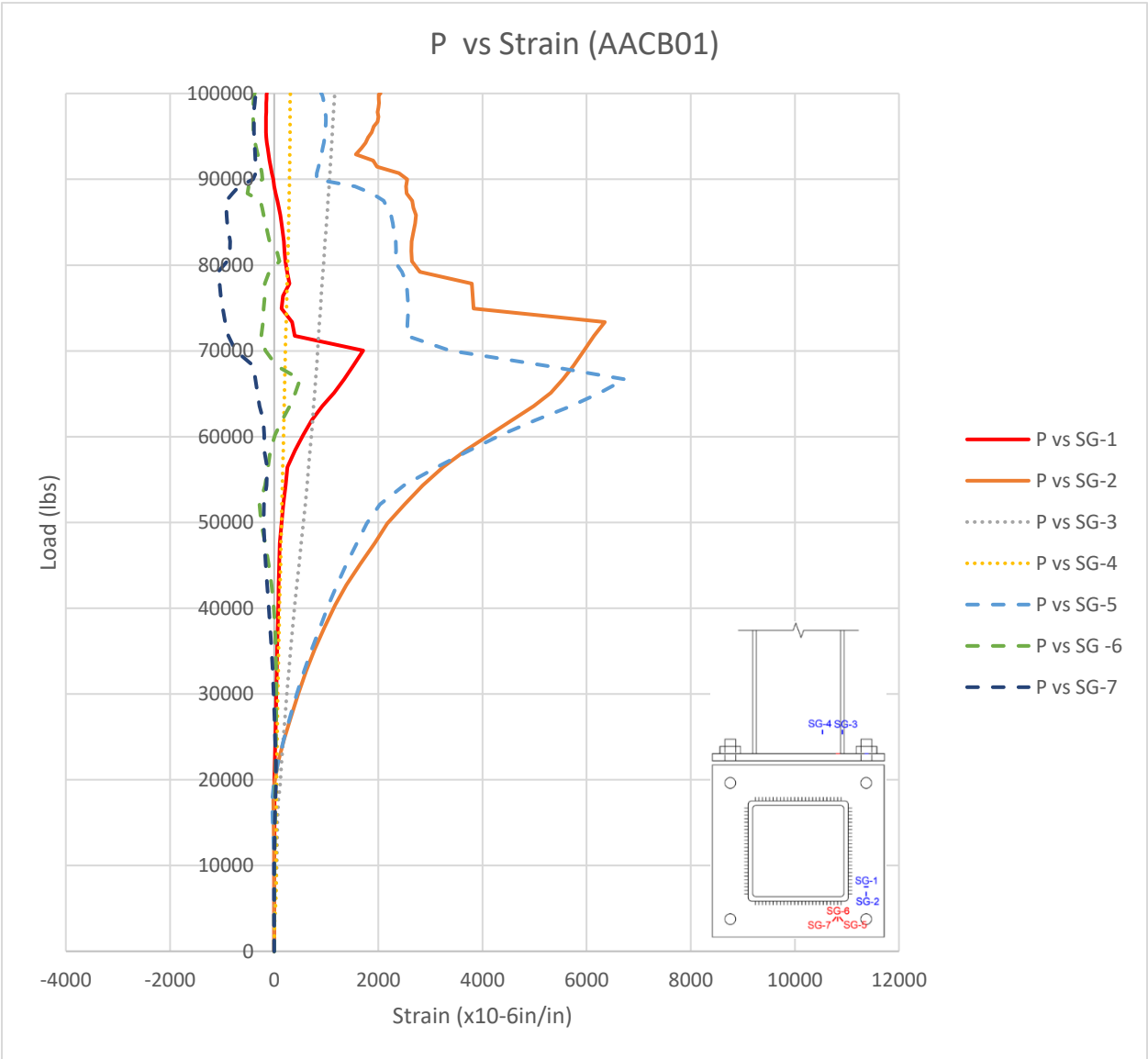
**Figure B3***Load versus Average Displacement for All Specimens*

Load versus Strain Plots

All-Around Weld and Corner Bolt Specimens (AACB)

Figure B4

Load versus Strain for Specimen AACB01



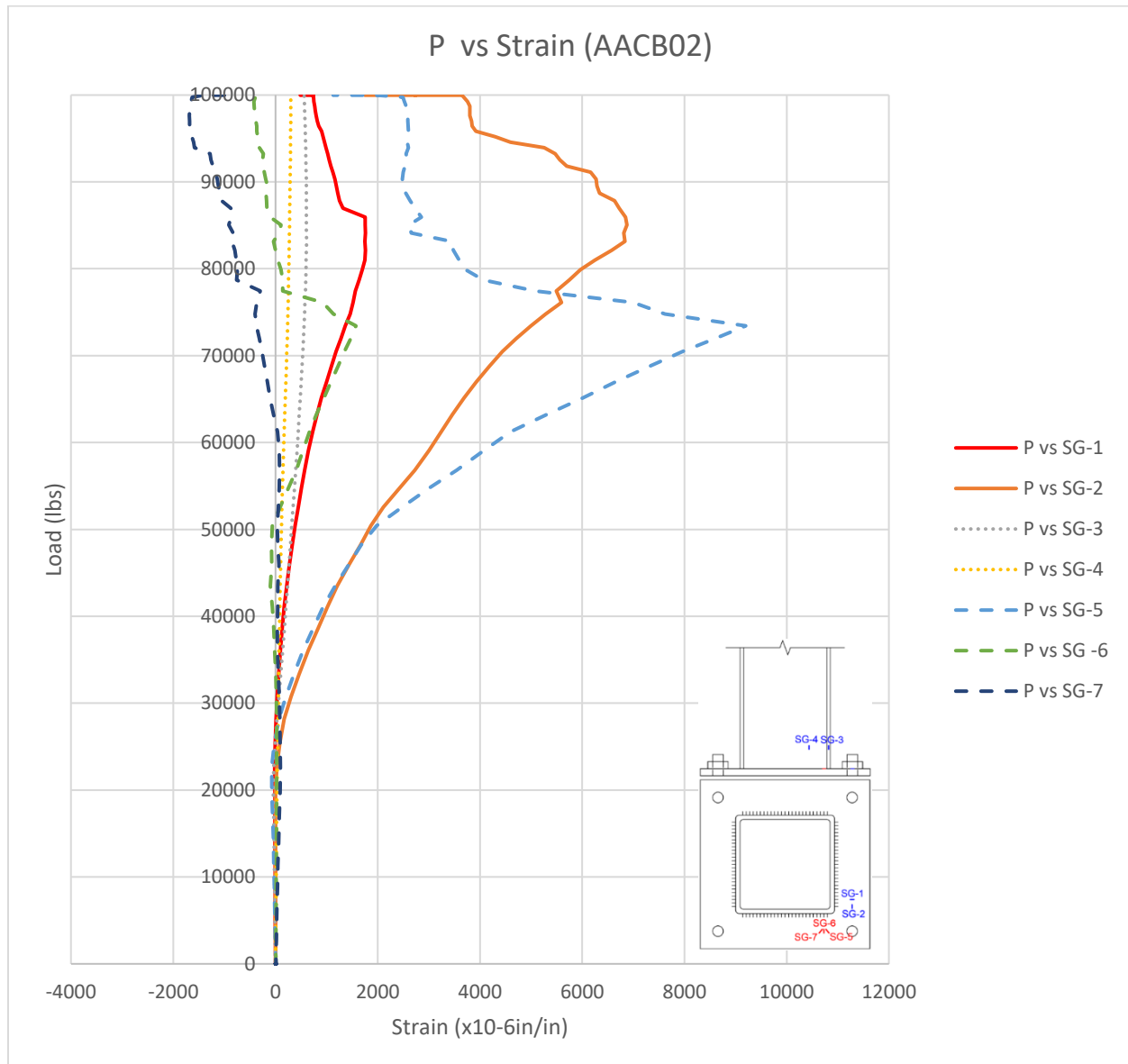
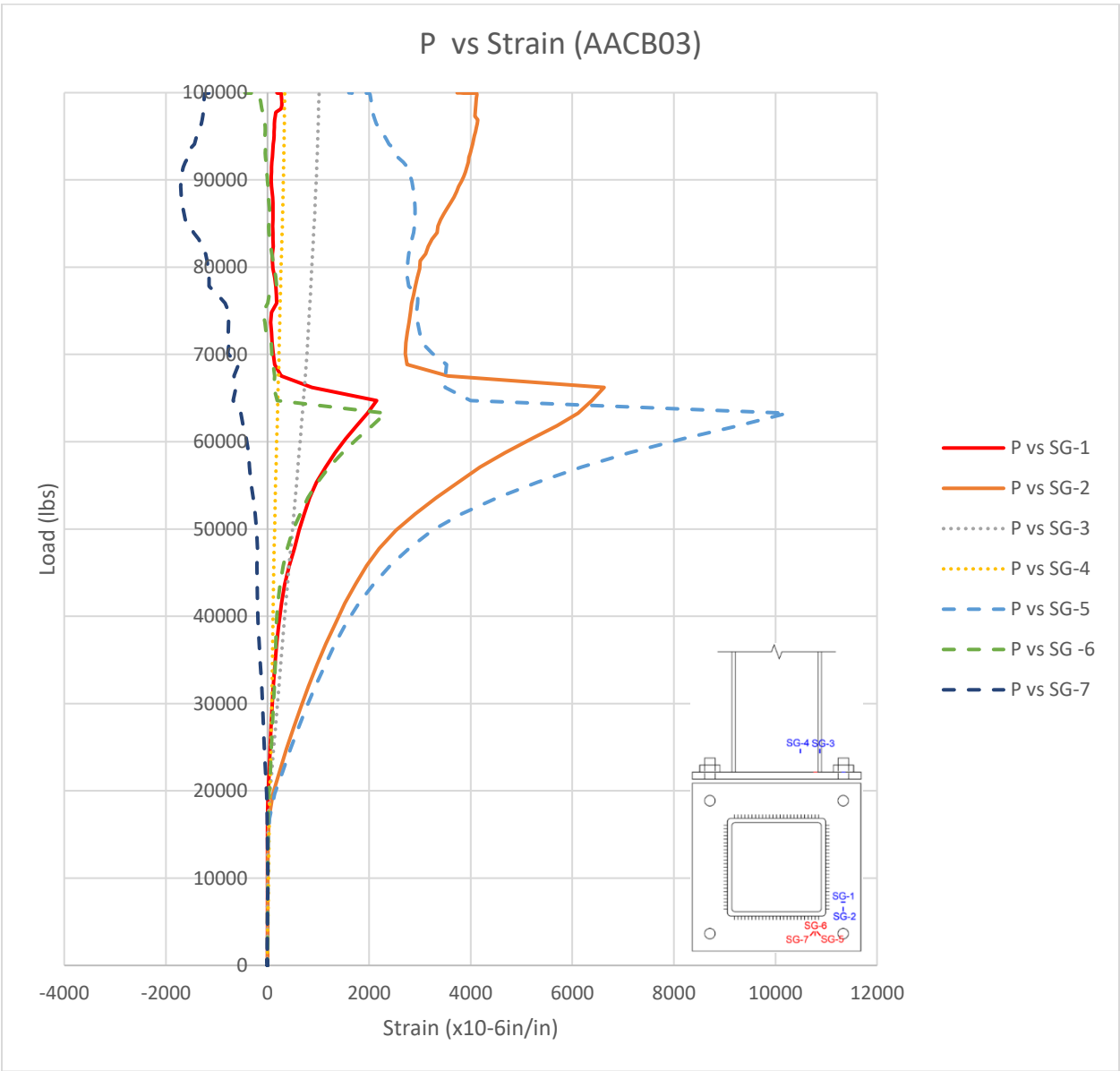
**Figure B5***Load versus Strain for Specimen AACB02*

Figure B6

Load versus Strain for Specimen AACB03





Workable-Flat Welds and Corner Bolt Specimens (WFCB)

Figure B7

Load versus Strain for Specimen WFCB01

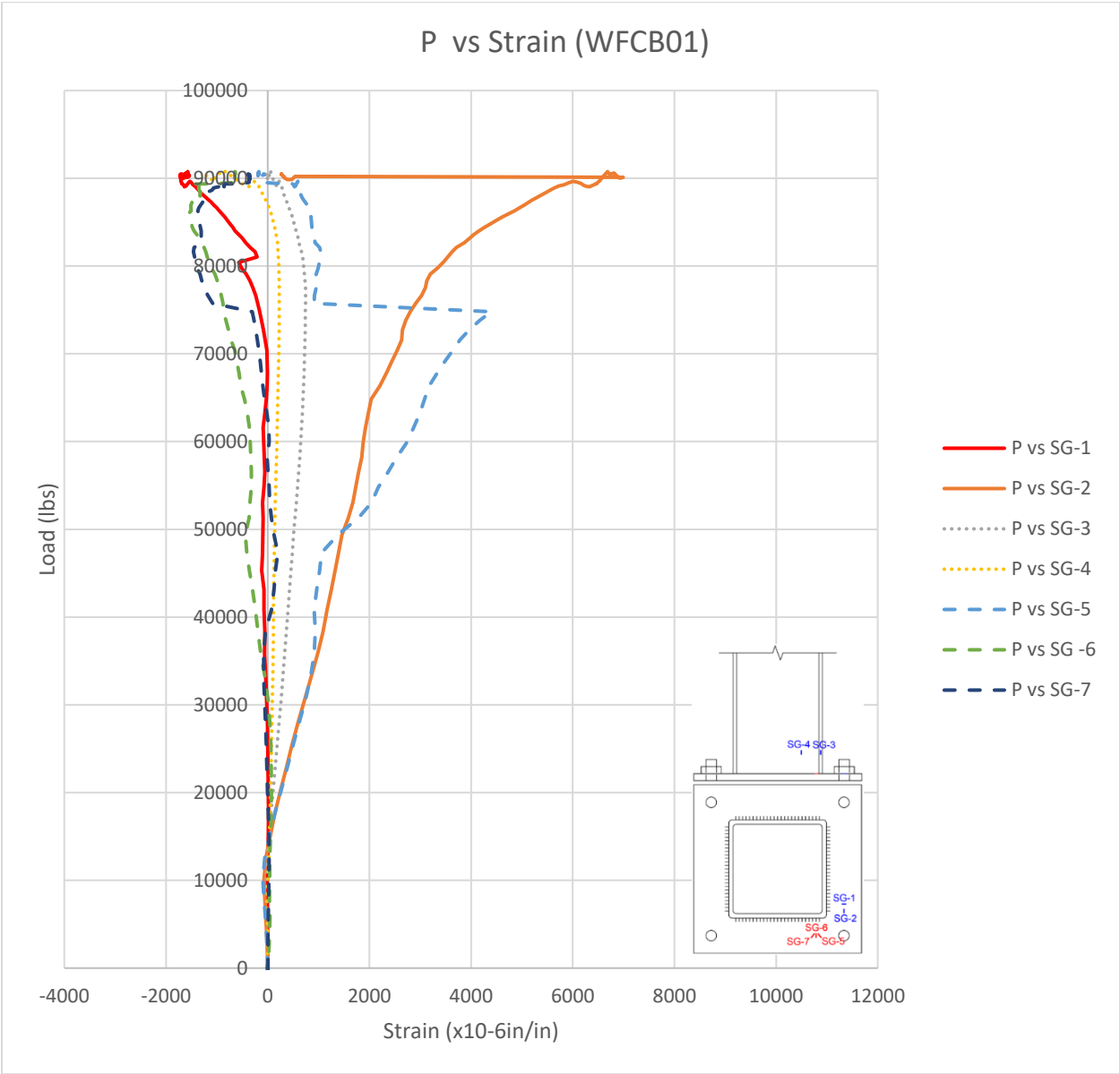


Figure B8

Load versus Strain for Specimen WFCB02

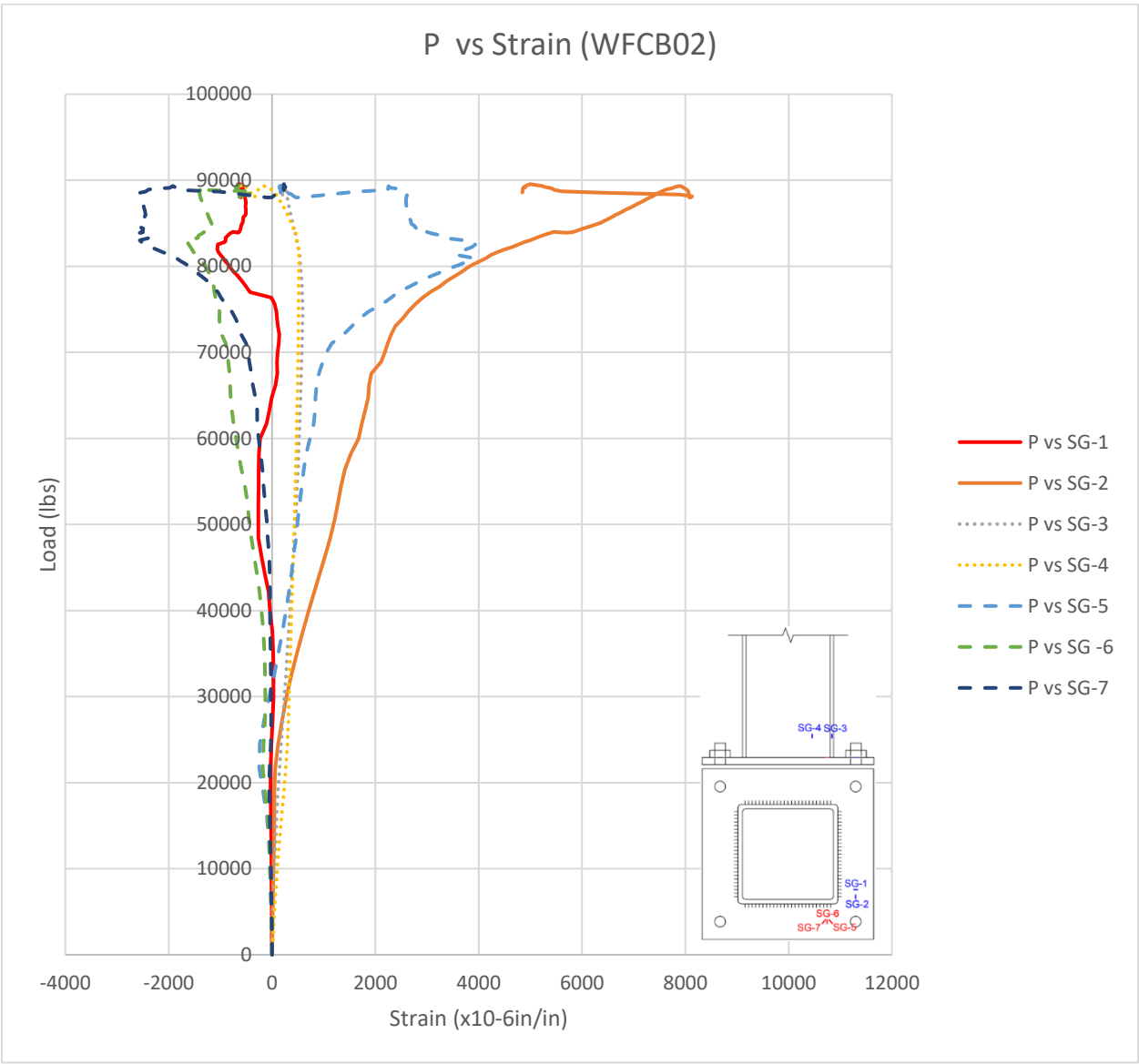
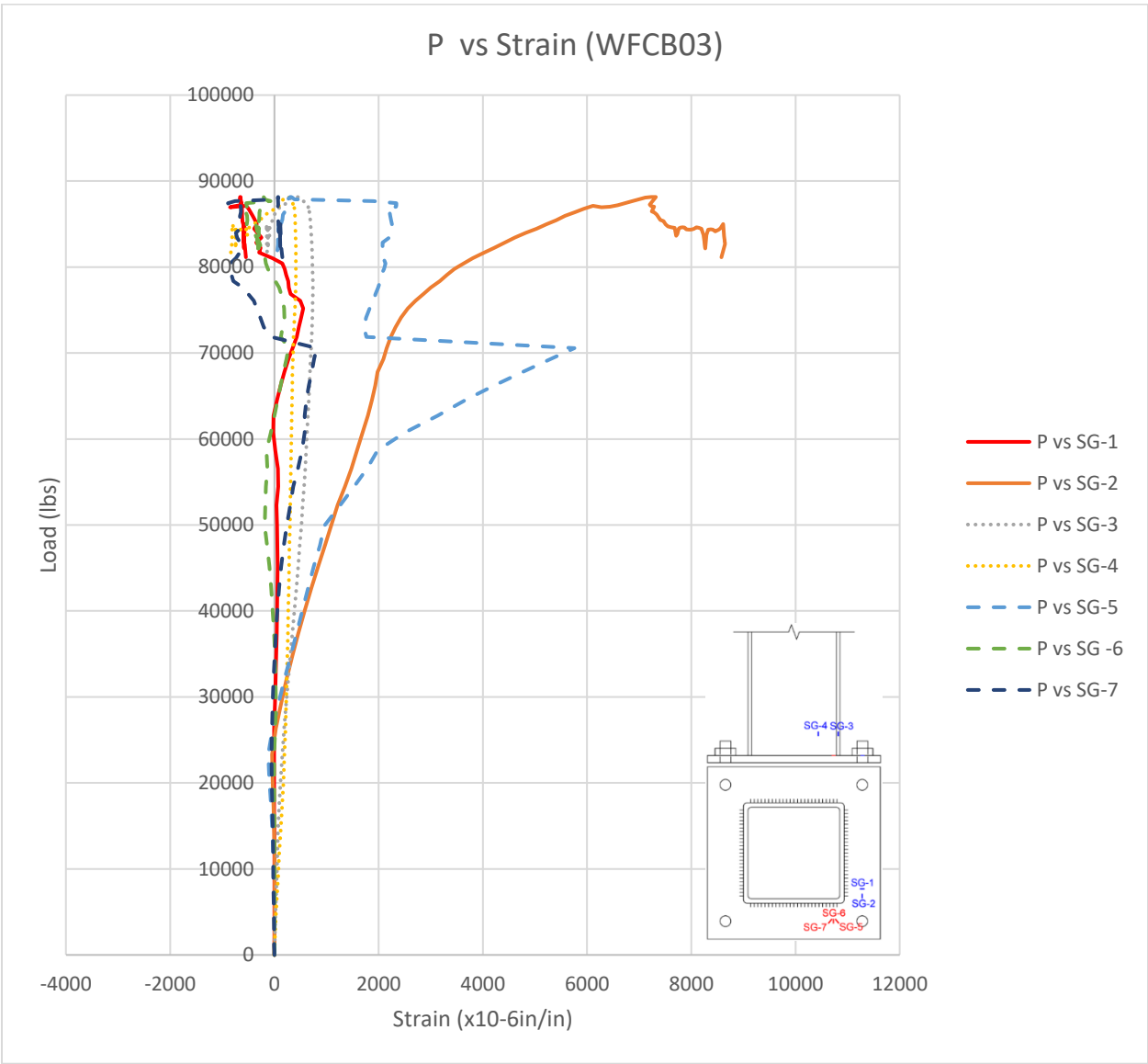


Figure B9

Load versus Strain for Specimen WFCB03



### Figure B10

**P vs Strain (AASB01)**

Load (lbs)

Strain (x10-6in/in)

Legend:

- P vs SG-1
- P vs SG-2
- P vs SG-3
- P vs SG-4
- P vs SG-5
- P vs SG-6
- P vs SG-7

Schematic of the test specimen showing strain gauges SG-1 through SG-8.

Figure B11

Load versus Strain for Specimen AASB02

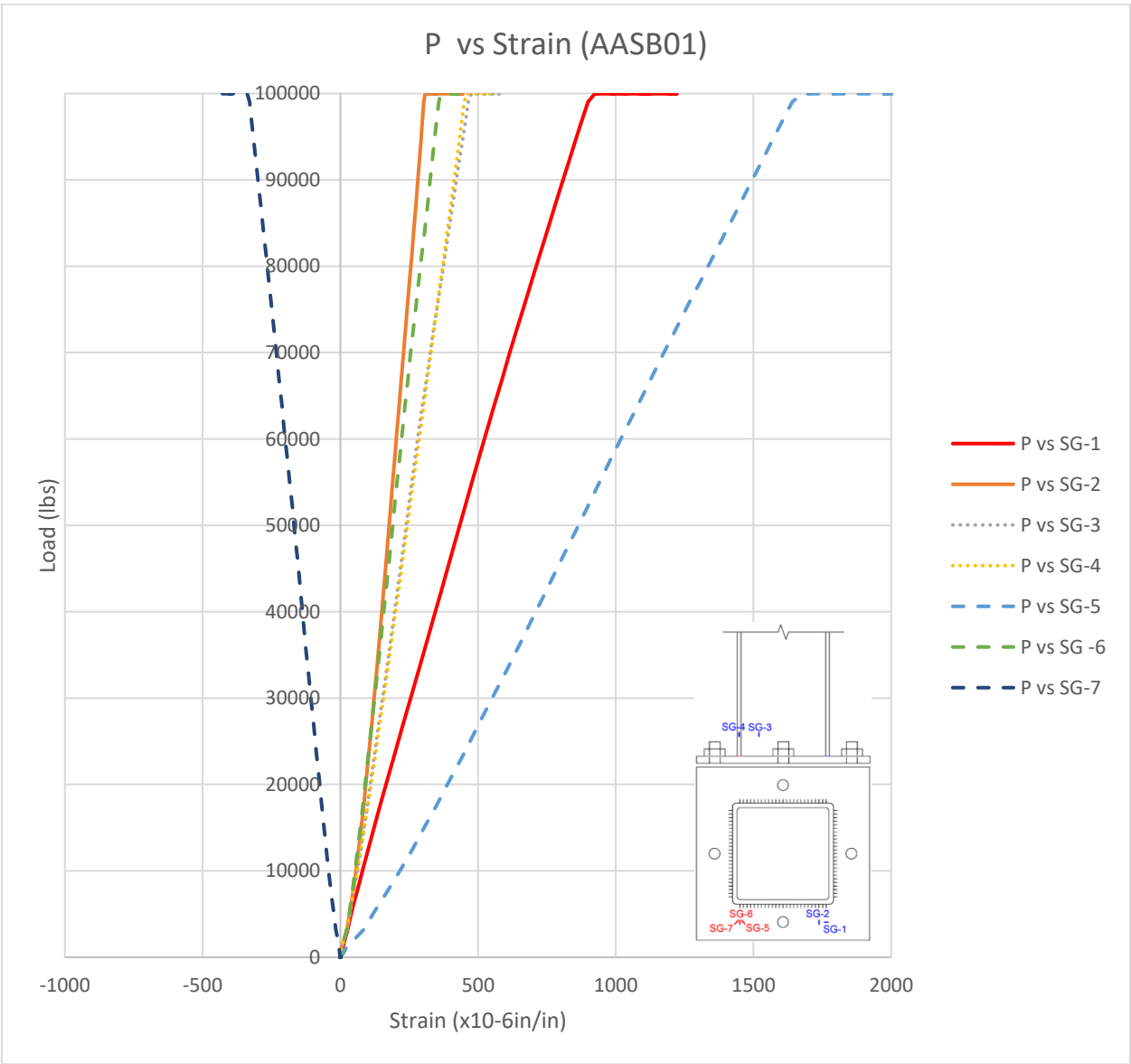


Figure B12

Load versus Strain for Specimen AASB03

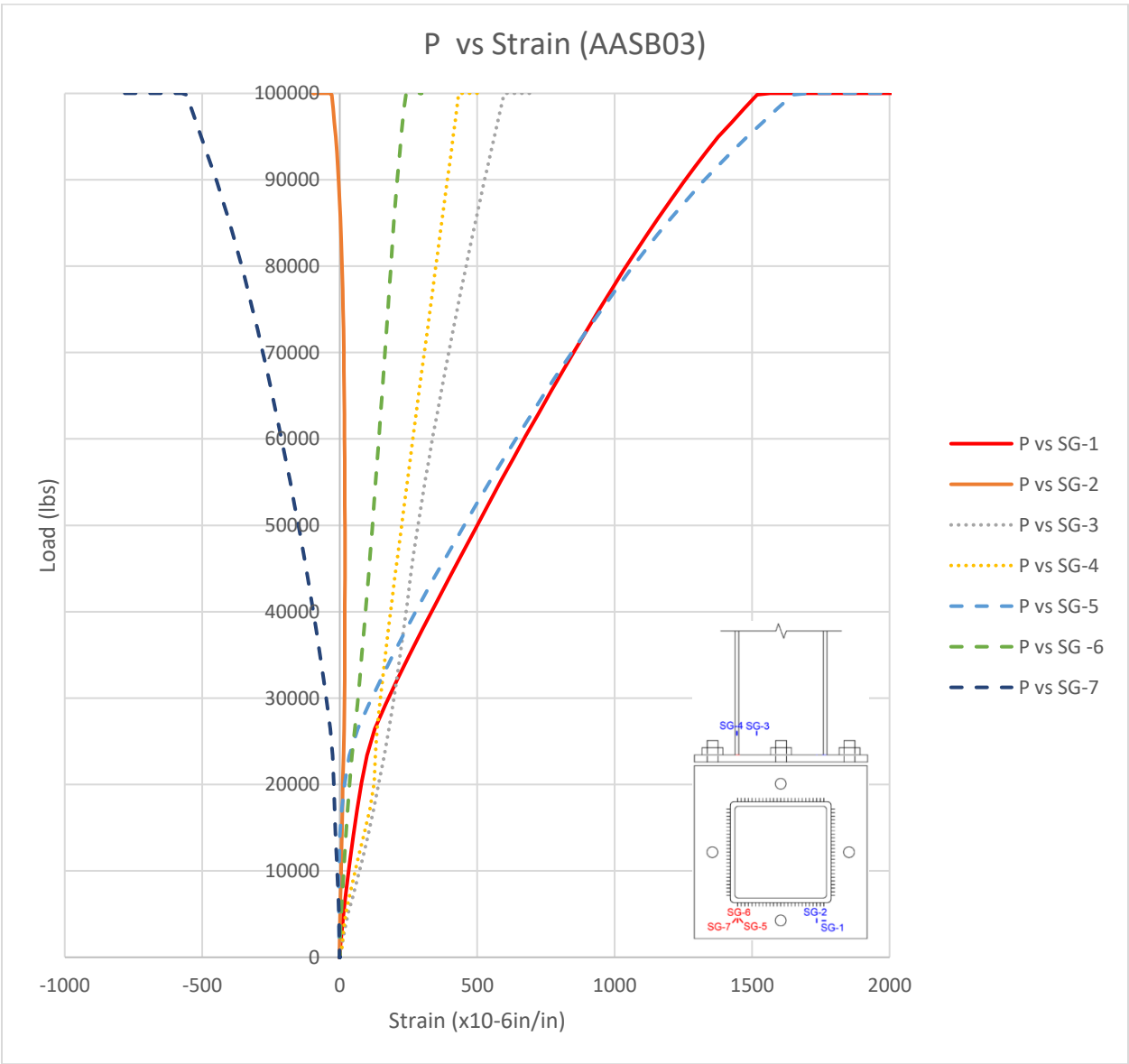
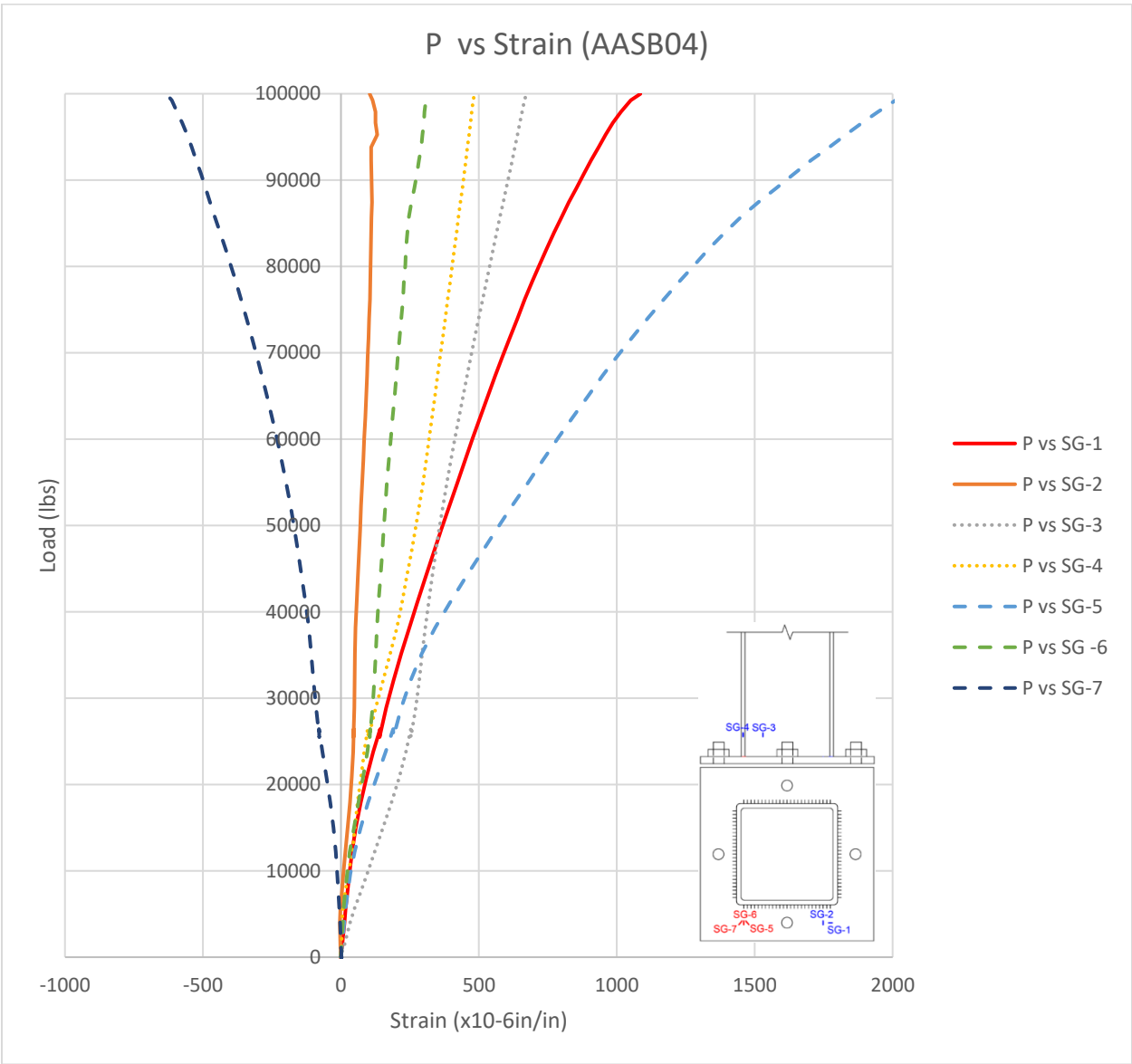


Figure B13

Load versus Strain for Specimen AASB04



Workable-Flat Welds and Side Bolt Specimens (WFSB)

Figure B14

Load versus Strain for Specimen WFSB01

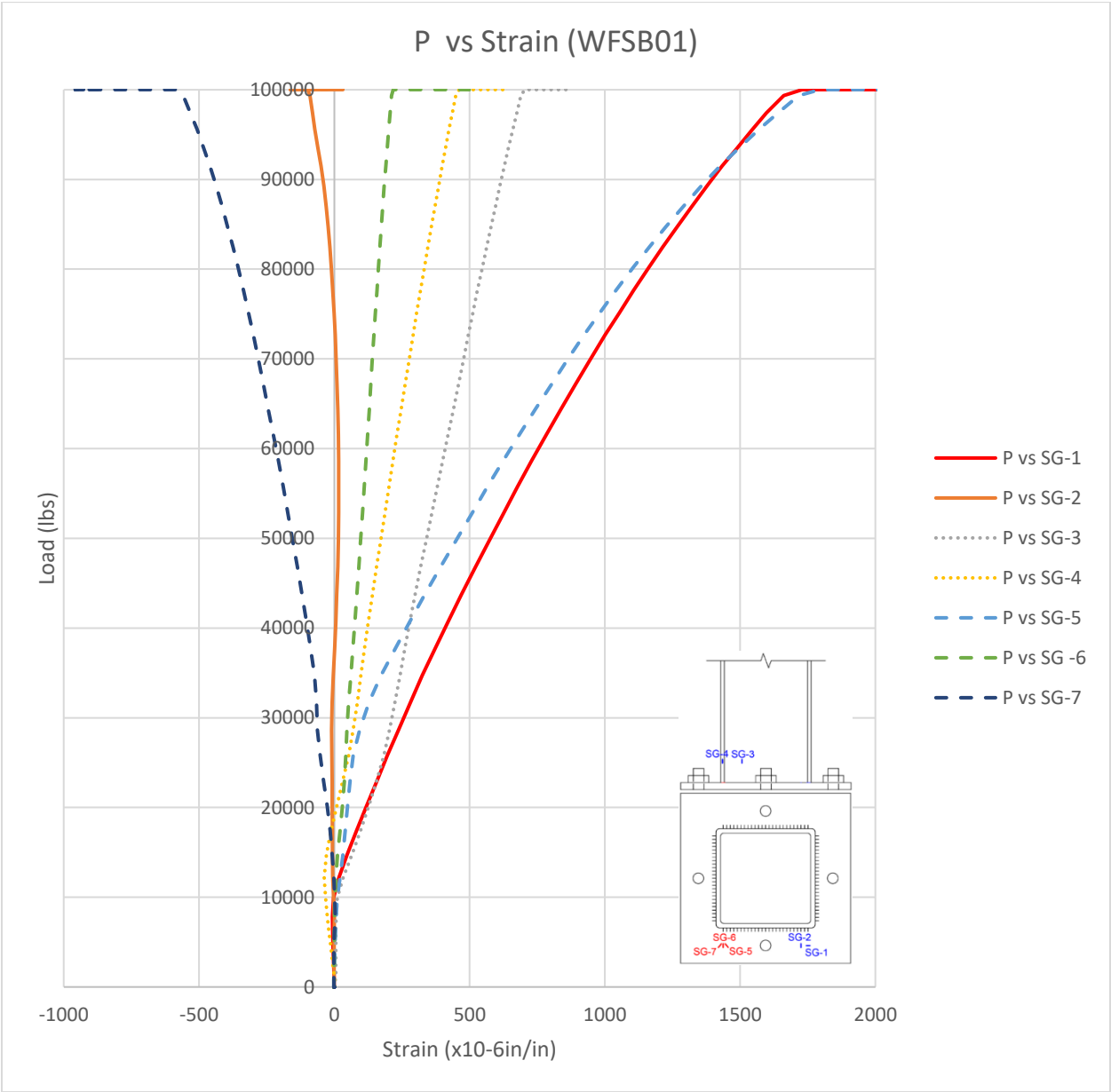
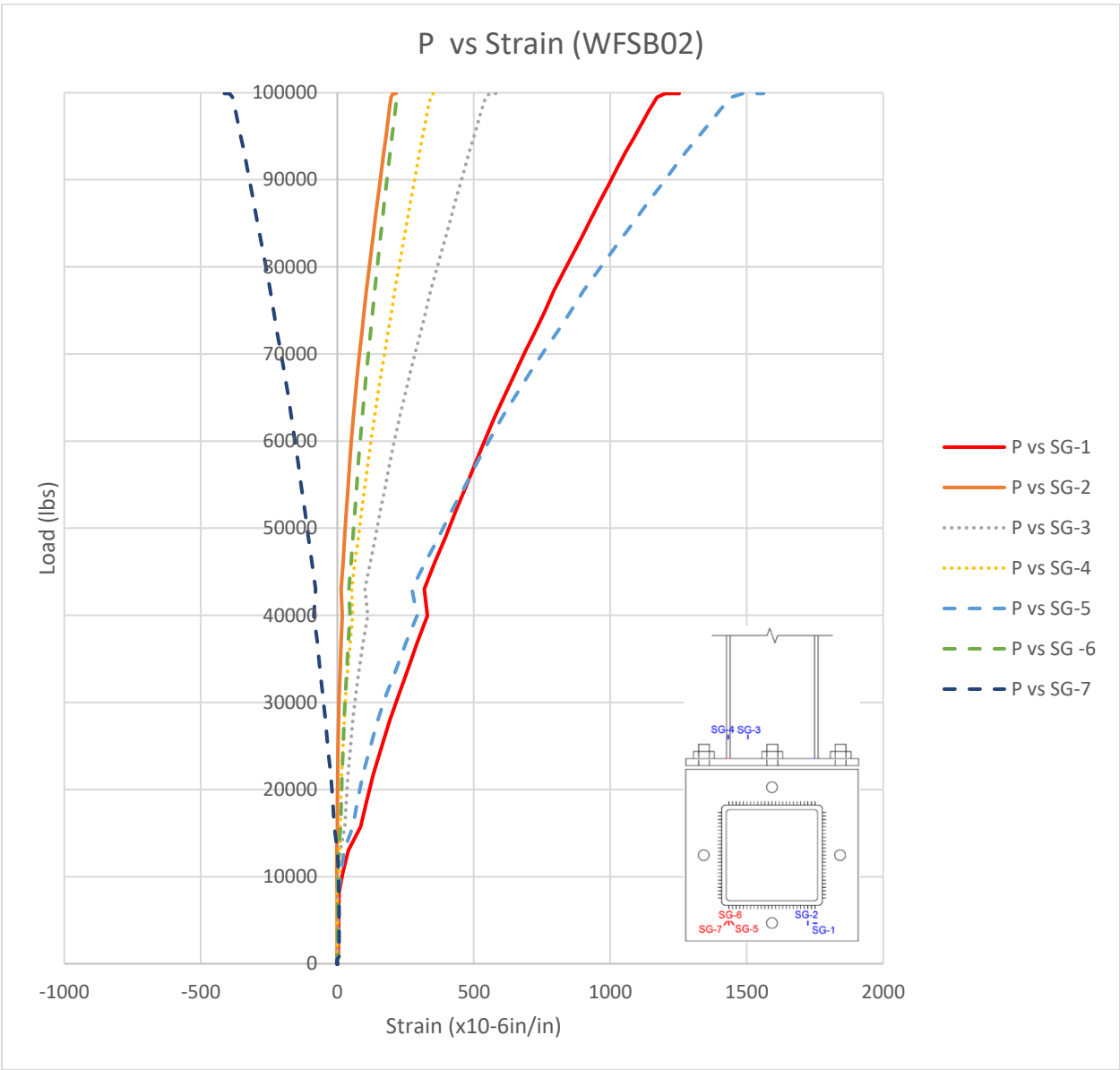




Figure B15

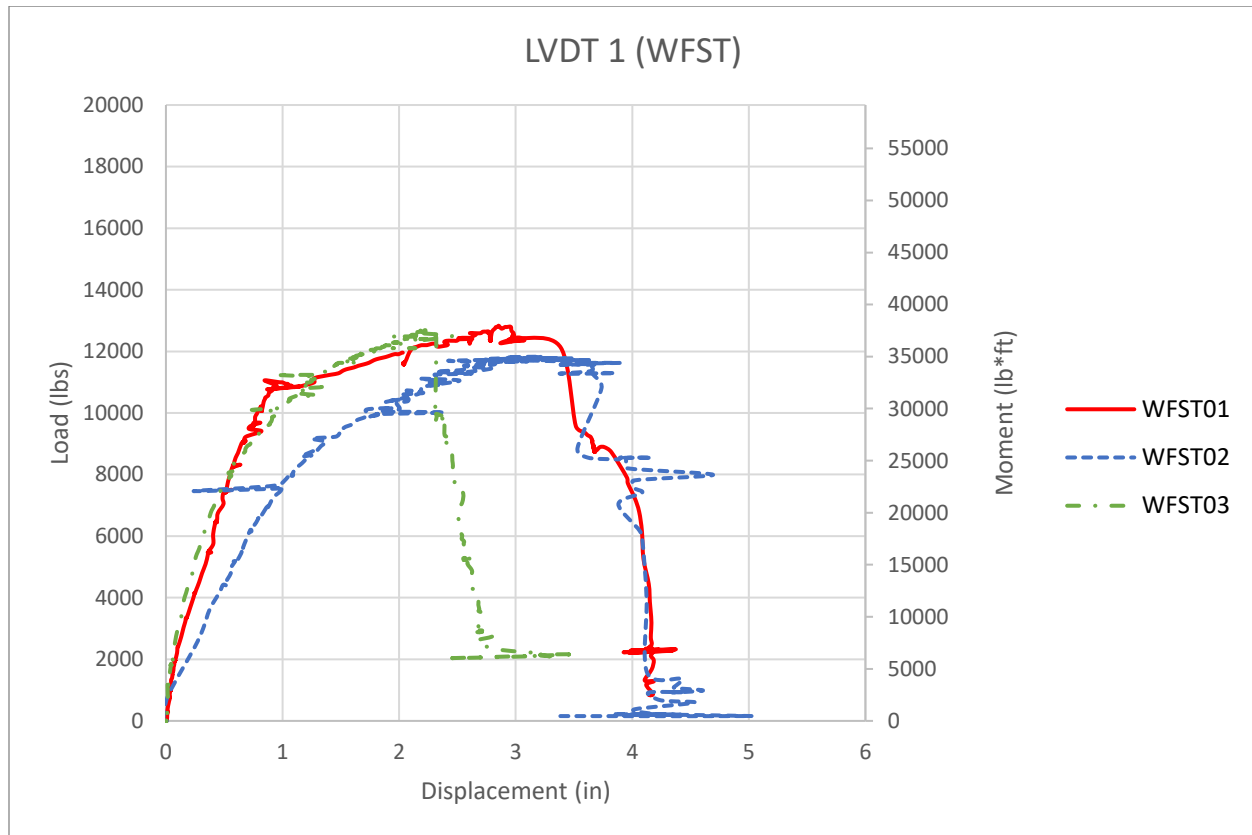
Load versus Strain for Specimen WFSB02



## **Appendix C – Flexural Tests Plots**

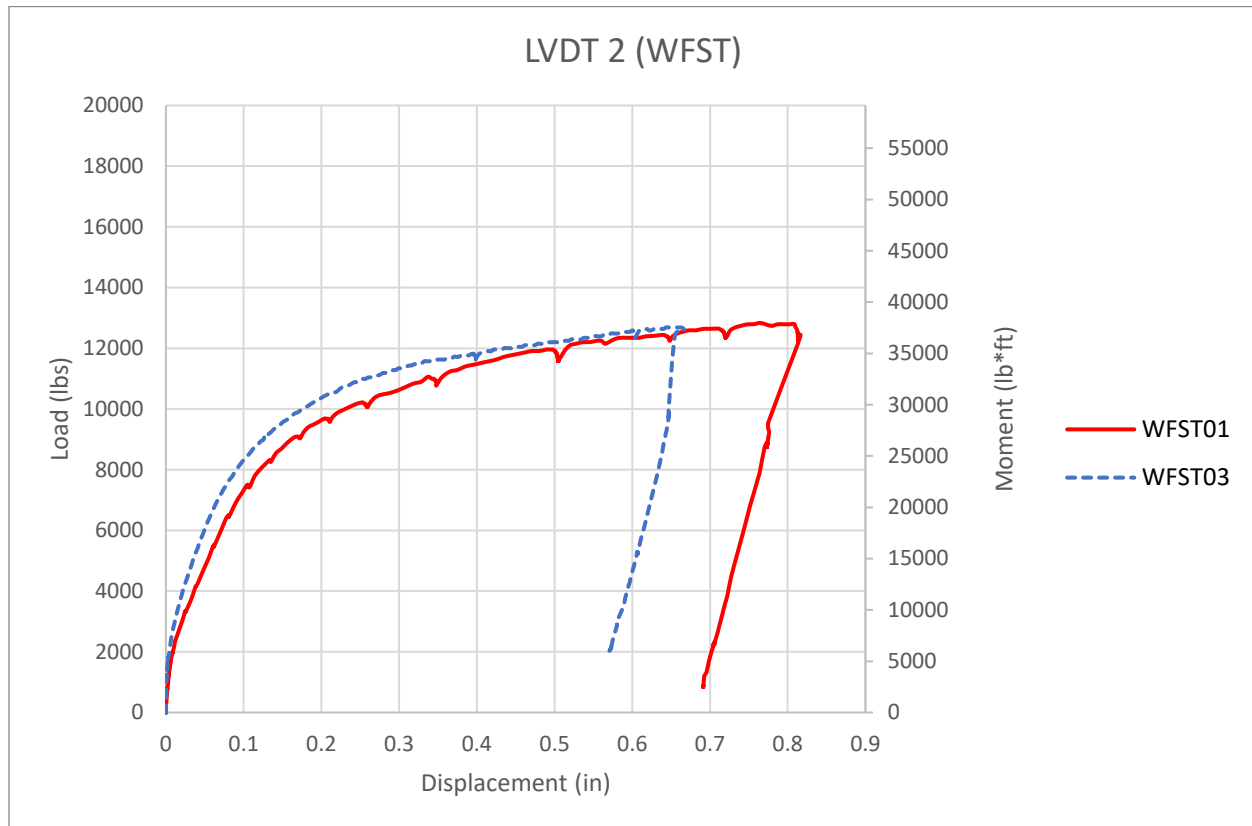
**Load versus Displacement Plots****Figure C1**

*Raw Load versus LVDT 1 Displacement for Specimen WFST Specimens*



**Figure C2**

*Raw Load versus LVDT 2 Displacement for Specimens WFST Specimens*



**Figure C3**

*Raw Load versus LVDT 1 Displacement for Specimens AAST Specimens*

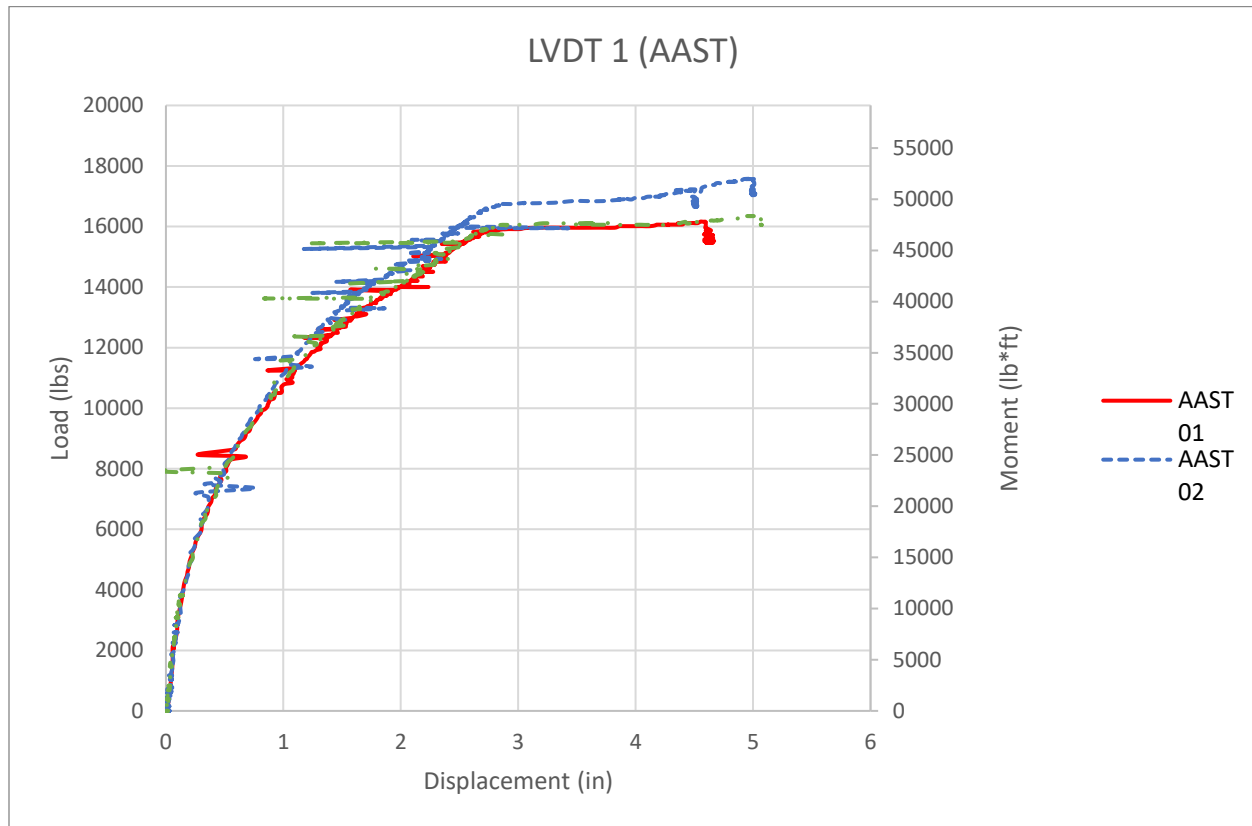
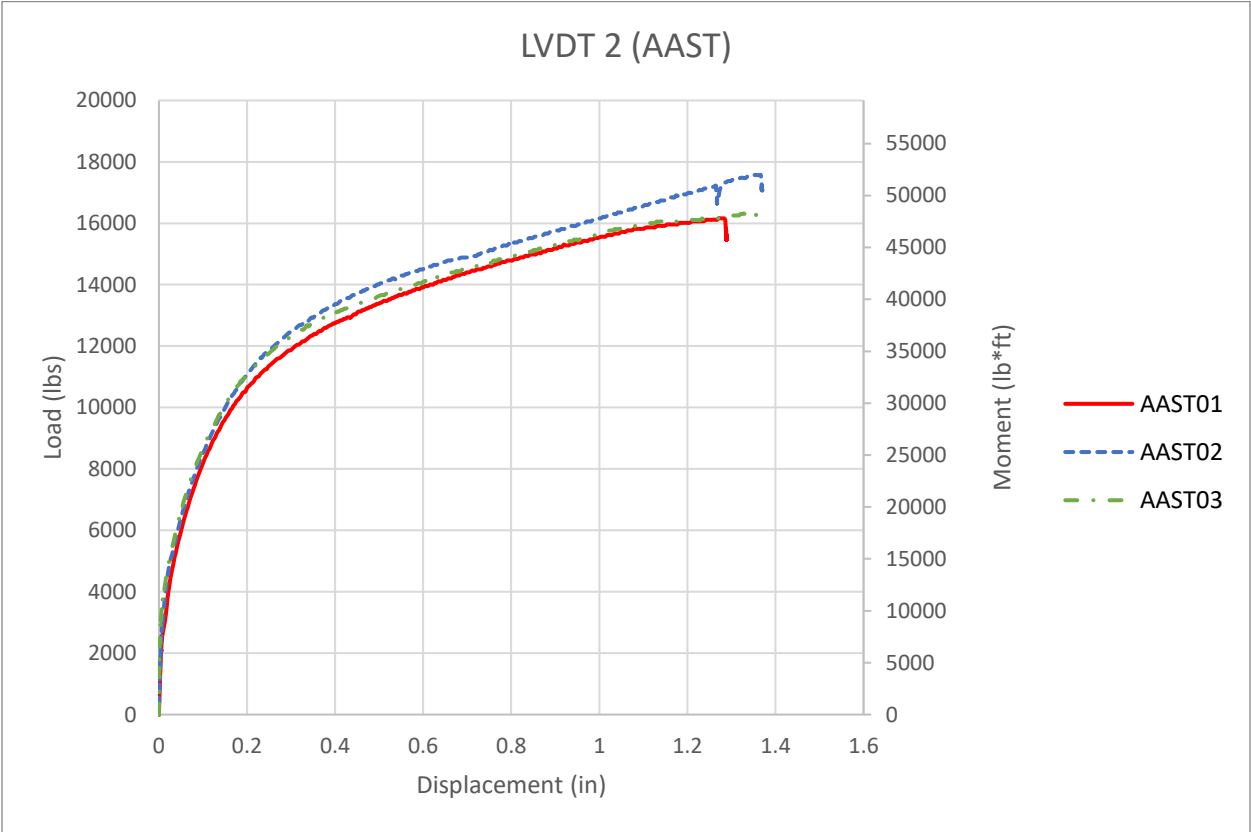


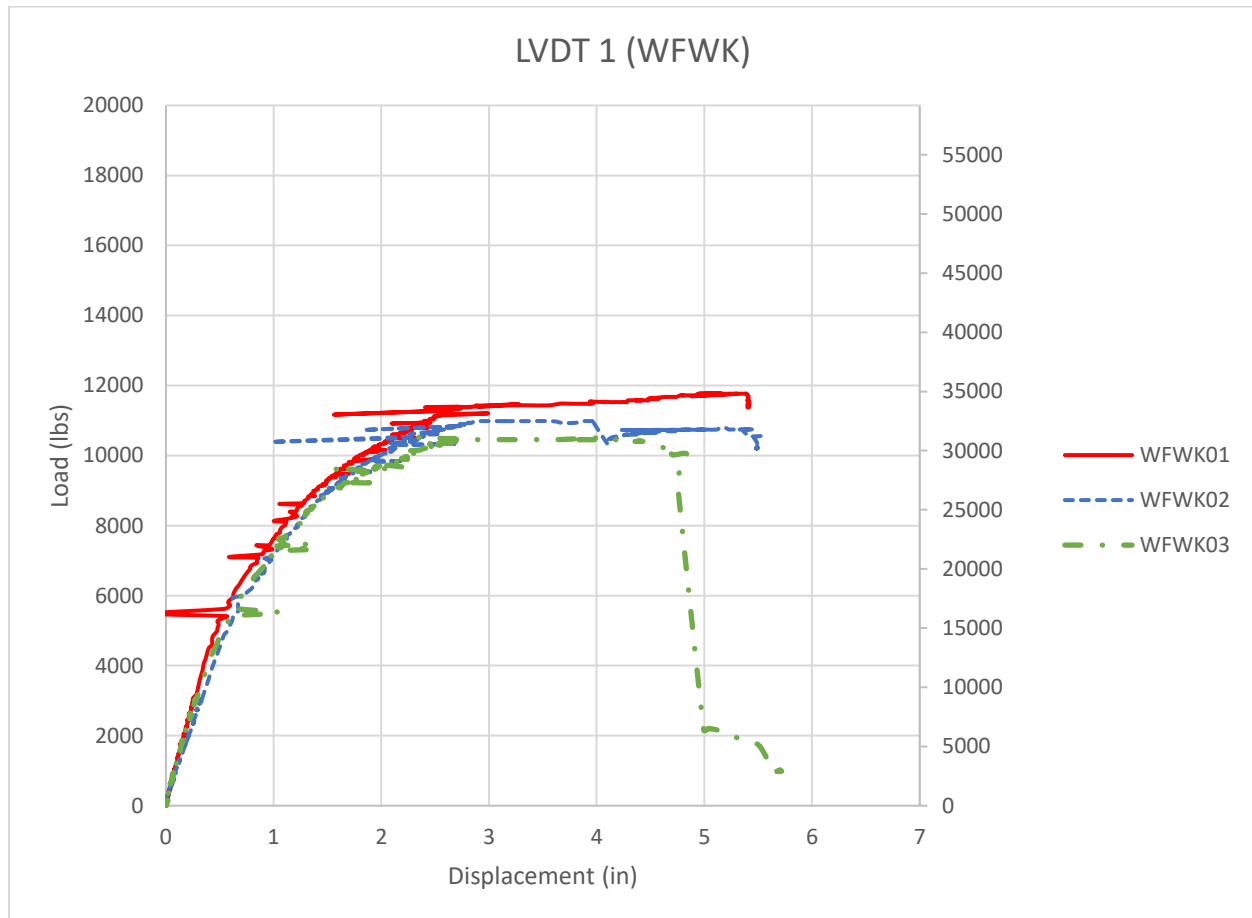
Figure C4

Raw Load versus LVDT 2 Displacement for Specimens AAST Specimens



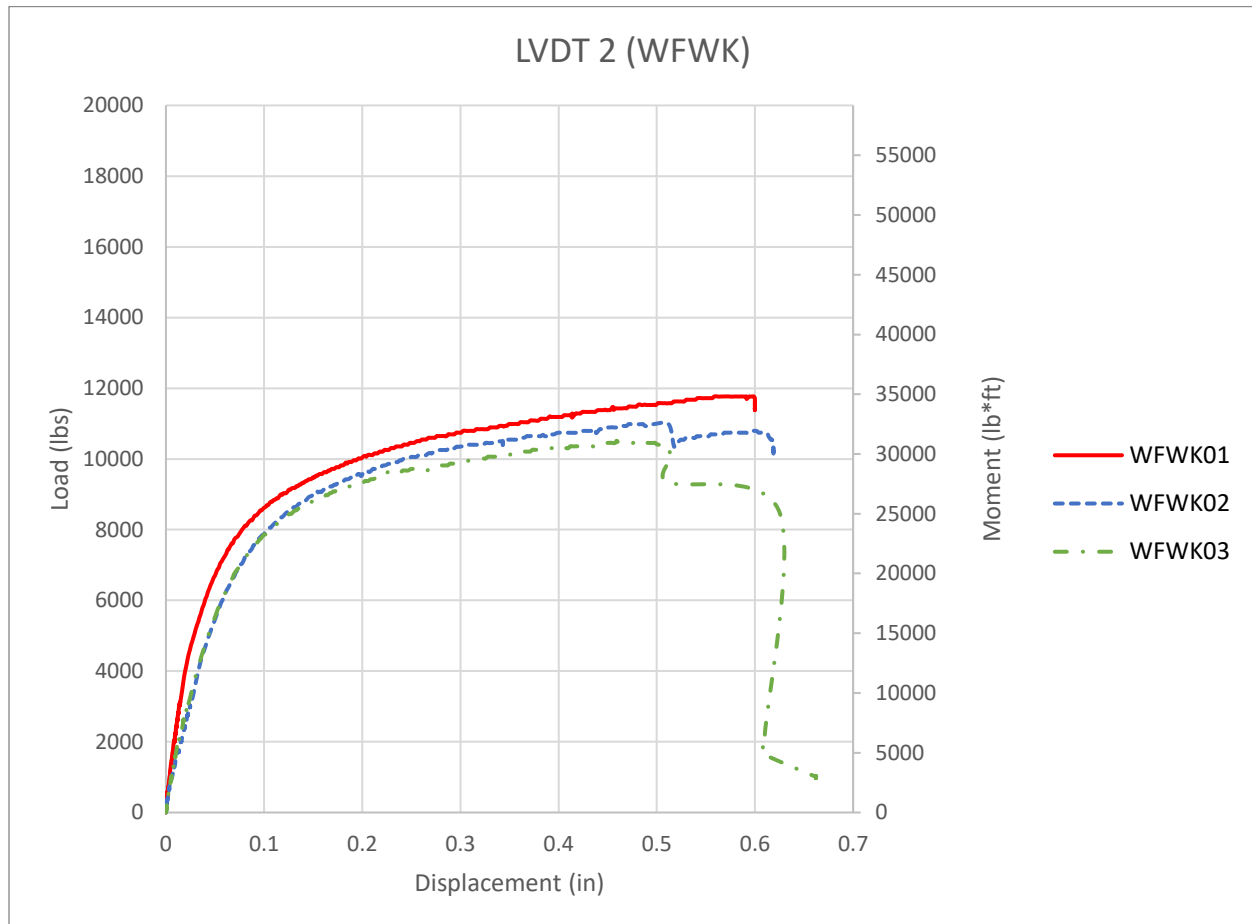
**Figure C5**

*Raw Load versus LVDT 1 Displacement for Specimens WFWK Specimens*



**Figure C6**

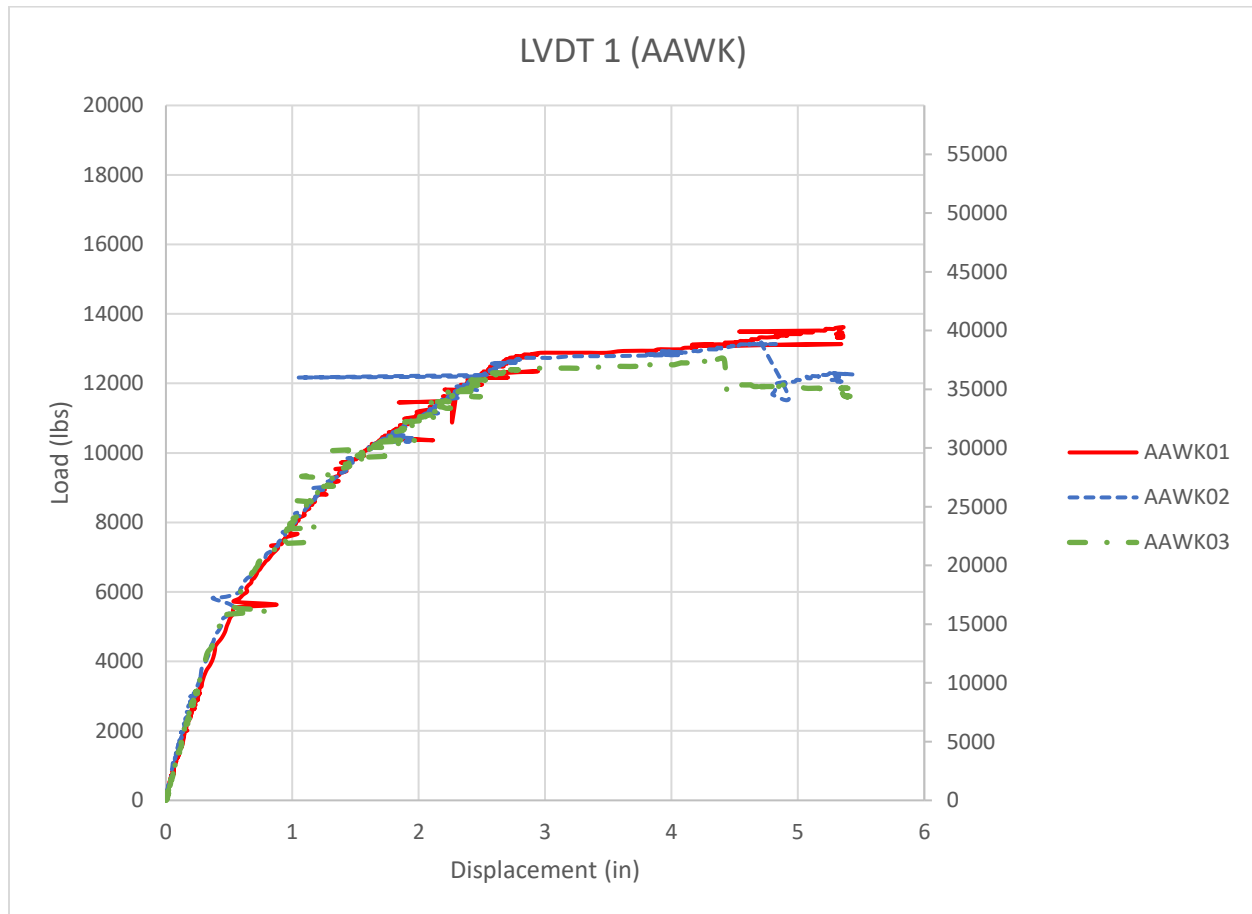
*Raw Load versus LVDT 2 Displacement for Specimens WFWK Specimens*





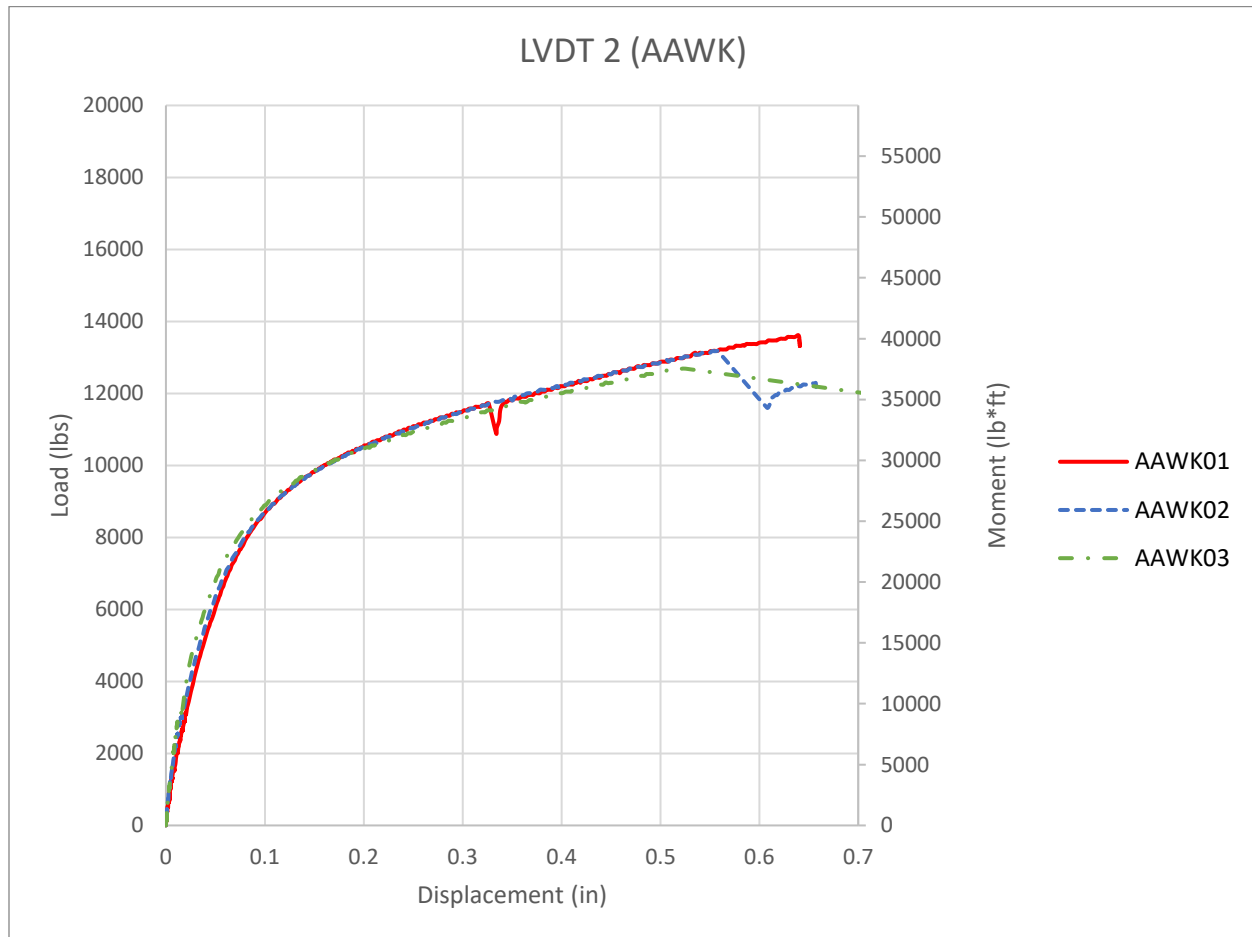
**Figure C7**

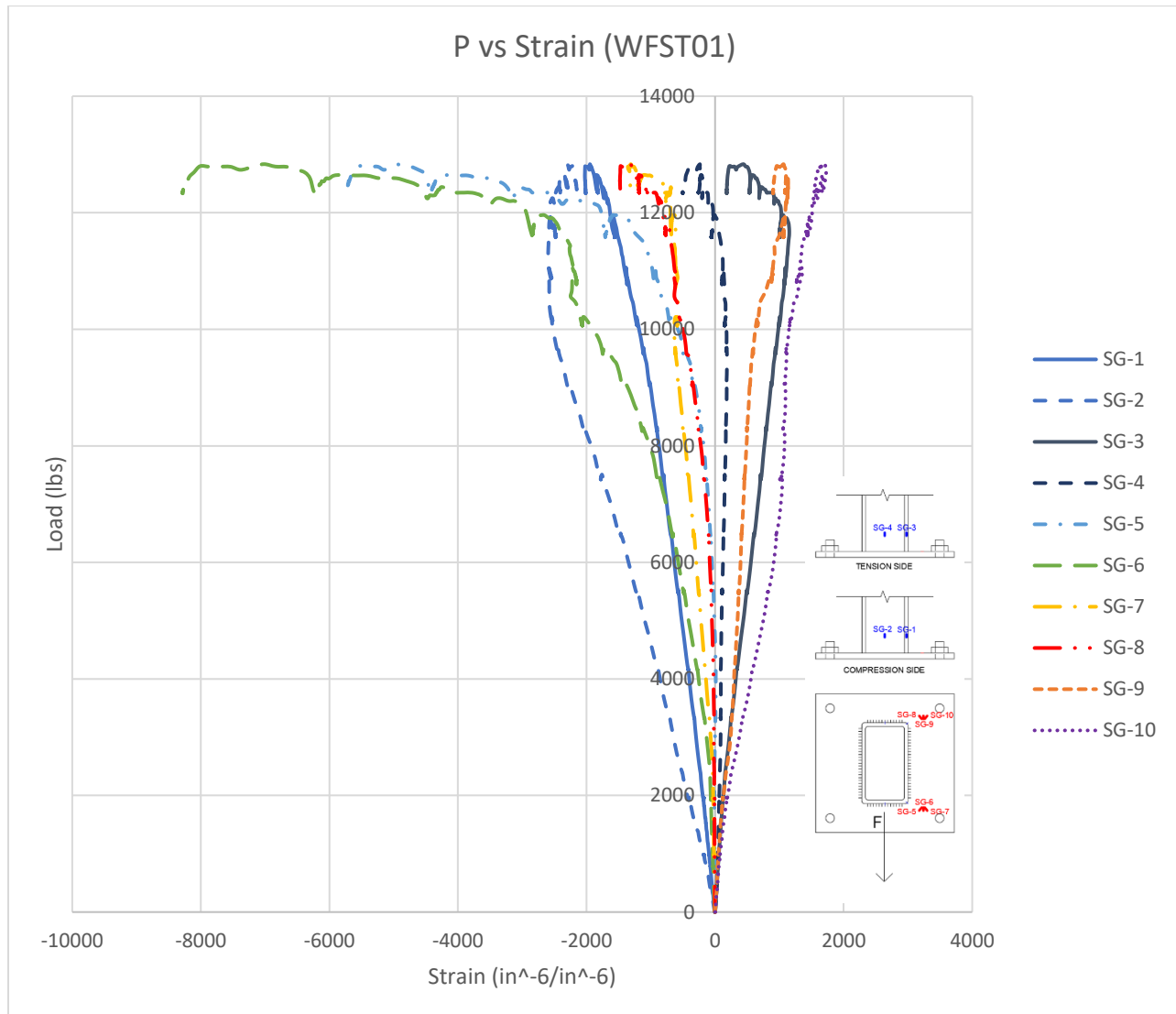
*Raw Load versus LVDT 1 Displacement for Specimens AAWK Specimens*



**Figure C8**

*Raw Load versus LVDT 2 Displacement for Specimens AAWK Specimens*



**Load versus Strain Plots****Figure C9***Load versus Strain for Specimen WFST01*

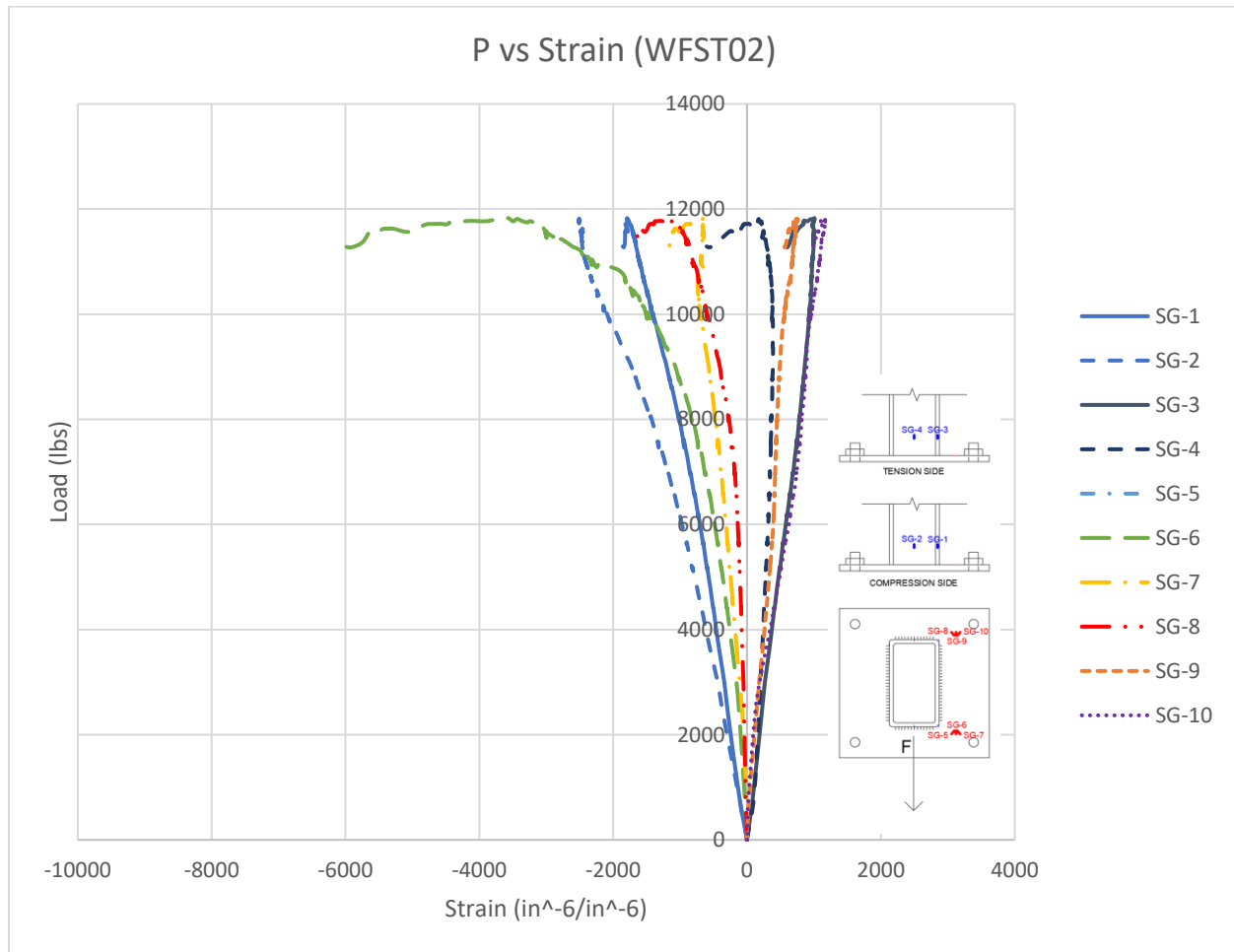
**Figure C10***Load versus Strain for Specimen WFST02*

Figure C11

Load versus Strain for Specimen WFST03

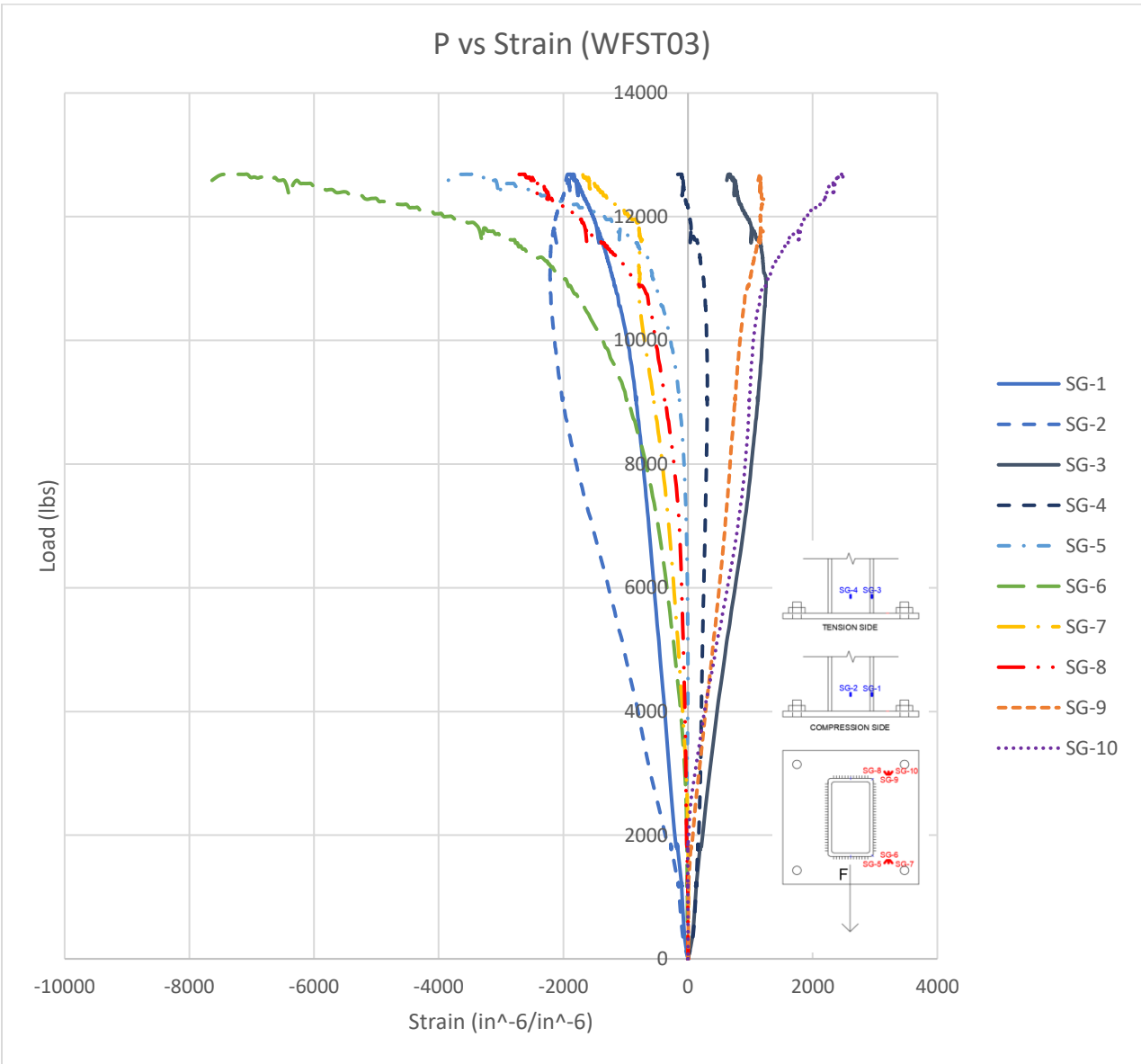


Figure C12

Load versus Strain for Specimen AAST01

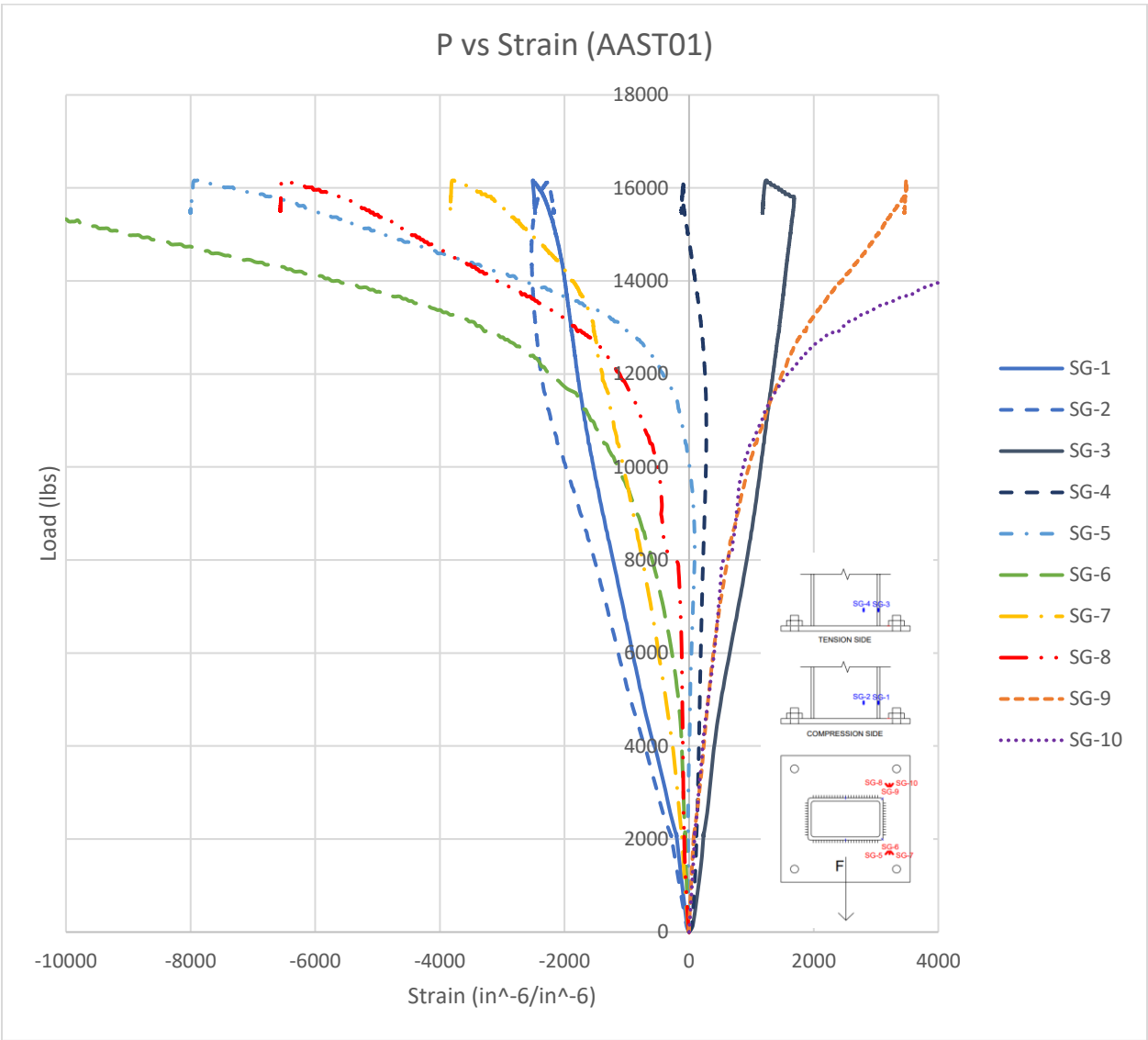
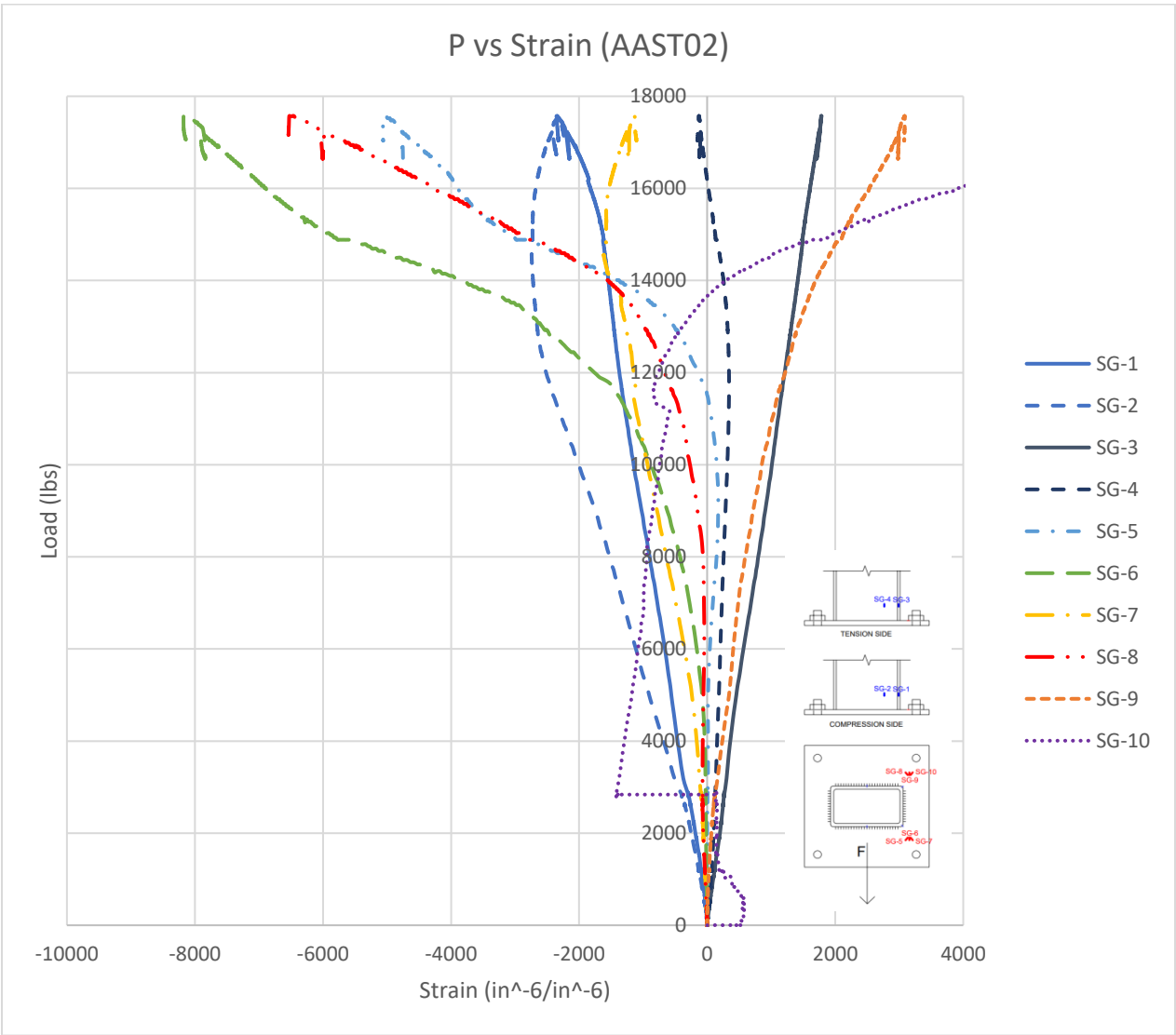


Figure C13

Load versus Strain for Specimen AAST02



*Load versus Strain for Specimen AAST03*

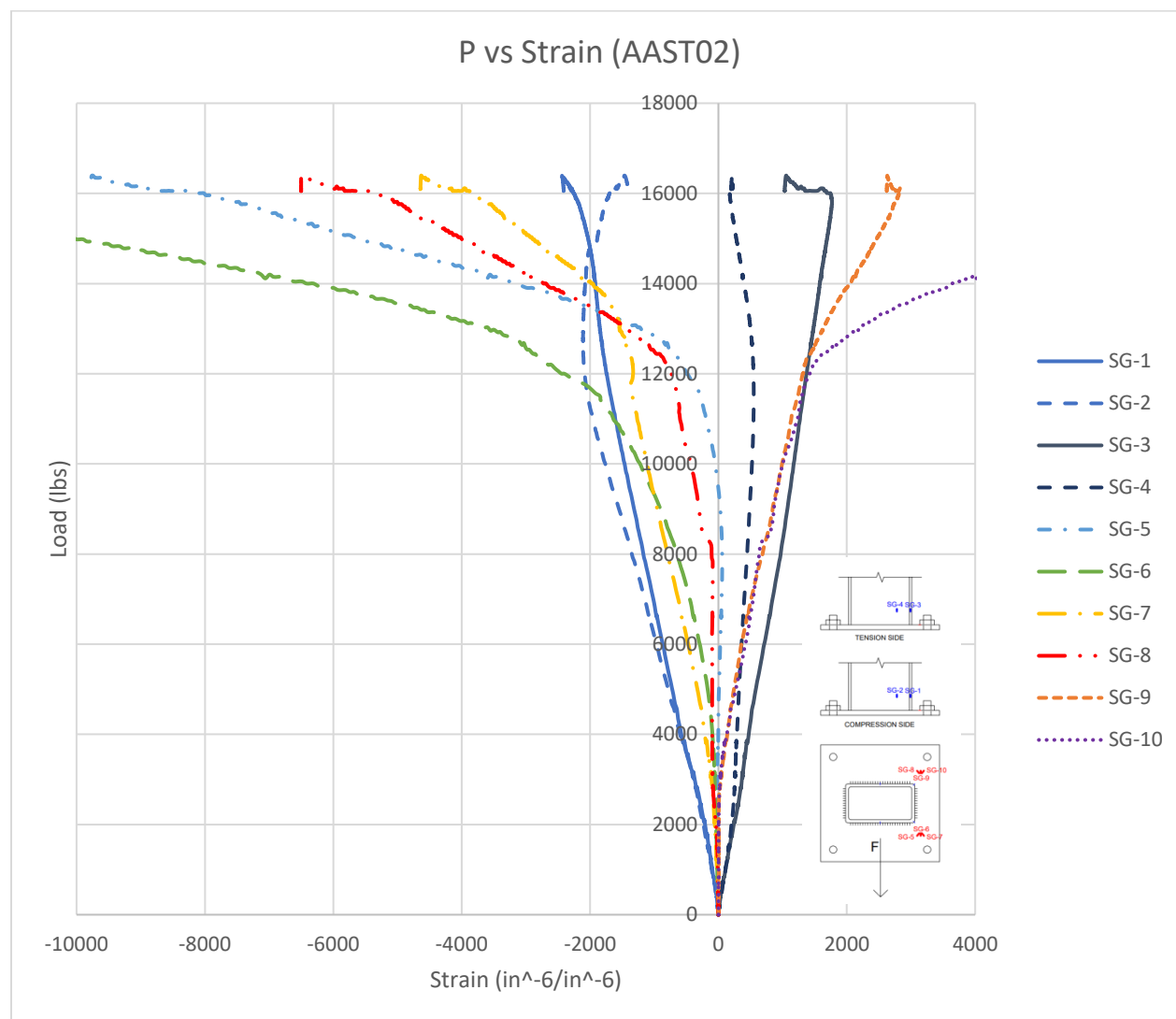




Figure C15

Load versus Strain for Specimen WFWK01

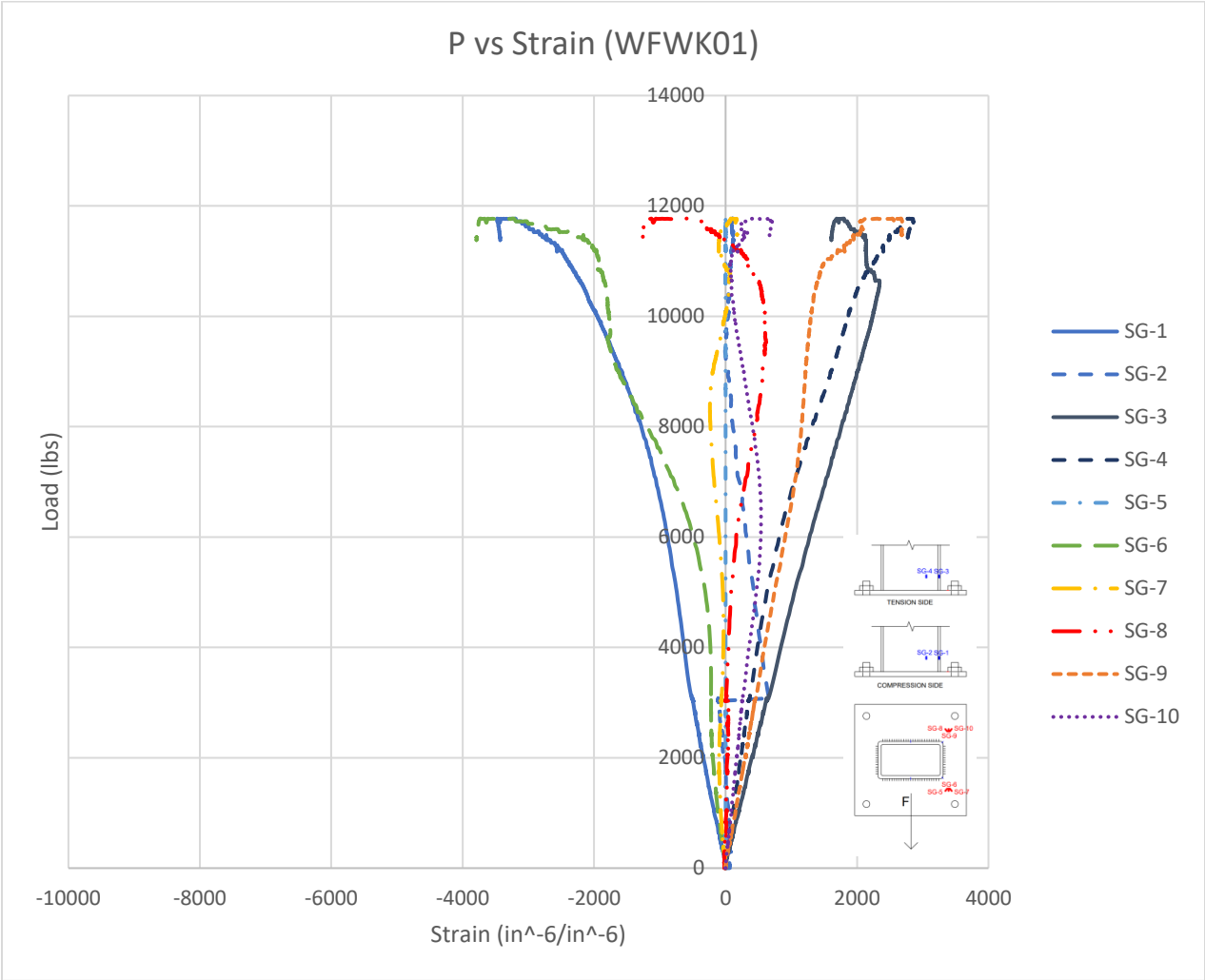


Figure C16

Load versus Strain for Specimen WFWK02

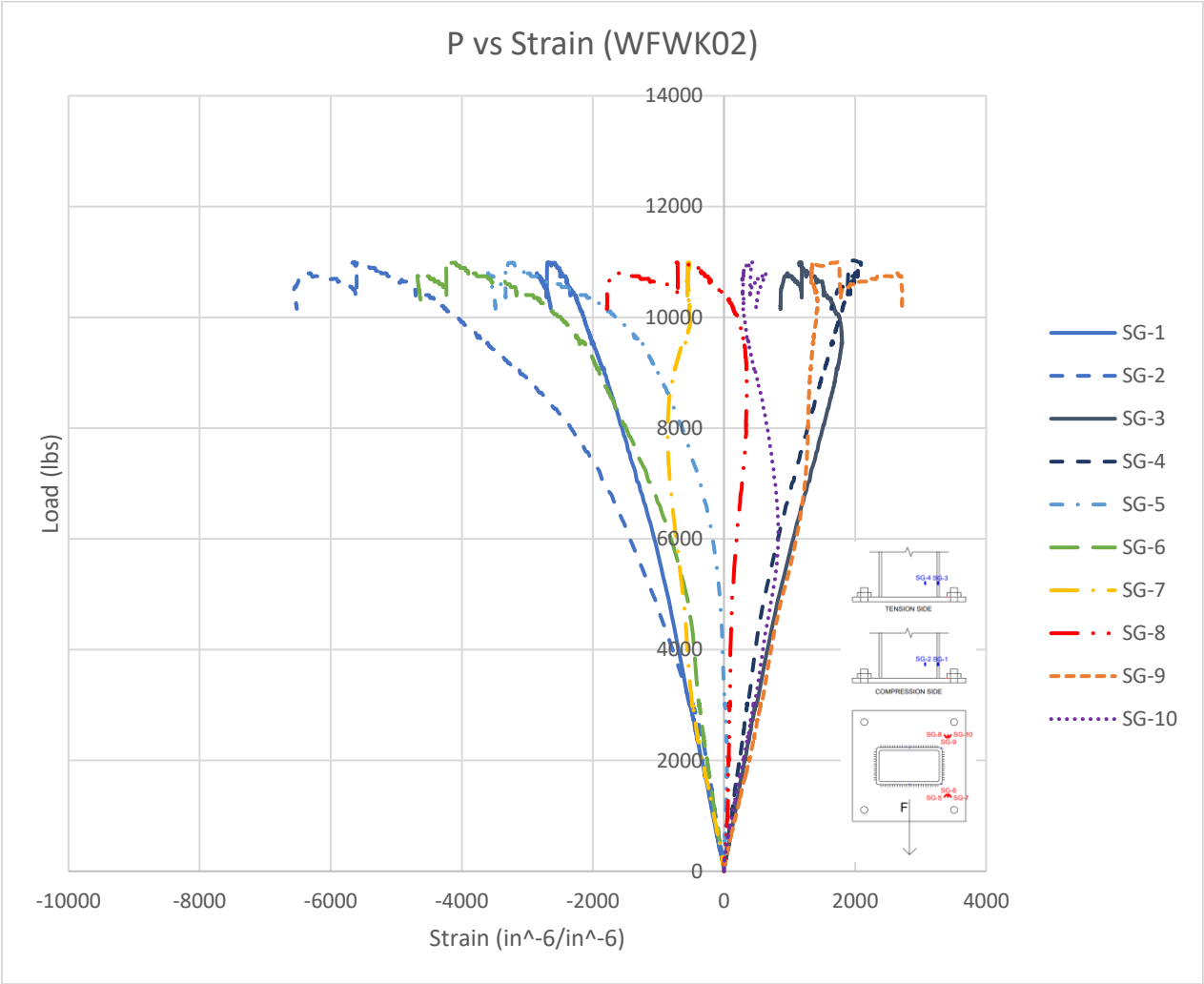


Figure C17

Load versus Strain for Specimen WFWK03

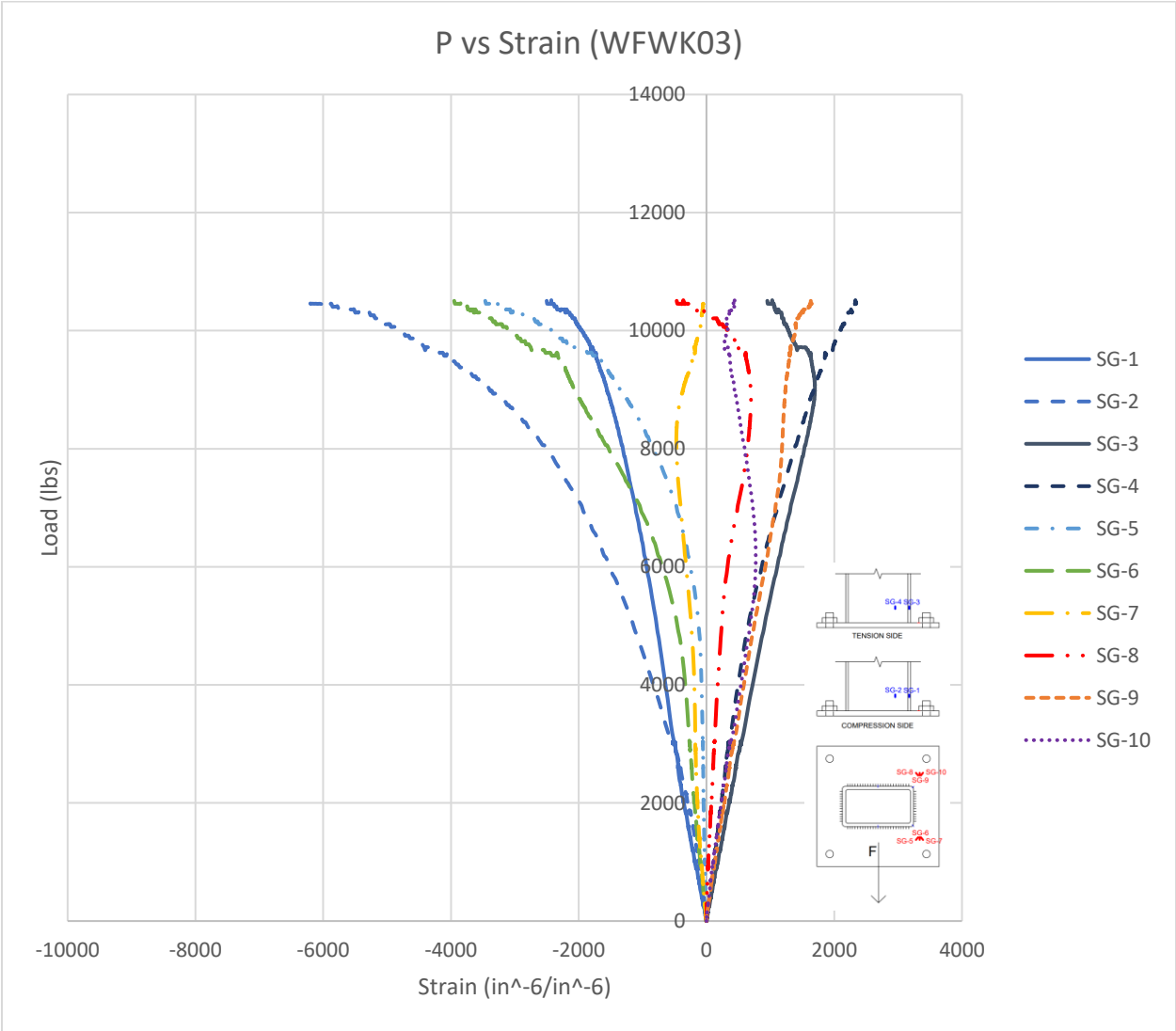


Figure C18

Load versus Strain for Specimen AAWK01

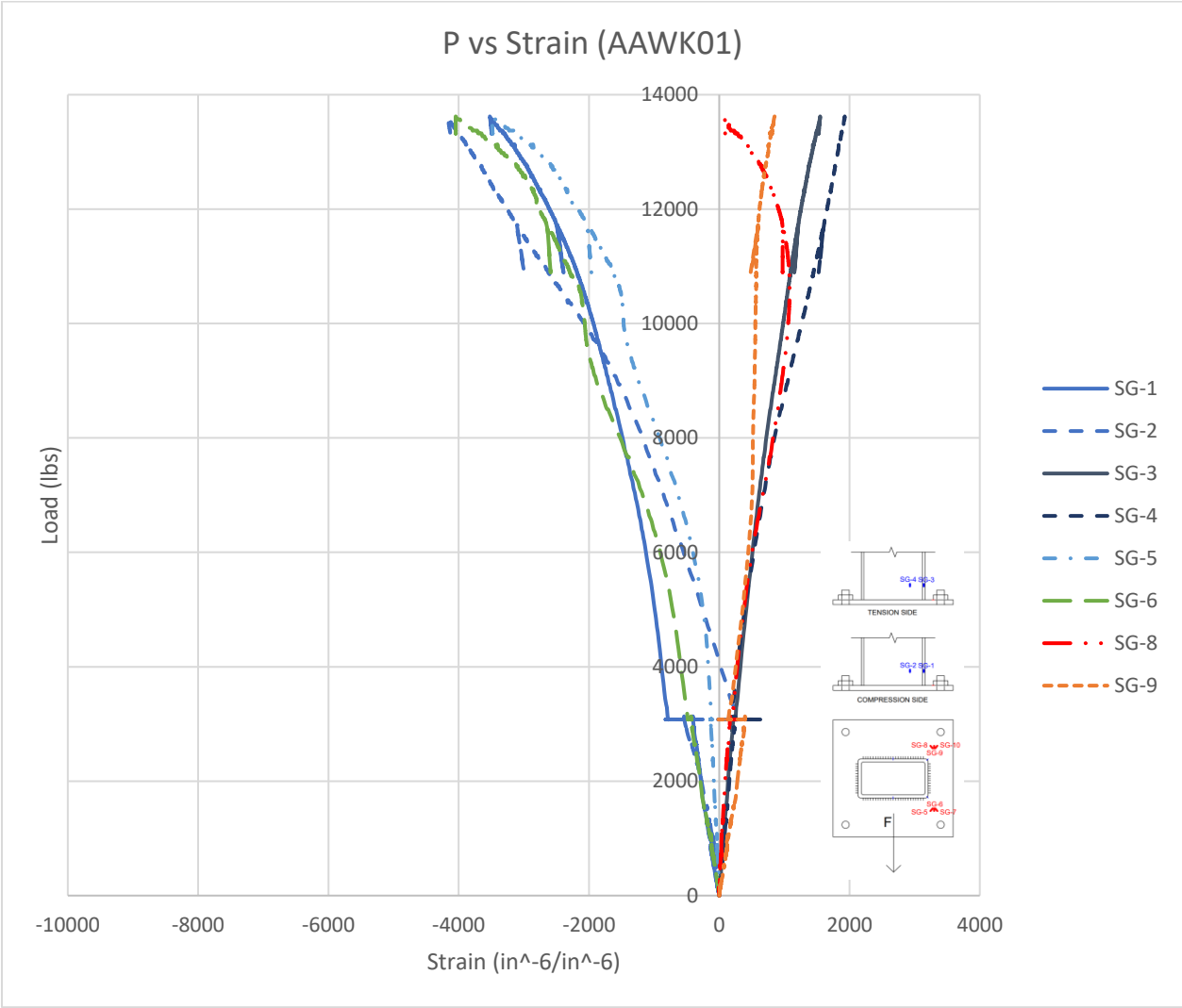


Figure C19

Load versus Strain for Specimen AAWK02

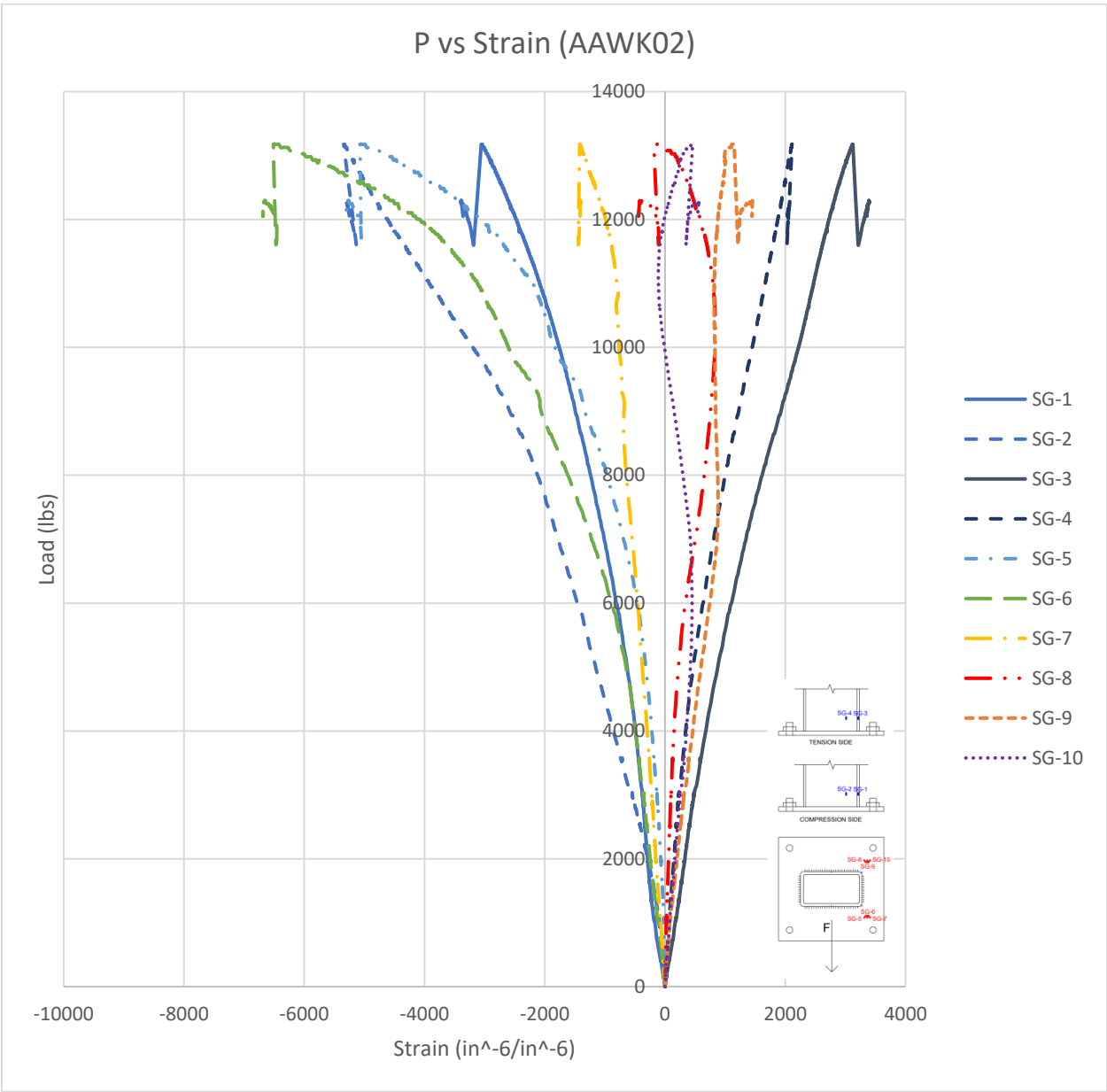
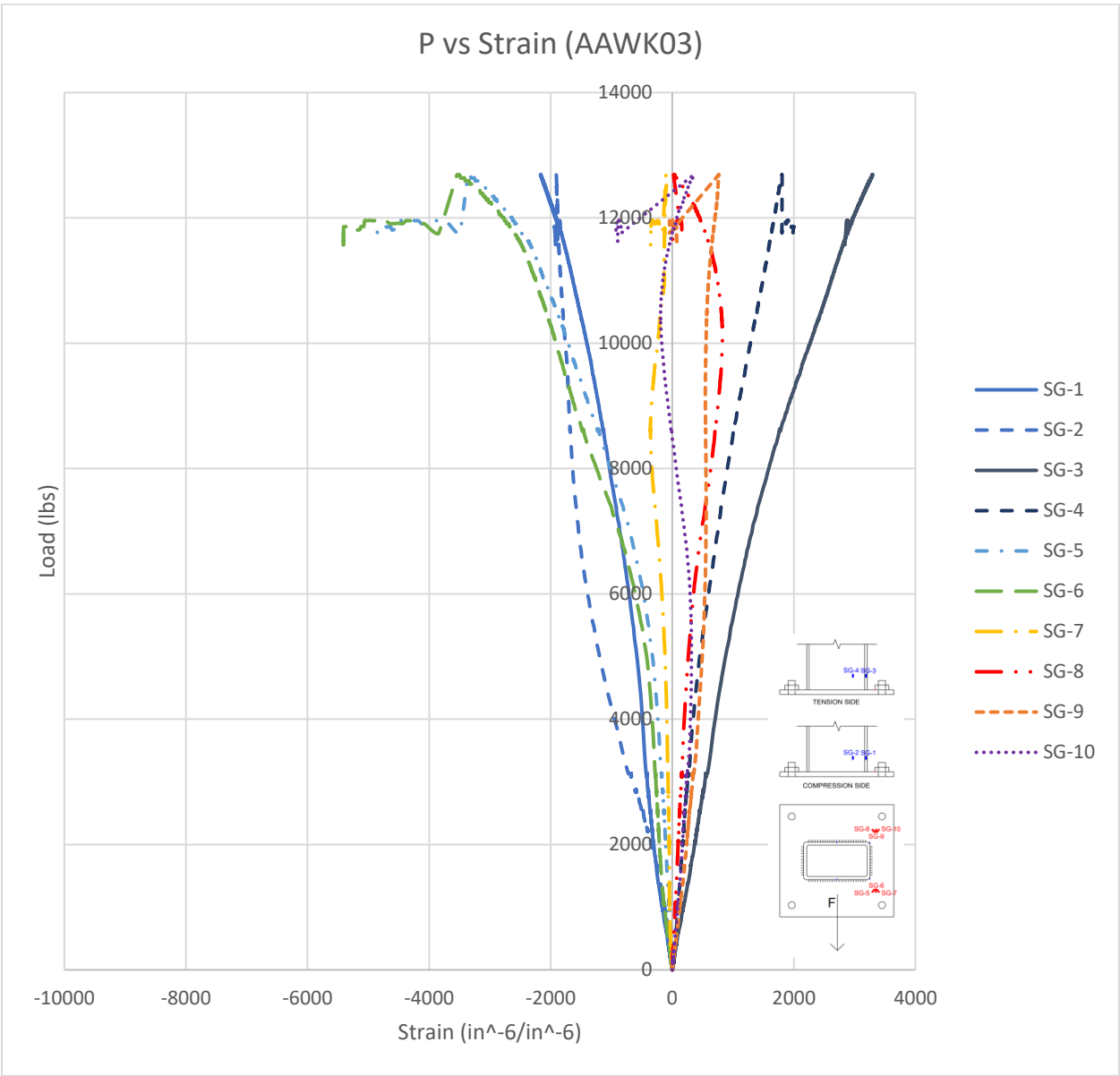


Figure C20

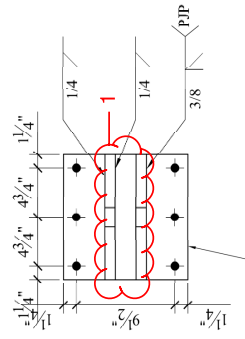
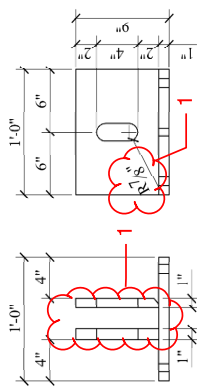
Load versus Strain for Specimen AAWK03



**Appendix D – Structural Drawings**

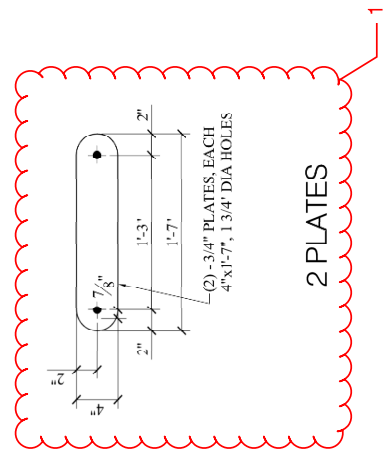






(2) - 1" PLATES, EACH  
8"X12", CENTERED ON  
12"X12"X1" PLATE  
SEPARATED BY 2" GAP

AXIAL SPECIMEN CAP PLATE ASSEMBLY  
1 ASSEMBLY



2 PLATES

# CONNECTION ASSEMBLIES

NOTE: ALL HOLES 13/16" DIA UNO  
ALL PLATE MATERIAL GRADE 50



Project  
CV8000 -  
RESEARCH  
AND  
PRESENTATION  
JAKE ZIETLOW

Client  
MILWAUKEE  
SCHOOL OF  
ENGINEERING  
Milwaukee, WI

Ver	Date	Description	Released	Course I/C	Section No.	Section Title
1	07/19	RELEASED	REVISIONS	CV 8000	101	
2	08/16				08/16/21	
					JAZ	
					PB	
					1" = 1'-0"	
					CONN ASSEMBLIES	
					Sheet No	S2

This drawing is part of a Milwaukee School of Engineering University academic project.

## **Appendix E – Shop Drawings**











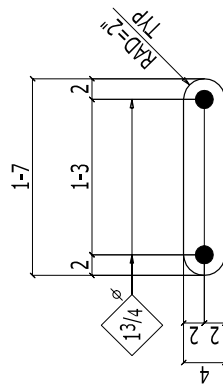












2 PLATES 100P010

# BILL OF MATERIAL

[illegible]

PAINT:  
NONE  
SSPC-SP1

PLOTTED Aug 18 2021

**ZALK JOSEPHS**  
STOUGHTON, WISCONSIN

SHOP NOTE:  
ALL RE-ENTRANT CORNERS  
SHALL BE SHAPED NOTCH  
FREE TO A RADIUS AT LEAST 1/2"

**ALL MATERIAL: A572-50**

**HOLE: 13/16" Dia  
(U.N.O)**

MSE  
CV8000-RESEARCH AND PRESENTATION

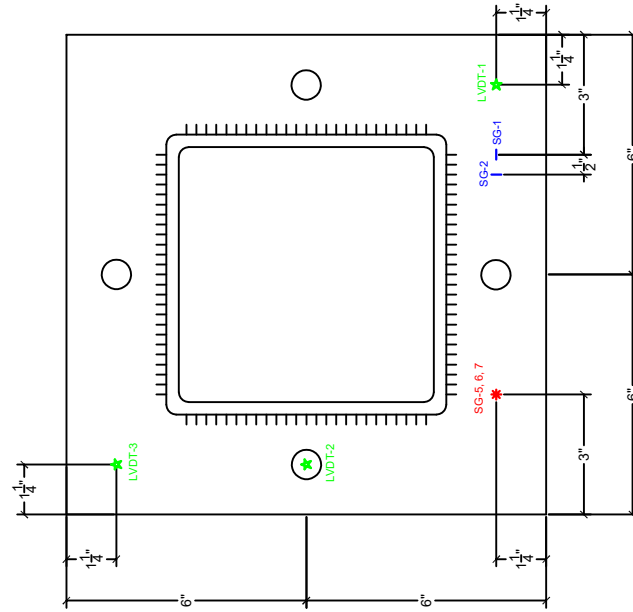
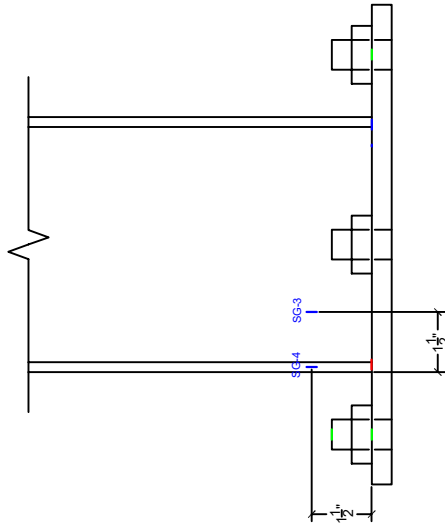
CONTR. NO.11201407	DWG. NO. 100P010	REV.
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FOR APPROVAL ONLY

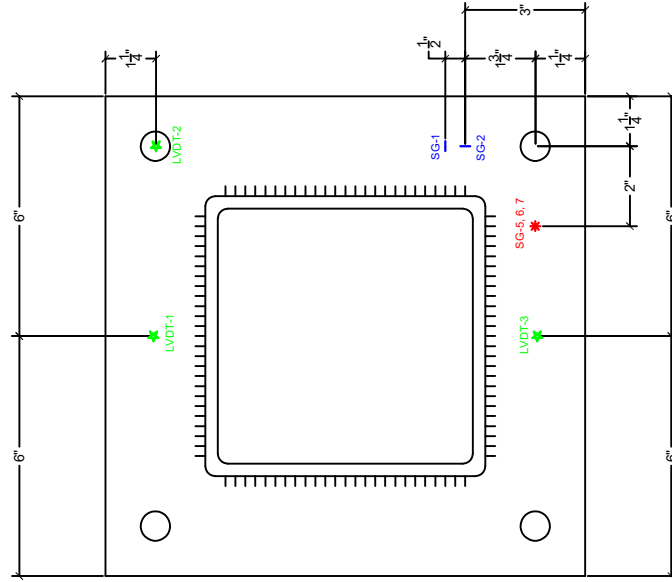
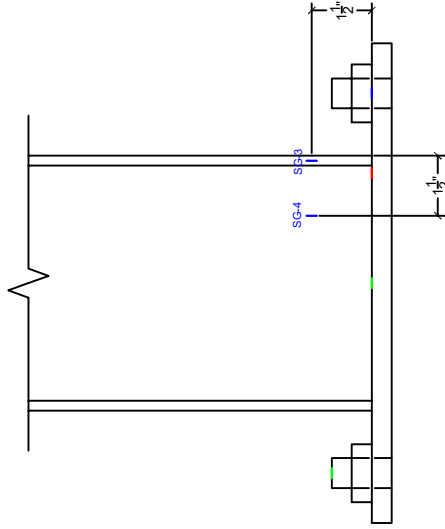
NOT CERTIFIED FOR FABRICATION OR FIELD USE

## **Appendix F – Instrumentation Plans**

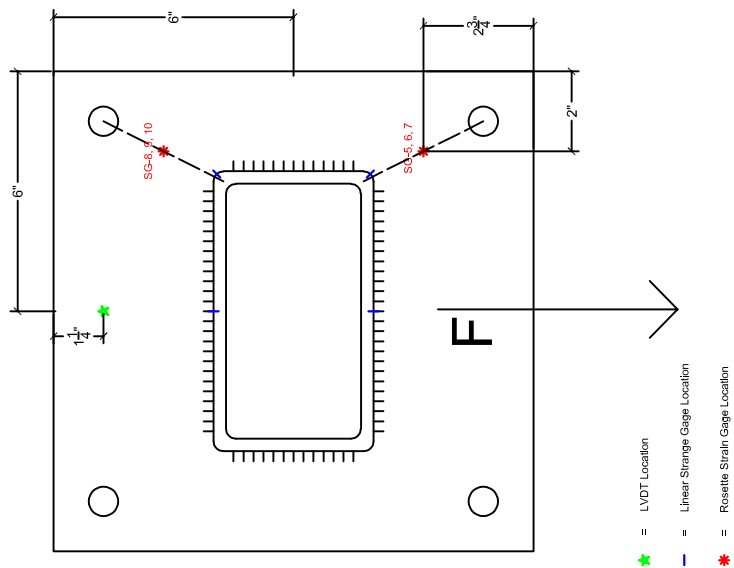
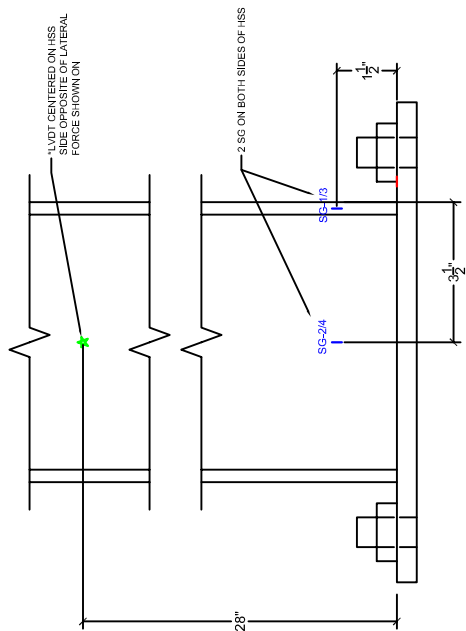
# Side Bolts



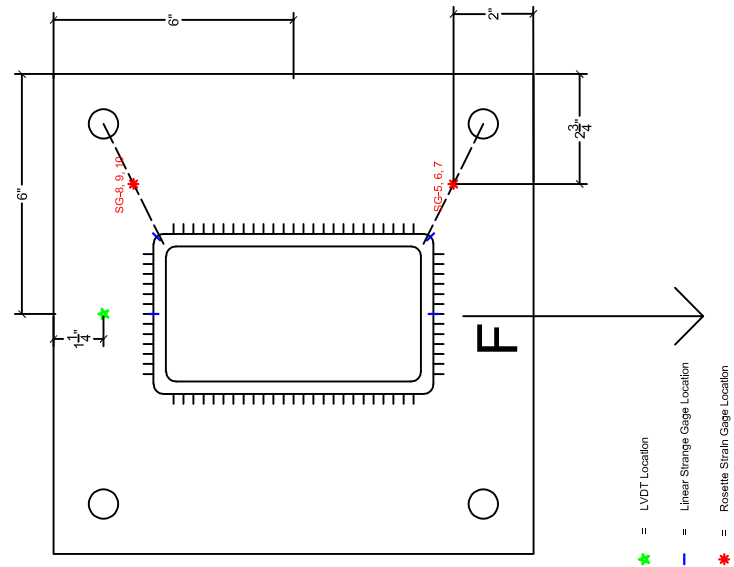
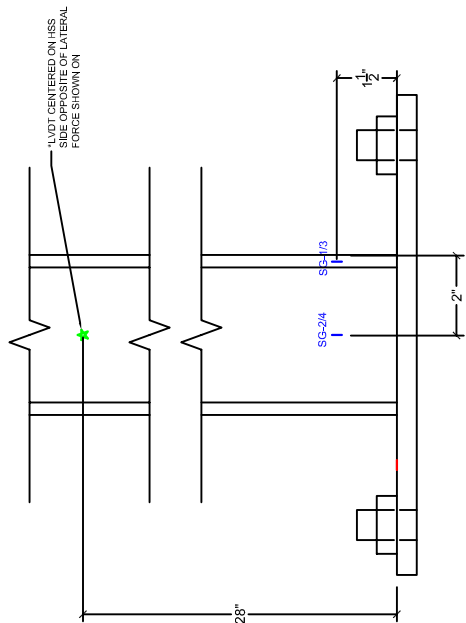
# Corner Bolts



# Weak Axis



# Strong Axis



## **Appendix G – Material Testing**



## Tensile Test Report (Page 1 of 2)

**MAI Report No:** 222-2-196 **Date:** June 27, 2022  
**Client:** Milwaukee School of Engineering **Contact:** Pouria Bahmani  
**P.O. No:** Verbal **Date Rec'd:** June 16, 2022  
**Description:** Plate and Tube Sections

Property	Plate 1	Plate 2	Plate 3	ASTM A36	ASTM A572, Grade 50
Test Bar Dimensions, Diameter, inch Gage Length, inches	0.496 2.0	0.495 2.0	0.496 2.0	D 4D	D 4D
Tensile Strength, psi	82,900	82,900	82,900	58,000 - 80,000	65,000 min.
Yield Strength, psi (1)	57,300	56,900	57,200	36,000 min.	50,000 min.
Elongation, %	30	30	29	23 min.	21 min.
Elongation, %	67	66	67	Not Specified	Not Specified

Property	Tube 1	Tube 2	Tube 3	ASTM A500, Grade B	ASTM A1085
Test Bar Dimensions Width, inch Thickness, inch Gage Length, inches	0.493 0.295 2.0	0.489 0.293 2.0	0.493 0.293 2.0	0.50 Material Thickness 2.0	0.50 Material Thickness 2.0
Tensile Strength, psi	65,500	67,200	67,700	58,000 min.	65,000 min.
Yield Strength, psi (1)	53,100	53,300	53,300	46,000 min.	50,000 - 70,000
Elongation, %	35	32	32	23 min.	21 min.

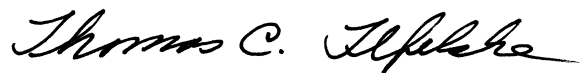
(1): at 0.2% offset

**Notes:** The tensile properties of the three plate samples are in conformance with both ASTM A36, "Standard Specification for Carbon Structural Steel," and ASTM A572, Grade 50, "Standard Specification for High-Strength Low-Alloy Columbium-Vanadium Structural Steels."

The tensile properties of the three tubes are in conformance with both ASTM A500, Grade B, "Standard Specification for Cold-Formed Welded and Seamless Carbon Steel Structural Tubing in Rounds and Shapes," and ASTM A1085, "Standard Specification for Cold-Formed Welded Carbon Steel Hollow Structural Sections (HSS)." It should be noted, however, that this specification also has specified Charpy V-Notch impact properties that were not determined.

The stress-strain curves for these samples are provided as separate Excel spreadsheets.

Respectfully submitted,

A handwritten signature in black ink, reading "Thomas C. Tefelske". The signature is written in a cursive, flowing style.

Thomas C. Tefelske,  
President


## Architectural Engineering

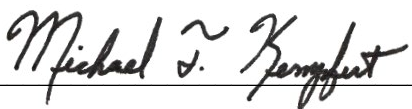
### Capstone Report Approval Form

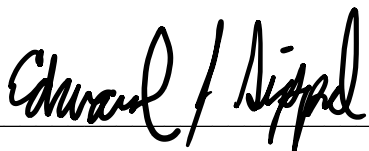
#### Master of Science in Architectural Engineering -- MSAE

#### Milwaukee School of Engineering

This capstone report, entitled “Experimental and Analytical Evaluation of the Flexural and Axial Capacity of Rectangular Steel Hollow Structural Section (HSS) End-Plate Connections,” submitted by the student Edward J. Nelson has been approved by the following committee:

Faculty Advisor:  Date: 05/08/2024  
Dr. Christopher Raebel, Ph.D.

Faculty Member:  Date: 05/08/2024  
Dr. Michael Kempfert, Ph.D.

Faculty Member:  Date: 05/08/2024  
Dr. Edward Sippel, Ph.D.