Tribological Models for Bearing and Cam Interfaces in a Variable Displacement Linkage Motor (VDLM)

by

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Abstract

The purpose of this project report is to report the final results for a capstone design project for the Milwaukee School of Engineering (MSOE) Master of Science in Engineering. (MSE) program. The purpose, background, and justification are described in this report. This report is the culmination of the proposal, progress, and final results for this project. The purpose of this project was to model the tribological aspects in a variable displacement linkage motor for off-highway vehicles using MATLAB and GT – SUITE simulation. Models for steady state friction torque, power losses, and film thickness for bearings and cam follower mechanism were developed in this project. The MATLAB programs developed have been included in this report. This project was sponsored by the Department of Energy and is part of an ongoing larger project led by the University of Minnesota, Minnesota (UMN). This project's tribological model will be integrated into the complete model of the motor by UMN.

Comparison of bearing power losses have been displayed for MATLAB and GT – SUITE. It can be observed that there was general agreement between the MATLAB results and the results obtained from the three-loss method in the GT – SUITE simulations. The total bearing loss for the current motor design was also analyzed in order to determine a good idea of the losses in the selected bearings. Film thicknesses for bearings and the cam were also evaluated. The power losses from the cam roller follower were additionally analyzed for this project.

Table of Contents

List of Figures

List of Tables

Nomenclature

Symbol	Definition	Unit
B	Bearing width	mm
b	Elastohydrodynamic half width contact	mm
C_S	Basic static load rating	N
$C_W, I_D, f_t, R_S, t, f_A$	Drag loss variables	–
D	Bearing outside diameter	mm
D	Bearing bore diameter	mm
d_m	Pitch diameter	mm
d_s	Seal counterface diameter	mm
E_e	Effective elastic modulus	N/mm^2
E_1, E_2	Elastic modulus of cylinder and plane	N/mm^2
F_a	Axial load	N
F_r	Radial load	N
F_r'	Cam rolling friction force	N
F_S	Static equivalent load	N
F_s'	Cam sliding friction force	N
F_β	Magnitude of Load factor	N
f_o	Type of bearing and bath factor	–
f_1	Bearing design and relative bearing load factor	–
F	Dimensionless load	–

Symbol	**Definition**	**Unit**
G	Dimensionless modulus	–
G_{rr}	Rolling frictional variable factor	m
G_{sl}	Sliding frictional variable factor	Pa.s
g_e	Elasticity number	–
g_h	Film thickness number	–
g_v	Viscosity number	–
H	Power loss	W
H_r	Cam rolling power loss	W
H_s	Cam sliding power loss	W
H	Dimensionless film thickness	–
h	Film thickness	m
h_c	Central film thickness	m
h_0	Minimum film thickness	m
HL	Oil level	mm
K_L	Roller bearing type geometric constant	–
K_{roll}	Rolling element constant	–
K_{S1}, K_{S2}	Seal and bearing type constants	–
K_Z	Bearing type geometric constant	–
L	Length of roller	mm
L_a	Asperity load ratio	–

Symbol	**Definition**	**Unit**
n	Bearing angular velocity	rpm
n_{avg}	Average velocity between the cam and roller	m/s
n_{cam}	Velocity of cam	m/s
n_{roller}	Velocity of roller	m/s
n	Dimensionless velocity	–
p	Pressure	Pa
p_h	Hydrodynamic pressure	Pa
Pe	Peclet number	–
q	Heat flux	W/m^2
R	Radius of roller	mm
R_{red}	Reduced radius of roller	mm
R_1, R_2, S_1, S_2	Geometric and load dependent constants for rolling and sliding	m
T	Total Torque	N.mm
T_{drag}	Torque due to drag	N.mm
T_{rr}	Torque due to rolling	N.mm
T_{seal}	Torque due to seal	N.mm
T_{sl}	Torque due to sliding	N.mm
ΔT	Flash temperature	°C
V	Elastic deformation	m
V_M	Drag loss factor	–

Symbol	Definition	Unit
X_S	Static equivalent load factor	–
x_0	Inlet coordinate along line of motion	m
x_c	Exit coordinate along line of motion	m
y, z	Applied load factor for radial deep grove bearing	–
α	Piezoviscosity coefficient	m^2/N
β	Seal exponent constant	–
η_{avg}	Average dynamic viscosity	Pa.s
η_0	Initial dynamic viscosity	Pa.s
μ_{bl}	Movement depending constant	m^2 / s
μ_{EHL}	Sliding friction coefficient in full film conditions	–
μ_{sl}	Sliding friction coefficient	–
v_o	Kinematic Viscosity	cSt
v_1, v_2	Poisson's ratio of cylinder and plane	–
ρ	Lubricant density	kg / m^3
ϕ_{bl}	Sliding friction weighting factor	–
ϕ_{ish}	Inlet shear heating reduction factor	–

Abbreviations

DOE	Department of Energy
GT – SUITE	Gamma Technologies Suite
MSOE	Milwaukee School of Engineering

MB	Main Bearing
RF	Roller Follower
RG	Rocker Ground
RM	Rocker Moving
UMN	University of Minnesota Twin Cities
VDLM	Variable Displacement Linkage Motor
WP	Wrist Pin

1. Introduction

Tribology is the branch of science and technology that deals with friction, lubrication, and wear of interacting surfaces in relative motion [1]. The efficiency of hydraulic systems is affected by friction in hydraulic motors, cylinders, and other components [2]. These losses generate heat and reduce the power available. Lubrication is an effective means of reducing friction and minimizing wear. It is crucial to focus on tribology in hydraulic systems because an understanding of tribological effects can help to reduce friction, which can result in economic savings and a reduction in emissions.

Working as a graduate research assistant at the Fluid Power Institute (FPI) of the Milwaukee School of Engineering (MSOE), the author is privileged to have been assigned to become a part of a project sponsored by the Department of Energy (DOE). The purpose of the project is to create an efficient, compact, and smooth variable propulsion motor. This project is led by the University of Minnesota (UMN) partnering with two corporations. This project entails the re-design of the radial embodiment of the Variable Displacement Linkage Pump (VDLP) as a motor, termed a VDLM, to propel off-highway vehicles [2]. A schematic of the VDLM can be seen in Figure 1. This motor uses an adjustable four-bar linkage that drives a cam to vary the displacement of the piston, resulting in a Variable Displacement Linkage Motor (VDLM). The displacement of the VDLM is changed by moving the adjustable ground pivot. Conceptually, the efficiency of this motor is high across a wide range of displacements, speeds, and pressures [2].

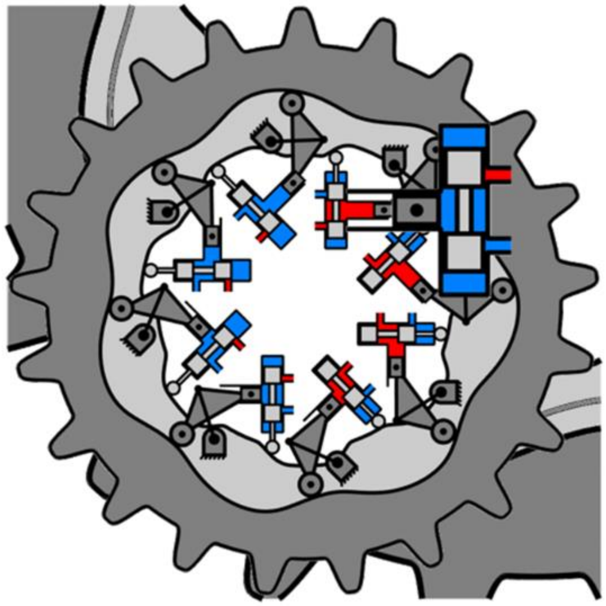

Figure 1: Variable Displacement Linkage Motor Mechanism [2].

Incorporating the architecture of the VDLP, the VDLM would enable drastic improvements in efficiency. It would reduce friction, which would in turn save fuel and increase torque output. The VDLM will incorporate varied displacement, which will increase transport speed and engine efficiency. UMN is responsible for the modeling of the mechanical system and MSOE has been given the task of tribological modeling of the components.

2. Description of Project

This project involved construction of mechanical friction models for tribological contacts of bearing and cam follower interfaces in the motor. The project consisted of two fundamental models including the lubricating film thickness model and the friction at surface interfaces model.

1. Lubricating film thickness model: Interface speeds and loads from the kinetic model created were used to estimate the lubricant film thickness at the cam-roller follower and rolling element bearing interfaces. Film thickness models for the bearing interfaces and cam follower were modeled throughout the anticipated operating range to determine the fluid film thickness using hydrodynamic and elastohydrodynamic lubrication principles. The lubrication regime was characterized based upon film thickness calculations (boundary, mixed or full film). Two types of lubrication regimes were investigated: hydrodynamic and elastohydrodynamic lubrication. In hydrodynamic lubrication, the bearing surfaces are fully separated by a lubricating film of liquid or gas. There are two conditions for the occurrence of hydrodynamic lubrication [1]:

    - The two surfaces must move relatively to each other with sufficient velocity for a load carrying lubricating film to be generated;
    - The surfaces must be inclined at some angle to each other.

    Hydrodynamic films usually have a considerable thickness – reaching 100 µm – and therefore prevent contact between the asperities of even the roughest surfaces. An example of hydrodynamic pressure generation and film thickness between two non-parallel surfaces is shown in Figure 2.

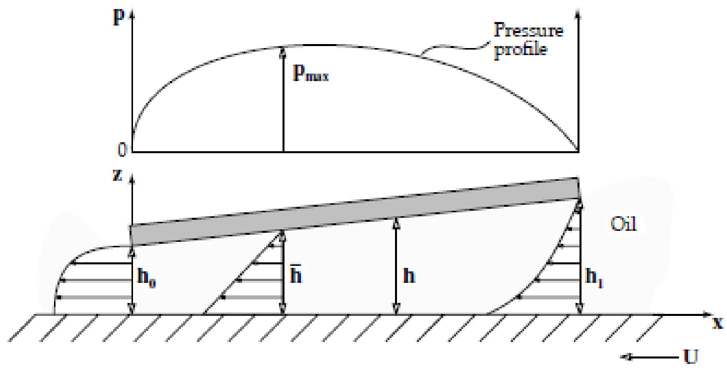

Figure 2: Hydrodynamic Pressure Generation and Film Thickness Between Two Surfaces [1].

Elastohydrodynamic lubrication can be defined as a form of hydrodynamic lubrication where the elastic deformation of the contacting bodies and the changes of viscosity with pressure play fundamental roles [1]. The lubricating films are very thin, in the range of 0.1 to 1 µm, but manage to separate the interacting surfaces, resulting in a significant reduction of wear and friction. Figure 3 depicts a plot between EHL pressure and film thickness.

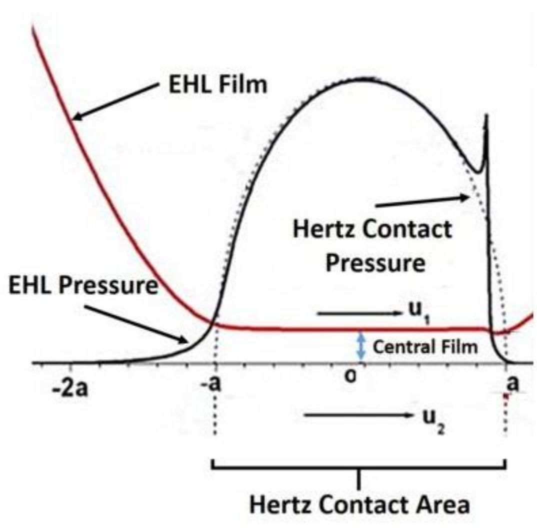

Figure 3: A Common Elastohydrodynamic Lubricated Pressure and Film Thickness Plot [1].

Elastohydrodynamic lubrication (EHL) is generally found in rolling elements of bearings and gears [3]. For a transient approach, the film thickness can be numerically solved by using a reduced Reynold's equation, which is based on the Navier – Stokes momentum equation. For EHL, the elastic deformation of the lubricated surfaces plays an important role and cannot be neglected [4]. So, the Reynolds equation, along with elastic deformation, pressure, and viscosity, must be solved numerically to obtain a transient film thickness. For this project, the line contact problem was solved.

2. Modeling friction at surface interfaces: The friction torque and power losses within individual tribological contacts were modeled as a function of angular position, angular velocity, radial loads, and viscosities of hydraulic fluids. The total power losses for all the interfaces were calculated through the model and finally incorporated into the VDLM kinematic model. The entire modeling effort was completed in two parts. In the first part, bearing power loss was analyzed, and in the second part, cam friction and power loss were analyzed. The rolling elements in both of these machine elements are cylindrical, and therefore, they were modeled as a line contact as shown in Figure 4.

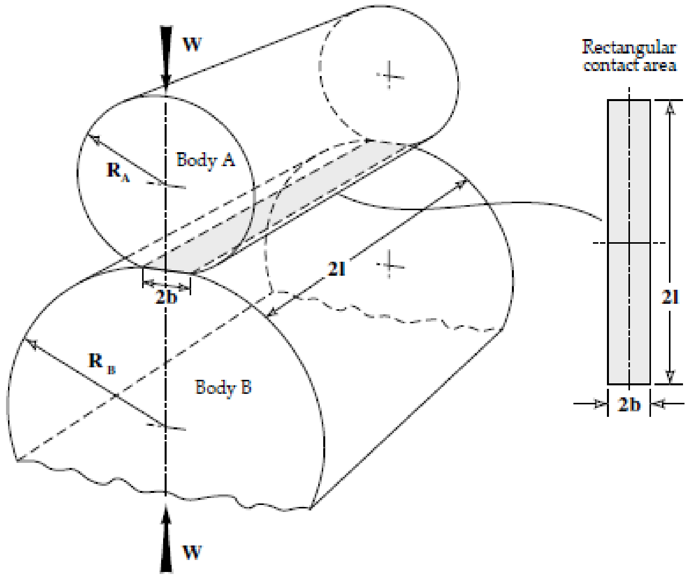

Figure 4: Geometry of the Contact Between Two Parallel Cylinders [1].

These models were created using MATLAB software. The author created different MATLAB programs for the different models incorporating numerical methods where applicable. Furthermore, the author created similar models in GT – SUITE, as well.GT – SUITE is a widely used simulation tool produced by Gamma Technologies LLC [5]. GT – SUITE simulations were used to validate the models developed in MATLAB. Similar results from both models were desired.

3. Justification

The VDLM architecture is similar to that of common radial piston motors but offers the advantage of incorporating a linkage system that facilitates variable displacement, minimizes friction during the power-stroke, and reduces torque ripple. These features are highly desirable in mobile hydraulic equipment. As a result, Bobcat plans to incorporate the VDLM in its Mini Track Loader. Currently, the Variable Displacement Linkage Motor is being developed by UMN, which has only published the proposal regarding the motor. This proposal, by Van De Ven [2], was used to identify the technical specifications required for this capstone project. Modeling the tribological elements of the motor and how they affect friction torque and power losses is crucial in the VDLM design process. The tribological models developed in this capstone project will be incorporated into VDLM mechanical and kinematic simulations developed by UMN. A step-by-step optimization process will be performed in order to optimize the reliability and efficiency of the motor. The resulting design will be used to produce a single-cylinder and half-scale prototype motors that will be evaluated in a dynamometer at MSOE. In terms of academic merit, predicting friction and power loss in antifriction bearings based upon the empirical models of Harris and Kotzalas [6] and SKF [7] is fairly straightforward. Determining the power losses in the cam/follower system is much more challenging because it requires a detailed analysis of the elastohydrodynamic conditions within the lubrication contact.

4. Background and Literature Review

Two types of bearings were analyzed: needle roller and tapered roller bearings. and they are shown in Figure 5.

Figure 5: SKF Radial Needle Roller and Tapered Bearings [8].

There are four locations where the needle roller bearings are used: Rocker Ground, Rocker Moving, Roller Follower, and Wrist Pin. These bearing locations are illustrated in Figure 6.

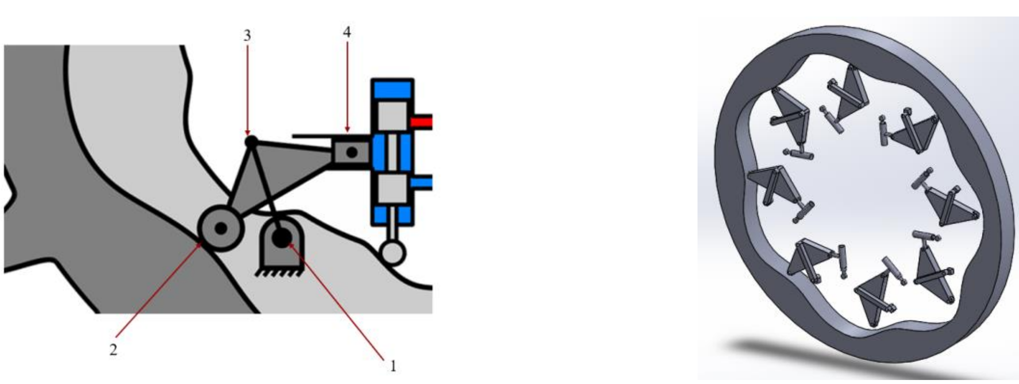

Figure 6: Locations of Rocker Ground (1), Roller Follower (2), Rocker Moving (3), Wrist Pin (4) Bearings [2].

There are five types of bearings present in the single cylinder prototype. The main bearing is a tapered roller bearing. The loads on the main bearing, rocker ground bearing, and rocker moving bearing are split between two bearings. Thus, there is a total of eleven bearings in the single cylinder prototype. The loads and angular velocities were exported from a MATLAB file provided by UMN (see Appendix H). Bearing specifications were found from Koyo product catalogs [9]. The friction torque and the power losses were calculated for all the bearings. The Harris and Kotzalas model was taken from *Rolling Bearing Analysis: Essential Concepts of Bearing Technology*, [6]. The equations are derived empirically from Palmgren utilizing various test conditions and bearing types. The bearings used in this project were needle roller bearings because of their uniqueness and design. For the same shaft diameter, needle roller bearings have lower friction than ball or roller bearings. The SKF models were taken from the publication, *SKF Model for Calculating Frictional Moment* [7].

4.1 Harris and Kotzalas Bearing Power Loss Model

<u>Friction Torque of Radial Needle Roller Bearing</u>

Empirical friction torque equations for a radial needle roller bearing were derived by Chiu and Meyers [10]. These models were collected under constant speed conditions with oil jet lubrication. As shown in Equation (1), friction torque, *M*, is a function of pitch diameter, kinematic viscosity, rotational frequency, and radial load:

$$T = d_m(4.5\times10^{-7}\nu_o^{0.3}n^{0.6} + 0.12F_r^{0.41}). \qquad (1)$$

The pitch diameter, d_m, was obtained from the Koyo bearing catalogs [9]. The viscosity, ν_o, was calculated using the 'Fluid Properties' code shown in Appendix E. The bulk fluid pressure was 5000 psi and the operating temperature was 50°C [11]. The angular velocity, n, and the radial load, F_r, were based upon MATLAB outputs from data sets provided by UMN.

Friction Torque of Radial Tapered Roller Bearing

The friction torque for radial roller ball bearing was derived using the method of Witte [12]. The torque due to applied load is given by Equation (2):

$$T = 3.76 \times 10^{-6} G (n\nu_0)^{1/2} \left(f_t \frac{F_r}{K} \right)^{1/3}. \tag{2}$$

The equivalent thrust load factor, f_t can be obtained from Figure 10.2 in Harris and Kotzalas [6]. The bearing geometry factor, G, is given by,

$$G = d_m^{3/2} D^{1/6} (Z.l)^{2/3} \left(\sin \alpha \right)^{-1/3}. \tag{3}$$

Values of d_m, D, Z, l and α were obtained from bearing catalogues [9].

4.2 SKF Bearing Power Loss Model

Rolling Friction Torque

This model was derived from advanced computational models developed by SKF [7]. These models were collected under constant speed conditions with oil jet lubrication. As shown in Equation (4), friction torque, T_{rr}, is a function of inlet shear heating reduction factor, geometric and load dependent factor, kinematic viscosity, and rotational frequency:

$$T_{rr} = \phi_{ish} G_{rr} (\nu\eta)^{0.6}. \tag{4}$$

The inlet shear heating reduction factor and the rolling frictional variable factor are given by Equations (5) and (6):

$$\phi_{ish} = \frac{1}{1+1.84\times10^{-9}(\eta d_m)^{1.28}\nu^{0.64}}. \tag{5}$$

$$\begin{aligned} &G_{rr} = R_1 d_m^{2.41} F_r^{0.31}, && \text{(Cylindrical roller)} \\ &\textit{and} \\ &G_{rr} = R_1 d_m^{2.38} (F_r + R_2 Y F_a)^{0.31}. && \text{(Tapered roller)} \end{aligned} \tag{6}$$

The rolling geometric constants R_1 and R_2 are available from Catalog Table 2c and Table 2d [7].

<u>Sliding Friction Torque</u>

The sliding torque is given by Equation (7):

$$T_{sl} = G_{sl}\mu_{sl}, \tag{7}$$

where G_{sl} is the sliding frictional variable factor and μ_{sl} is the sliding friction coefficient and can be defined as follows:

$$\begin{aligned} &G_{sl} = S_1 d_m^{0.9} F_a + S_2 d_m F_r, && \text{(Cylindrical roller)} \\ &\textit{and} \\ &G_{sl} = S_1 d_m^{0.82} (F_r + S_2 Y F_a). && \text{(Tapered roller)} \end{aligned} \tag{8}$$

S1 and S2 are the sliding geometric constants available from Table 2c and Table 2d [7].

The sliding friction coefficient is given by

$$\mu_{sl} = \phi_{bl}\mu_{bl} + (1-\phi_{bl})\mu_{EHL}, \tag{9}$$

where the weighting factor for the sliding friction coefficient is given by

$$\phi_{bl} = \frac{1}{e^{2.6\times10^{-8}(\eta\nu)^{1.4} d_m}}, \tag{10}$$

and the μ_{bl} and μ_{EHL} values can be found in the literature under sliding frictional moment [7].

Drag Friction Torque

Bearings lubricated by the method of oil bath are either partially or fully submerged. The drag losses that occur when bearings are rotating in oil contribute to the total frictional losses. This is given by

$$T_{drag} = 4V_M K_{roll} C_w B d_m^4 n^2 + 1.093\times10^{-7} n^2 d_m^3 \left(\frac{n d_m^2 f_t}{\nu}\right)^{-1.379} R_s. \tag{11}$$

The rolling element constant and drag loss variables are

$$K_{roll} = \frac{K_L K_z (d+D)}{D-d} 10^{-12}, \tag{12}$$

and

$$\begin{aligned} & C_w = 2.789\times10^{-10} l_D^3 - 2.786\times10^{-4} l_D^2 + 0.0195 l_D + 0.6439, \\ & l_D = 5\frac{K_L B}{d_m}, \\ & f_t = \begin{cases} \sin(0.5t), & when\ 0 \le \mathrm{t} \le \pi \\ 1, & when\ \pi \le t \le 2\pi \end{cases}, \\ & R_s = 0.36 d_m^2 (t - \sin t) f_A, \\ & t = 2\cos^{-1}\left(\frac{0.6 d_m - H}{0.6 d_m}\right) \text{ when } \mathrm{H} \ge 1.2\mathrm{d}_m,\ use\ \mathrm{H} = 1.2\mathrm{d}_m, \\ & f_A = 0.05\frac{K_Z (D+d)}{D-d}. \end{aligned} \tag{13}$$

where V_M is the drag loss factor found in Diagram 4 in the SKF model [7]. K_Z and K_L are bearing and roller bearing type geometric constants.

Seal Friction Torque

Bearings are sometimes fitted with contact seals that generate more friction losses than the bearing [7]. The frictional moment of seals can be estimated using Equation (14):

$$T_{seal} = K_{S1} d_s^{\beta} + K_{S2}, \tag{14}$$

where K_{S1} and K_{S2} are constants depending on seal type, bearing type and size. Here, β is the exponent depending on seal type and bearing type and not on the bearing size, and d_s is the seal counterface diameter. The values of these constants can be found in Table 3 of "SKF Model for Calculating Frictional Moment" [7].

<u>Total Torque</u>

The total torque is the sum of the rolling, sliding, and drag friction torque and is given by the following equation:

$$T = T_{rr} + T_{sl} + T_{drag} + T_{seal}. \tag{15}$$

<u>Bearing Power Loss</u>

Based on the principle that torque times rotational frequency equals power, the power loss of the bearings was determined using Equation (16) in watts (W) [6]. Thus,

$$H = 1.047 \times 10^{-4} T.n. \tag{16}$$

4.3 Cam Power Loss Model

During EHL conditions, viscosities change as an exponential function of pressure and it is important to consider this in any EHL analysis. For this model, an average value of the viscosity change is considered [13] and is given as follows:

$$\mu_{avg} = \mu_0 \exp\left\{(\ln \mu_0 + 9.67)\left[-1 + (1 + 5.1\times10^{-9} p_h)^Z\right] - K_T \Delta T\right\}, \tag{17}$$

where the hydrodynamic pressure is given by

$$p_h = p(1 - L_a / 100), \tag{18}$$

and L_a , asperity load ratio, is given by

$$L_a = 0.005F^{-0.408}N^{-0.088}G^{0.103}[\ln(1+4470\bar{\sigma}^{6.015}V^{1.168}F^{0.485}N^{-3.741}G^{-2.898})]. \tag{19}$$

The dimensionless parameters are defined as follows:

$$\begin{aligned} F &= F / BE^{'}R, \\ N &= \mu_0 u_r / E^{'}R, \\ G &= E^{'}\alpha, \\ V &= \upsilon / E^{'}. \end{aligned} \tag{20}$$

The flash temperature, ΔT , is given by

$$\Delta T = \frac{2bq}{\sqrt{\pi}\left[k_1\sqrt{1+Pe_1}+k_2\sqrt{1+Pe_2}\right]}, \tag{21}$$

where

$$\begin{aligned} b &= \sqrt{8RF / \pi BE^{'}}, \\ Pe &= u_s b\rho C_p / 2k. \end{aligned} \tag{22}$$

The heat flux is defined as

$$q = f_c u_s p(L_a / 100) + u_s \Lambda_{\lim} p_h. \tag{23}$$

Once the average viscosity is obtained, the friction forces for sliding and rolling can be obtained and then the power losses [14]. The equations are as follows:

$$\begin{aligned} F_s &= \frac{2L\mu_0 R}{b}\frac{\mu_{avg}(n_{roller}-n_{cam})}{H}, \\ F_r &= \frac{4.485LR(2\mu_0\alpha n_{cam} / R_x)^{0.67}}{2\alpha}, \end{aligned} \tag{24}$$

$$\begin{aligned} H_s &= F_s\left|n_{cam}-n_{roller}\right|, \\ H_r &= F_r\left|n_{cam}+n_{roller}\right|. \end{aligned} \tag{25}$$

The MATLAB code for cam power loss can be found in Appendix F.

4.4 Minimum Film Thickness Model

The equations used for calculating the steady state minimum film thickness were taken from Kilundu and Devos [15], and are as follows:

$$F = \frac{F}{E_e R B}, \tag{26}$$

$$\hat{G} = \alpha E_e, \tag{27}$$

$$n = \frac{\eta_0 n}{E_e R}, \tag{28}$$

$$g_v = \frac{F^{1.5} \hat{G}}{n^{0.5}}, \tag{29}$$

$$g_e = \frac{F^{1.5}}{n^{0.5}}, \tag{30}$$

$$g_h = Z g_v^m g_e^n. \tag{31}$$

The minimum film thickness is given by

$$h_0 = R \frac{g_h n}{F^{\frac{4}{3}}}. \tag{32}$$

4.5 GT – SUITE Model

Bearing models were developed using a simulation software, GT – SUITE and the models were compared to the previously developed MATLAB models. All the types of bearings, Main, Roller Follower, Rocker Ground, Rocker Moving, and Wrist Pin, were evaluated. A schematic of the bearing model is shown in Figure 7. It includes speed, oil type, bearing mass, forces, and bearing geometry.

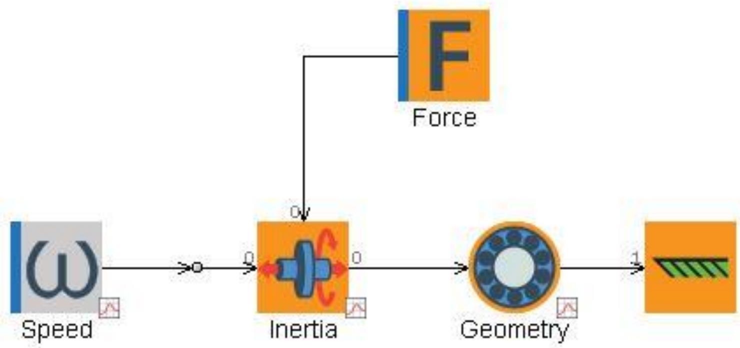

Figure 7: GT – SUITE Bearing Model.

The model was evaluated under the following operating conditions:

1) Speed – 200 RPM.
2) Temperature - 50°C.
3) Fluid – Group 1 HM 46.
4) Pressure – 3000 psi.

Force and speed inputs from UMN were entered into the model for each bearing and then evaluated, as shown in Figure 8 and 9. Geometric characteristics (D, d, B, # rollers) for the bearings selected by UMN were entered into the model and are shown in Figure 10. The effect of seals in friction were also incorporated in the model where applicable. There are three types of power loss calculation methods in GT – SUITE and all three were used in this project, including the methods described in ISO14179 – US, ISO14179 – DE, and SKF, as seen in Figure 11. ISO 14179 consists of two parts, US and DE [16]. The US part is the American proposal and it utilizes an analytical heat balance model to calculate the thermal transmittable power for a single or multiple stage gear drive lubricated with mineral oil [16]. The DE part is based on a German proposal where the thermal equilibrium between power loss and dissipated heat is calculated and it is an iterative method [16]. It was observed from the results that the seal losses were not incorporated in the DE method. The Fluid Properties model (Appendix E) was utilized to gather kinematic and dynamic viscosity values for temperature ranging from 10°C to 100°C and pressure ranging from 250 psi to 5000 psi. An example of dynamic input viscosity is shown in Figure 12 and the detailed inputs can be found in Appendix J.

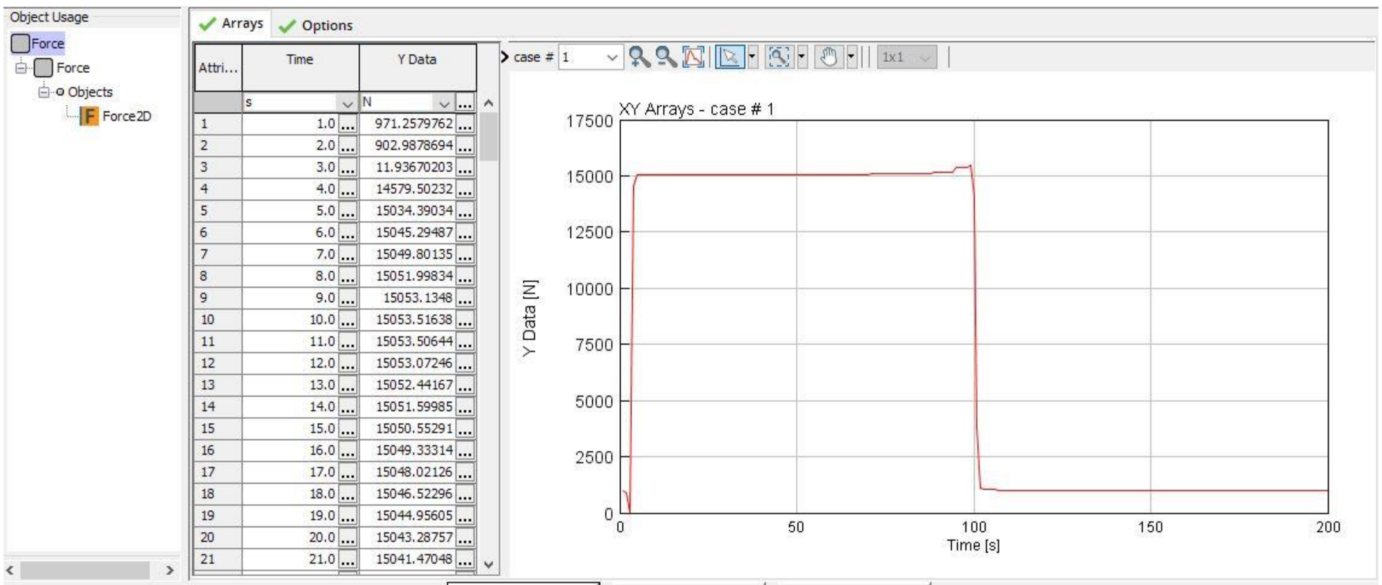

Figure 8: GT – SUITE Force Input.

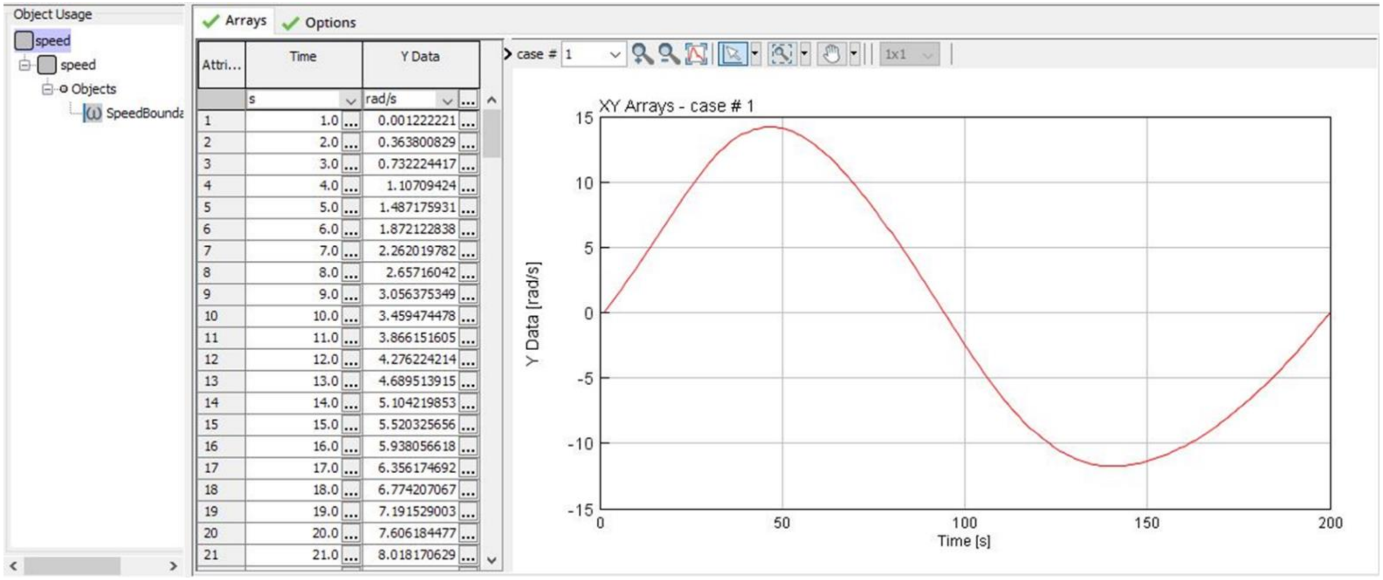

Figure 9: GT – SUITE Speed Input.

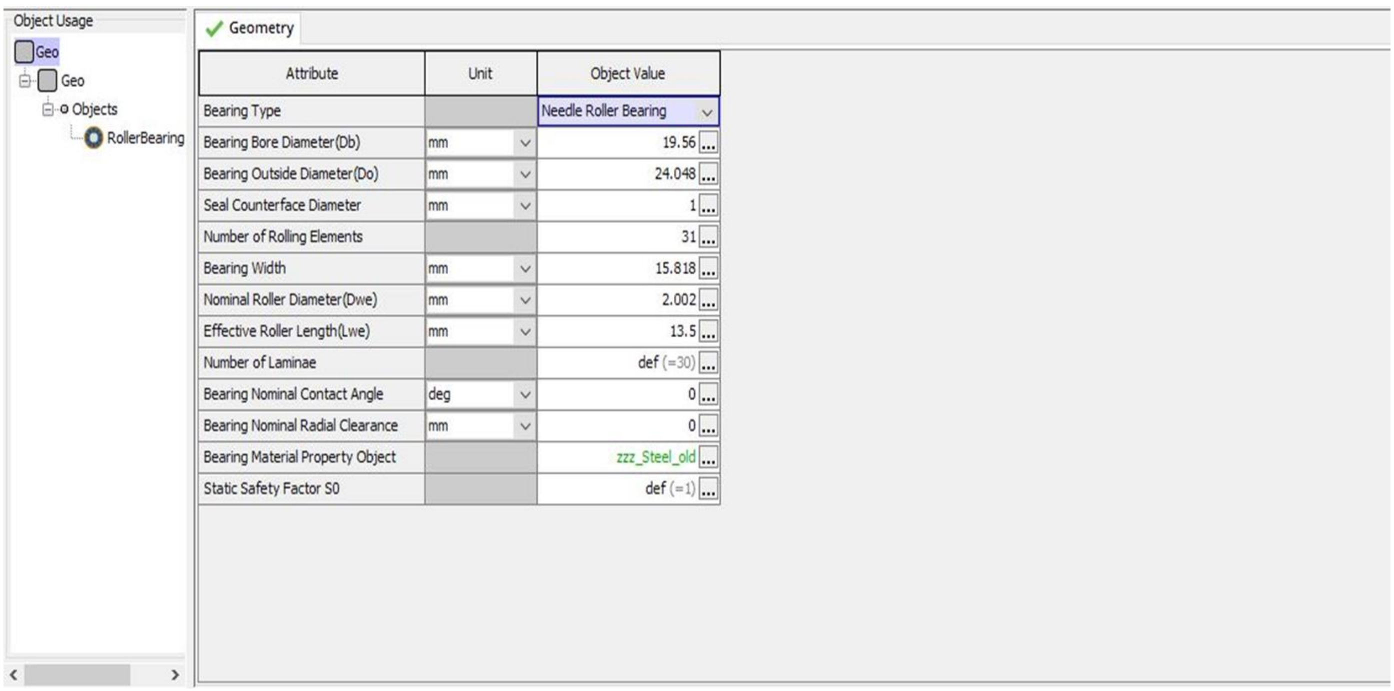

Figure 10: Bearing Specification Input.

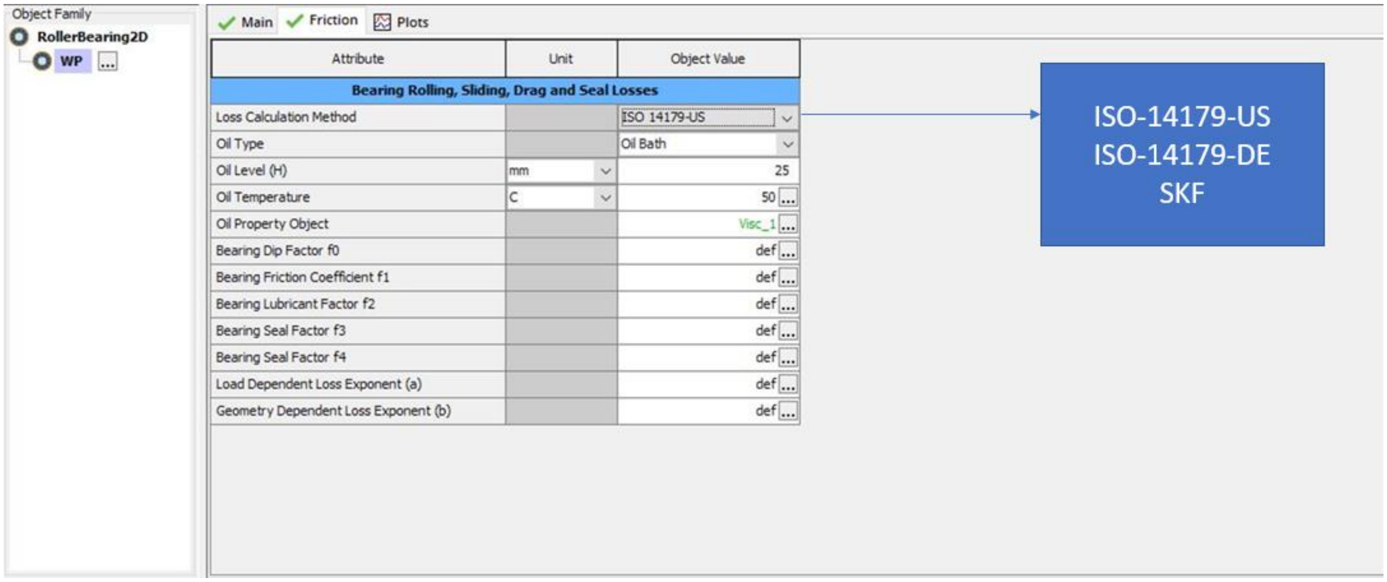

Figure 11: GT – SUITE Power Loss Methods.

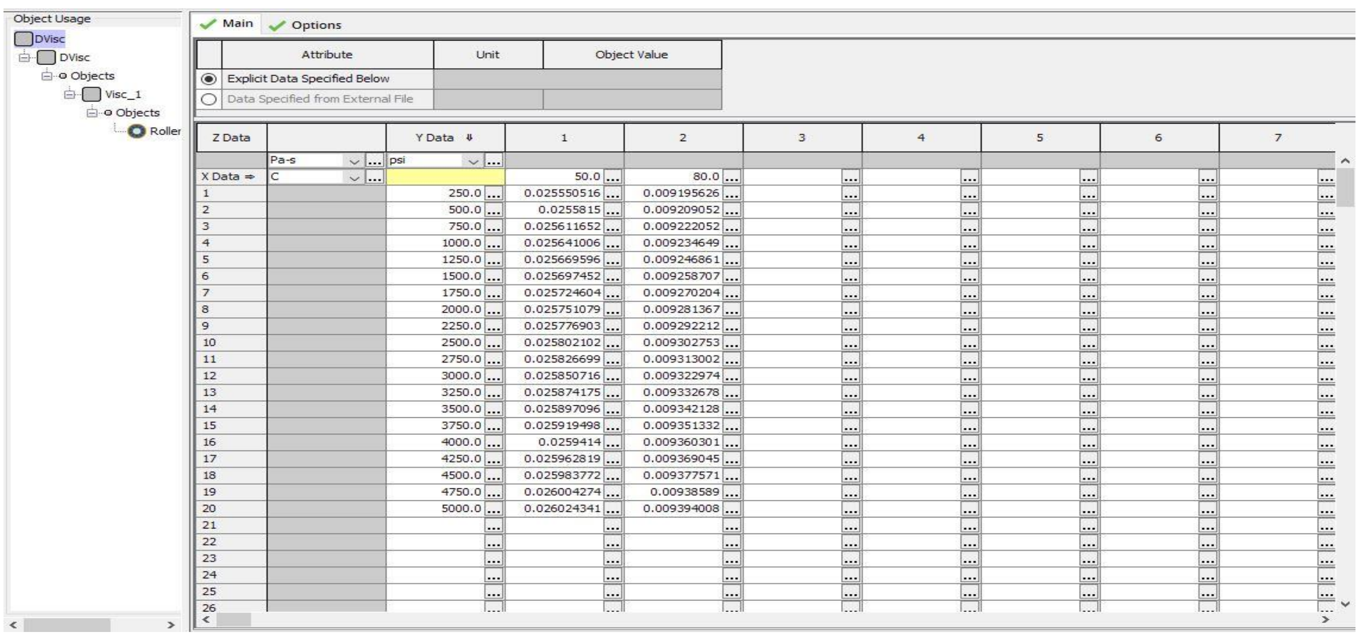

Figure 12: Dynamic Viscosity Input.

The bearings were procured from a local vendor, Allied Bearings, and the bearing dimensions were physically measured by the author with the help of a caliper. The dimensions were measured five times and the average value was taken for the analysis in GT – SUITE. The bearing pictures have been included in the GT – SUITE results section.

5. MATLAB Models

Two generations of MATLAB models were developed in this project. The first generation will be discussed first followed by the second generation and the new updates in the latter generation.

First Generation: General script files called 'Torque_Power_Harris' and 'Torque_Power_S' were created for both types of bearings (Appendix A and B). The inputs and outputs are shown in Figure 13.

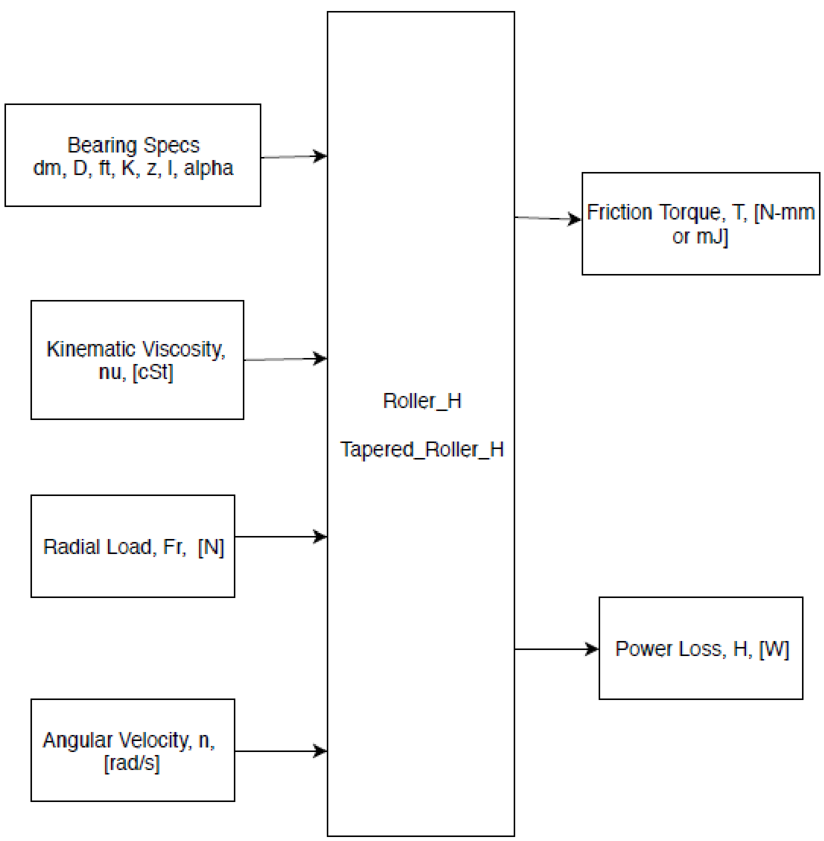

Figure 13: Input/ Output Flowchart for Calculation of Friction Torque and Power Loss.

The general script files obtain the pitch diameter input from the bearing specifications, oil viscosity input from the ‘FluidProperties’ script (Appendix E), radial load and the rotational frequency input from the data provided by UMN (Appendix H). The analysis was performed for a fraction of the cycle, 100 points.

Two different functions for each model were created for the needle roller and tapered roller bearings. The functions included the equations mentioned above and were called upon execution of the script. The script started with the description of the code and followed by the inputs provided. The ‘FluidProperty’ script developed by Pawan Panwar [11] was used to acquire the kinematic viscosity for the given temperature. Thus, for any temperature, friction torque and power losses can be calculated for all the bearings. Elementwise calculations were carried out for calculating the friction torque and power losses. The results obtained were plotted as well in a single figure using subplot function. This procedure was followed for all the bearings. A final graph was plotted showing the combined losses of all the bearings. The results can be calculated for speeds of 1, 10, 100, and 200 rpm. A similar approach as that of the Harris and Kotzalas model is used for calling the functions and the script file in the SKF model.

Second Generation: A new approach was used to create MATLAB models and it was in a structured form. This was done according to the guidelines of UMN and the project. UMN system modeling for the motor used very long scripts and functions, and in order to be efficiently running the system design code. UMN approached MATLAB modeling in a structured format. Similarly, the author updated the SKF bearing and the cam power loss models to comply with a structured format (Appendix G). A script file “TP_S” was created to call all the required input functions and calculation functions. Packed input functions for bearing specifications, forces, angular velocities, and fluid properties were created. Similar to the first-generation code, two calculation functions, ‘Roller_S’ and ‘Tapered_Roller_S’; were created to calculate friction torque and power losses in a roller type bearing and a tapered roller type bearing. The packed input functions were unpacked in the calculating functions and then all the functions were called in the main script file. The input functions were created for the ease of use for UMN as they would mainly require the generic functions to integrate bearing power losses in their code and compile the results as whole. The operating conditions for the structured code are similar to the first generation and can be run for any set of temperature and pressure values for motor speeds of 1, 10, 100, 200 rpm.

6. Results and Discussion

6.1 Bearing Power Loss

The analysis done was for a four – lobe, seven – piston motor. The results shown in Figure 14 and 15 indicate that at 200 RPM, 3000 psi, and an oil temperature of 50°C, the power losses were less than 100 watts and 130 watts. Figure 14 shows the results from the Harris and Kotzala model and Figure 15 shows results from the SKF model. The first fifty points are for motoring phase and the second fifty points are for pumping phase. It is evident that losses are higher in the motoring phase and it is as expected. As seen in the Harris and Kotzalas model, the roller follower bearing had the maximum contribution towards total power losses. However, it can be noted that for the SKF model, the main bearing had a higher power loss than the other bearings. This is as expected since the main bearing has a pre-load axial component incorporated even when there is no radial load. This pre-load factor gives rise to a higher power loss.

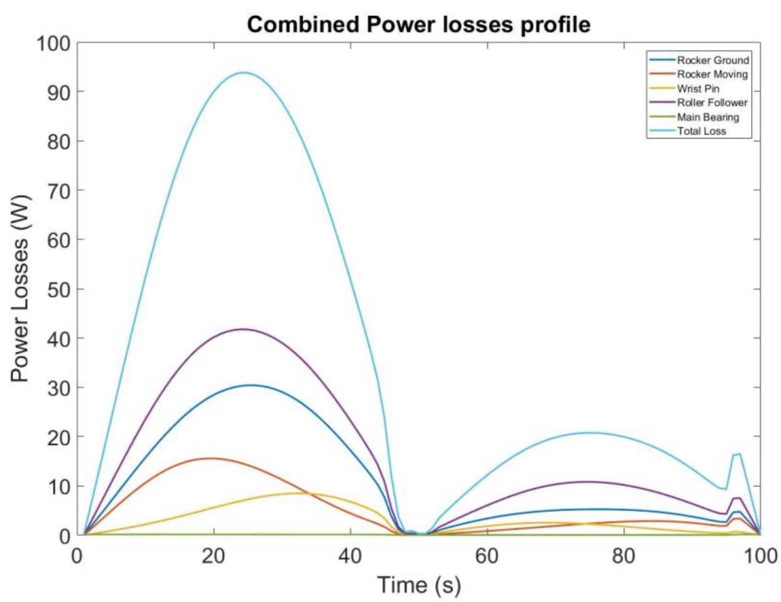

Figure 14: Combined Bearing Power Loss (Harris and Kotzalas Model).

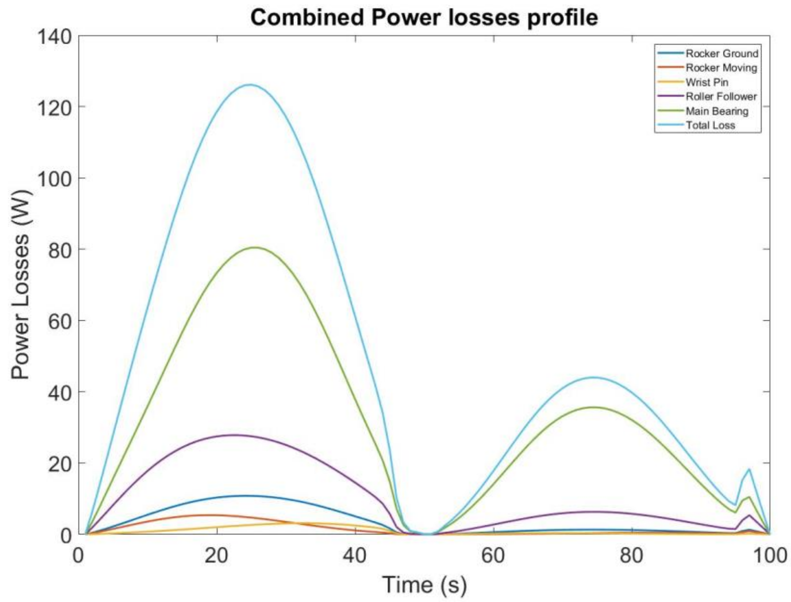

Figure 15: Combined Bearing Power Loss (SKF Model).

6.2 Structure Bearing Power Loss and Data Update

The team at UMN have been optimizing their motor design to get a much more efficient motor with better forces and angular velocities, and in turn, different bearing sizes. Figure 16 shows the old and new SKF bearing losses. It can be noted that the curves are much flatter than the previous models (Figure 14 and 15) and this is due to the fact that the data for forces and angular velocities were optimized by UMN. The most updated data can be found in Appendix H.

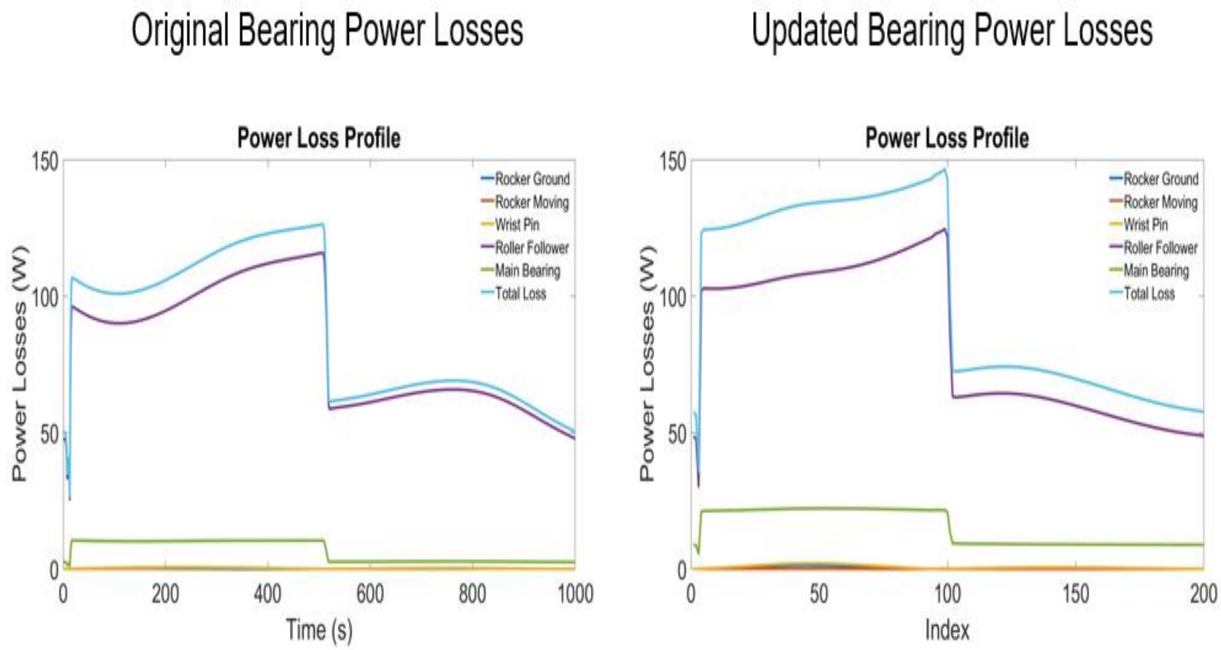

Figure 16: Total Bearing Loss Comparison.

It can be noted that the losses are higher for the updated bearing power losses on the right. There are several reasons for this result. The design for the older model included 14 RG, 14 RM, 7 WP, 7 RF, and 1 MB, and the newer model consisted of more bearings, 14 RG, 28 RM, 14 WP, 7 RF, 2MB – therefore, increasing the overall power loss. The design optimizations also included slightly larger bearings to handle the optimized forces and angular velocities – therefore, also increasing the overall power loss. Table 1 shows a snapshot of the increase in the dimensions of the bearings and a detailed list of specifications can be found in Appendix I. The new bearings also had different SKF constants, which affected the overall power loss.

Table 1: Bearing Specifications.

	Original	**New**	**Original**	**New**
	Pitch Diameter (mm)		Width (mm)	
Roller Follower (1)	29.5	31	19	19
Rocker Ground (2)	19	26.5	12	15
Rocker Moving (4)	19	19	12	16
Wrist Pin (2)	15.1	21	15.9	16
Main Bearing (2)	61	107.5	15	23

6.3 Average Losses with Seal

Seal losses were included in the power loss models, which increased the power losses significantly. The single cylinder prototype analysis consisted of 2 MB, 1 RF, 2 RG, 4 RM, and 2 WP. The multi cylinder (7 cylinders) model analysis consisted of 2 MB, 7 RF, 14 RG, 28 RM, and 14 WP. The average power losses for single bearings, single cylinder prototype, and multi cylinder prototype are shown in Table 2. The quantity of bearings is shown in curved brackets () after the power loss values for ease of comparison.

Table 2: Average Bearing Power Loss Comparison with Seal

	Single Bearing Power Loss (W)	Single Cylinder Power Loss 1 linkage (W)	Multi Cylinder Power Loss 7 linkages (W)
MB	16.3 (1)	27.9 (2)	27.9 (2)
RF	41.5 (1)	41.5 (1)	291 (7)
RG	0.81 (1)	1.41 (2)	9.93 (14)
RM	0.01 (1)	0.015 (4)	0.11 (28)
WP	0.30 (1)	0.34 (2)	2.44 (14)

It can be noted that the roller follower had the most contribution towards power losses of the bearings followed by the main bearing. The radial load is divided by the number of bearings for each type of bearing, which distributes the load evenly to the bearings, reducing power losses.

6.4 Total Losses for Single Cylinder with Seal

The total power loss for the single cylinder prototype was evaluated and is shown in Figure 17. All the types of bearings have been shown and the losses include rolling, sliding, drag, and seal losses. It can be seen that power losses are lower than 100 watts.

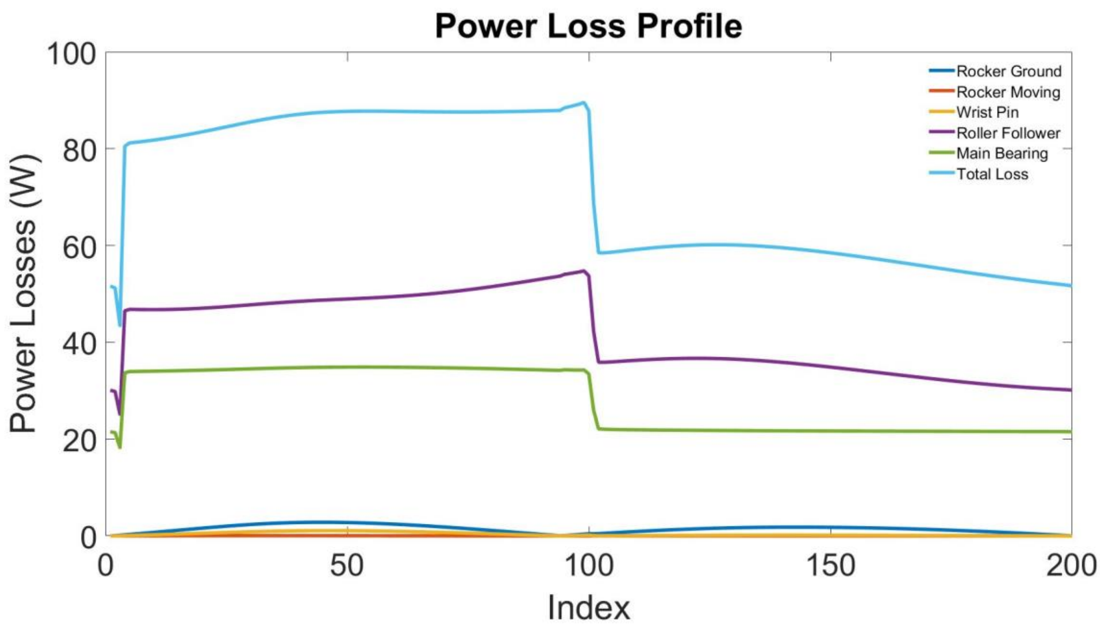

Figure 17: Total Power Losses for a Single Cylinder Prototype.

The roller follower contributes the most in the single cylinder prototype, followed closely by the main bearings. It can be observed that the other bearings have a very low power loss profile compared to the roller follower and main bearing.

6.5 Total Losses for Multi Cylinder with Seal

The total power loss for the multi (seven) cylinder prototype was also evaluated and is shown in Figure 18. Similar to the single cylinder prototype, all the types of bearings have been shown and the losses include rolling, sliding, drag, and seal losses. It can be seen that power losses are lower than 350 watts.

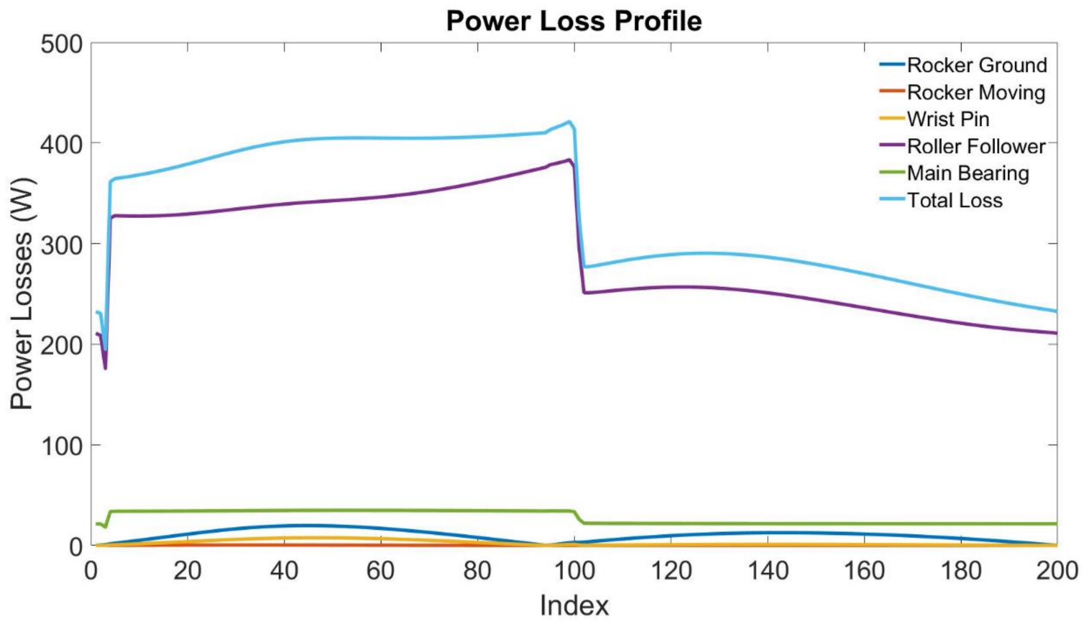

Figure 18: Total Power Losses for a Multi Cylinder Prototype.

Similar to the single cylinder prototype, the roller follower contributes the most in the multi cylinder prototype. It is followed by the main bearings and rocker ground bearings, which are much lower. It can be observed that the rocker moving, and wrist pin bearings have a very low power loss profile compared to the roller follower.

The average power loss for the single cylinder and multi cylinder prototypes are shown in Table 3.

Table 3: Total Average Power Loss Comparison.

	Single Cylinder	Multi Cylinder
Average Power Loss (W)	71.2	331

The single cylinder prototype has an average power loss of about 71 watts and is much lower than the multi cylinder prototype as the multi cylinder prototype has many more bearings included in it.

6.6 Bearing Film Thickness

The film thickness has a hydrodynamic and elastohydrodynamic regime, as shown in Figures 17 and 18, for the rocker ground bearing. For the elastohydrodynamic part, the film thickness results range from 1 to 5 μm, which is normal for elastohydrodynamic lubrication.

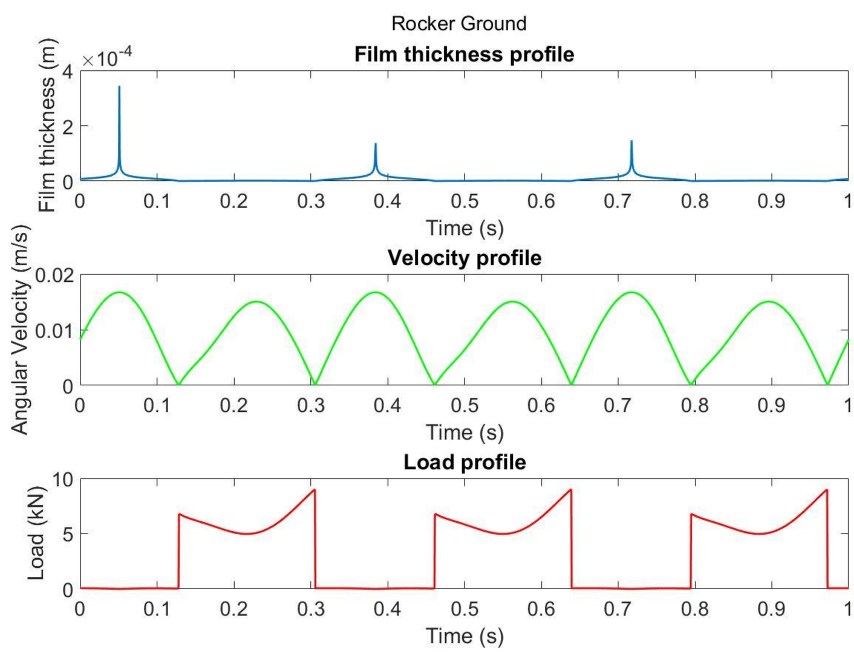

Figure 19: Rocker Ground Bearing Film Thickness (Hydrodynamic).

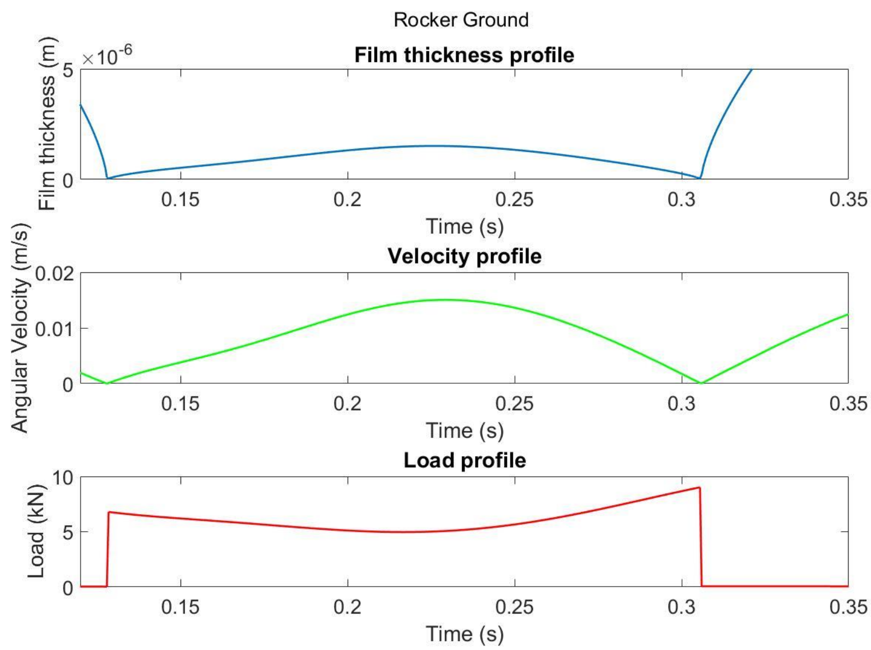

Figure 20: Rocker Ground Bearing Film Thickness (Elastohydrodynamic).

The film thicknesses for rocker moving and wrist pin can be similarly obtained (see Appendix C).

6.7 Cam Film Thickness

For the cam follower interface, a range of 0.1 to 0.3 μm were obtained, as seen in Figure 19. The film thicknesses obtained are within the acceptable range for a cam follower interface. Cam follower film thicknesses are compressed by the extreme loads experienced in the interface.

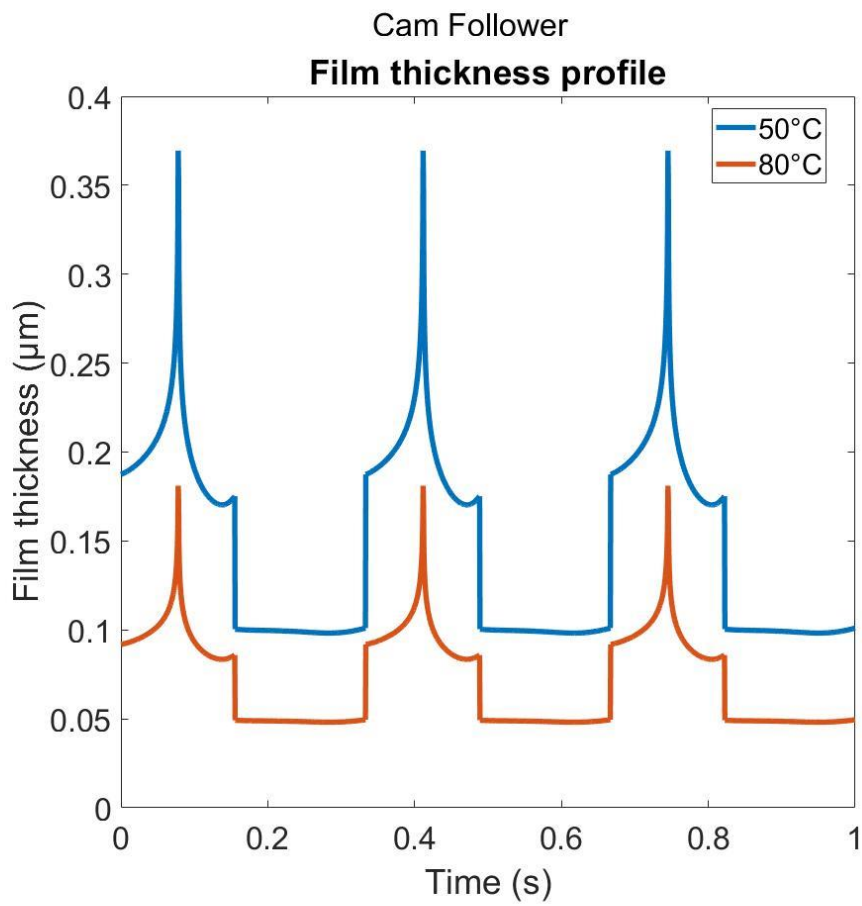

Figure 21: Cam Follower Interface Film Thickness.

6.8 Cam Power Loss

The cam power loss between one roller follower and the cam is shown in Figure 20. The rolling losses are almost constant and are about 2.5 W. The sliding power losses are lower than rolling losses and are about 1 W for the motoring cycle. The total power loss ranges from 3 to 3.5 W for the motoring and pumping cycle with respect to the latest data set.

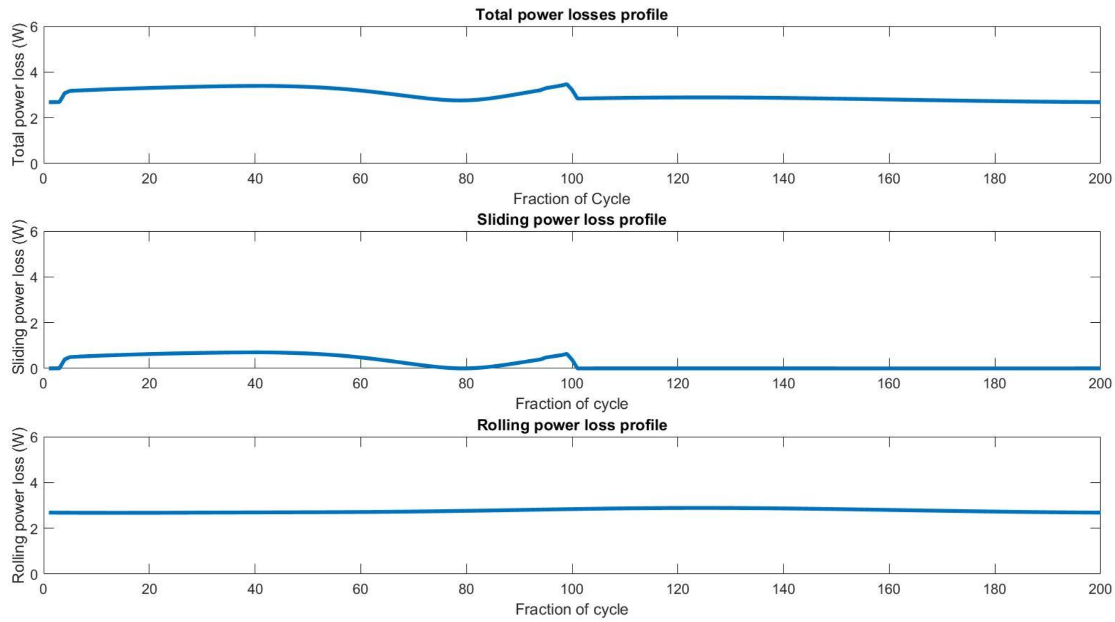

Figure 22: Cam Roller Follower Power Loss.

6.9 GT – SUITE Results

The results first show the plot from MATLAB model followed by the three GT – SUITE loss calculation method plots. This analysis was done for a single bearing for the ease of comparison. The wrist pin bearing, the rocker moving bearing, the rocker ground bearing, the roller follower bearing, and the main bearing each were analyzed with the MATLAB models developed for this project, as well as GT – SUITE simulations employing three loss calculation methods based on ISO 14179 – US, ISO 14179 – DE, and the SKF approach.

1) Wrist Pin Bearing Results

The model results for the wrist pin bearing are shown in Figure 21. The MATLAB and GT – SUITE SKF results shown on the left exhibit good agreement. The ISO results, both US and DE, are slightly higher. The ISO methods generate a higher estimate of rolling friction. As shown in Figure 22, the wrist pin bearing is splash lubricated and does not have seals. As a result, the friction is relatively low and there is good agreement between all four models. The power losses in the full complement wrist pin bearings are very low. They range from 0.1 to 1.4 W. The models agree within a fraction of a watt.

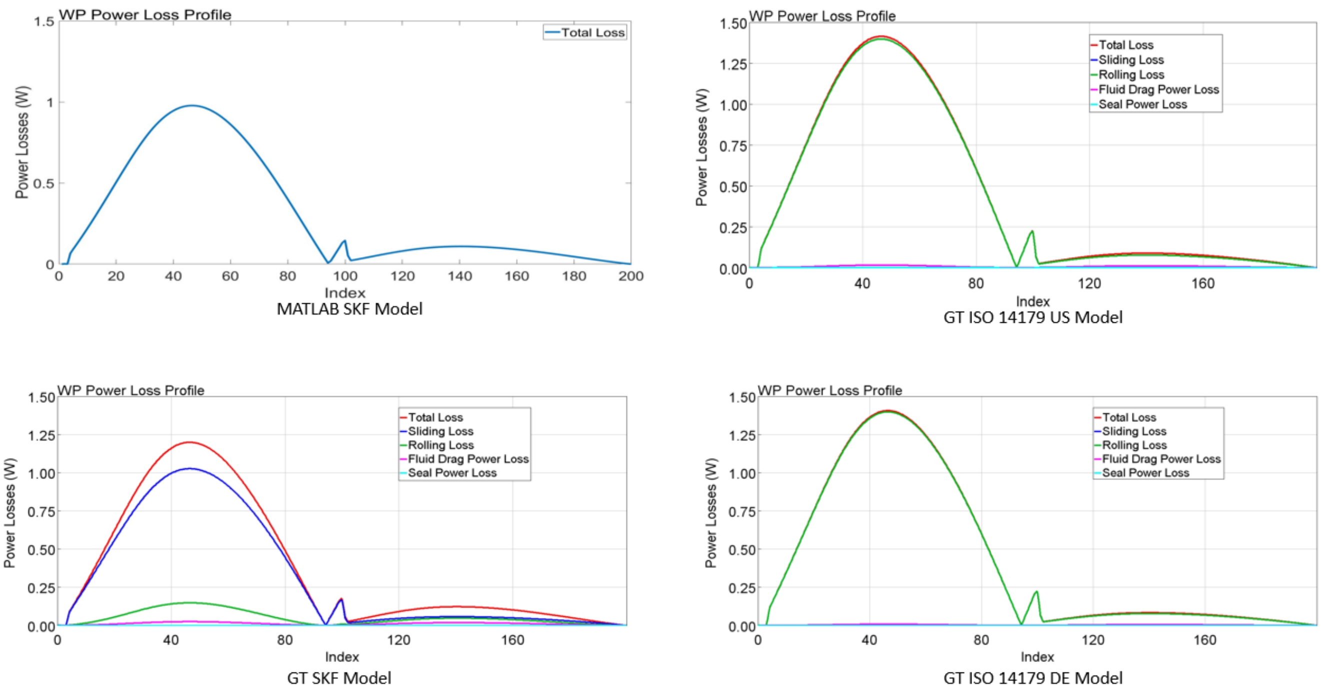

Figure 23: Full Complement Wrist Pin Power Loss Comparison.

Figure 24: Schaeffler HN1816 Wrist Pin Bearing.

2) Rocker Moving Bearing Results

Similar results were observed with the rocker moving bearing, which does not incorporate a seal and is splash lubricated. The results are shown in Figure 23 and the bearing is shown in Figure 24. The power losses in the full complement rocker moving bearing are low, as well. They range from 0.06 to 0.1 W. The velocities are very low in the rocker moving and wrist pin bearings. As a result, the predicted power losses are less than 0.1 W.

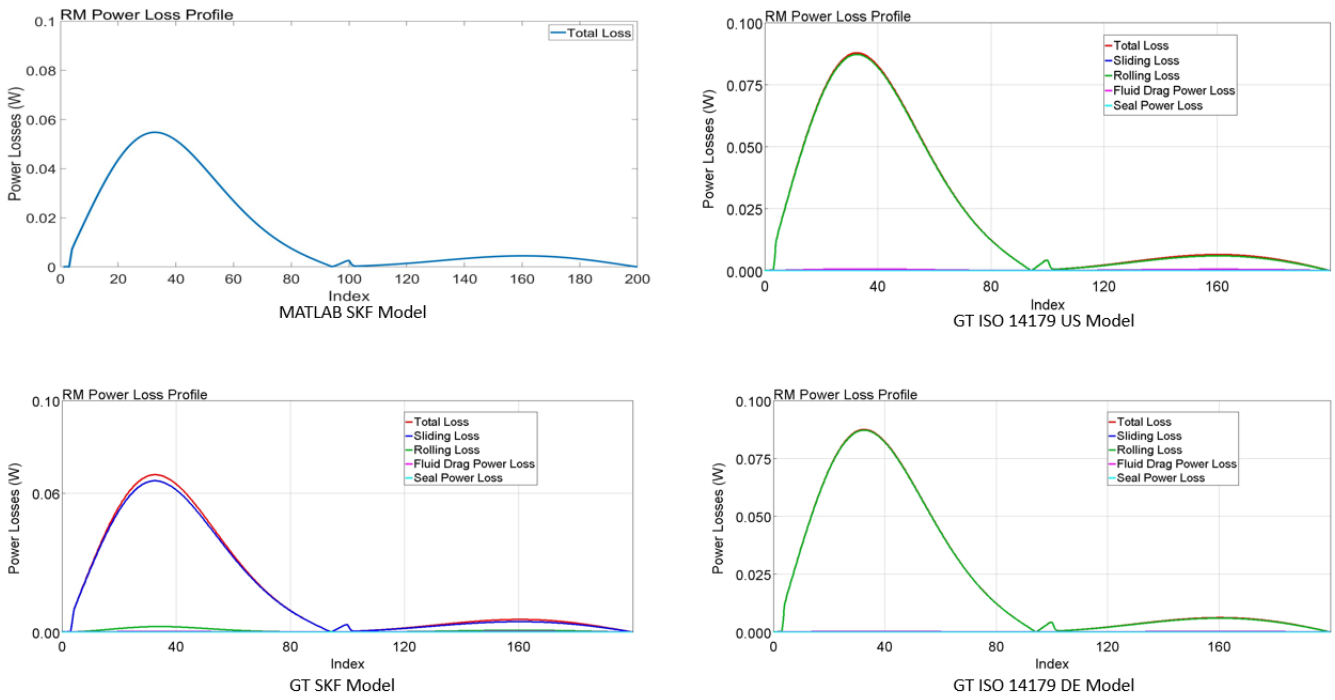

Figure 25: Full Complement Rocker Moving Power Loss Comparison.

Figure 26: Schaeffler HN1516 Rocker Moving Bearing.

3) Roller Follower Bearing Results

The model results for the roller follower bearing are shown in Figure 25. This bearing incorporates seals, as shown in Figure 26. Note that the MATLAB SKF and GT – SUITE ISO DE results, on the left, show lower power losses than the GT – SUITE SKF and GT – SUITE ISO US results. This is because the later models incorporate seal friction terms. The GT – SUITE US and GT – SUITE SKF models include the seal friction in the roller follower and the power losses come out to be lower than 60 W. The rolling loss seems to be the major contributor to the roller follower losses. The velocity of the roller follower is very high, and therefore, the results show higher losses.

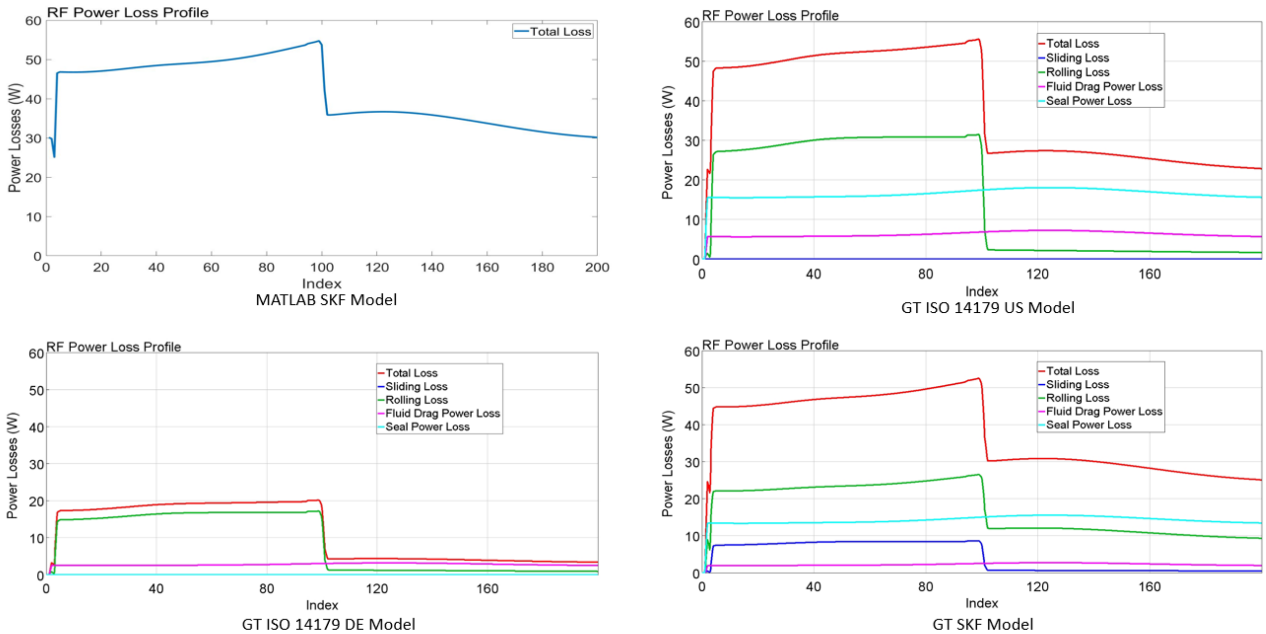

Figure 27: Sealed Full Complement Roller Follower Power Loss Comparison.

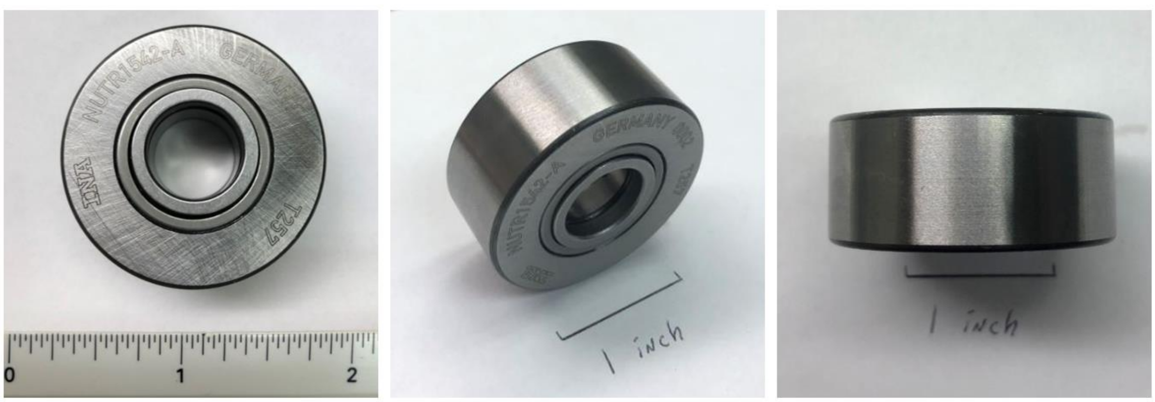

Figure 28: Schaeffler NUTR1542 Roller Follower Bearing.

4) Rocker Ground Bearing Results

The RG bearing is also sealed, and similar results with the roller follower can be observed where the presence of seals increases power losses. The results are shown in Figure 27 and the bearing is shown in Figure 28. In Figure 28, the graphs on the left show the MATLAB SKF and GT – SUITE ISO DE results and they are lower than 1 W. The graphs on the right show the GT – SUITE ISO US and GT – SUITE SKF results and the power losses come out to be lower than 2 W. The velocities are much lower, and hence, the power losses are much lower.

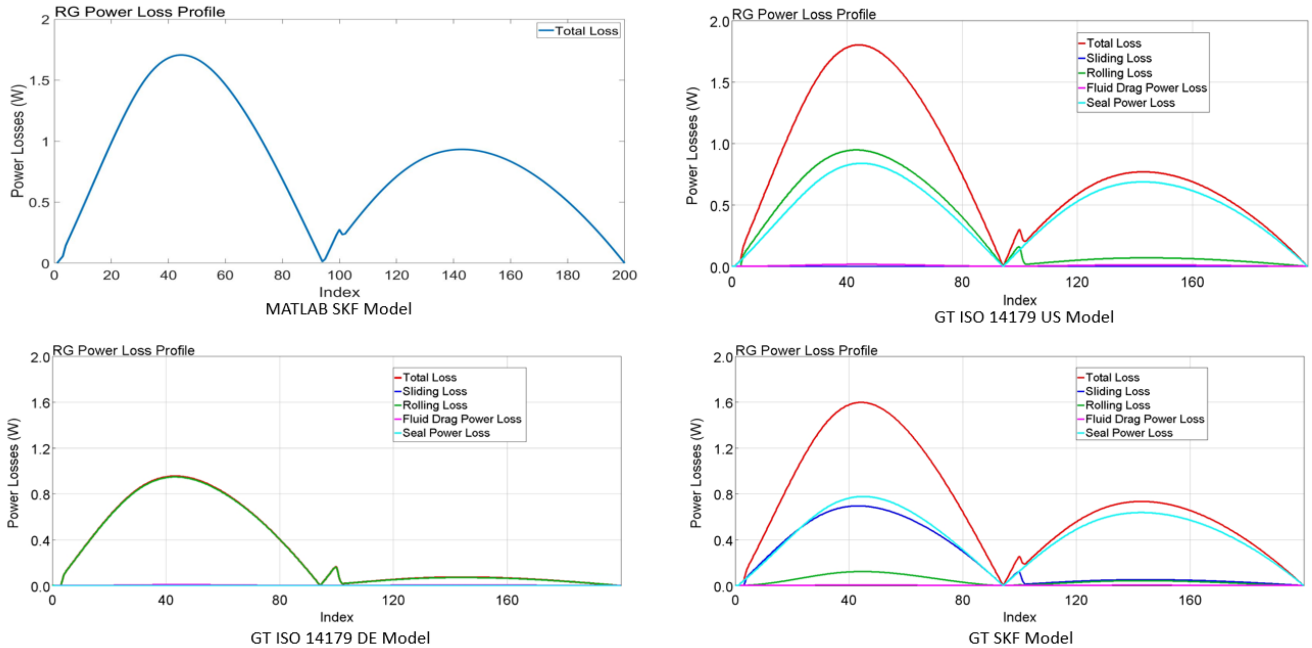

Figure 29: Sealed Full Complement Rocker Ground Power Loss Comparison.

Figure 30: Schaeffler NATR10-PP Rocker Ground Bearing.

5) Main Bearing Results

The main bearing is a tapered roller bearing. The power losses associated with the bearing, and determined by the MATLAB and GT – SUITE models using the ISO DE method, are shown on the left side. Both results are slightly lower and around 15 W, but the general shapes are in agreement. The power losses during motoring are greater than the power losses during pumping. The ISO US and GT – SUITE SKF method includes seal friction and power losses are less than 20 W. The power losses are higher than the ISO DE and MATLAB SKF models as they exclude seal friction. Since it is independent of load and only depends on sliding speed, the seal friction is a constant. While the main bearing does not have a built-in seal, a seal is required to retain fluid within the motor case. The results are shown in Figure 29 and the bearing is shown in Figure 30.

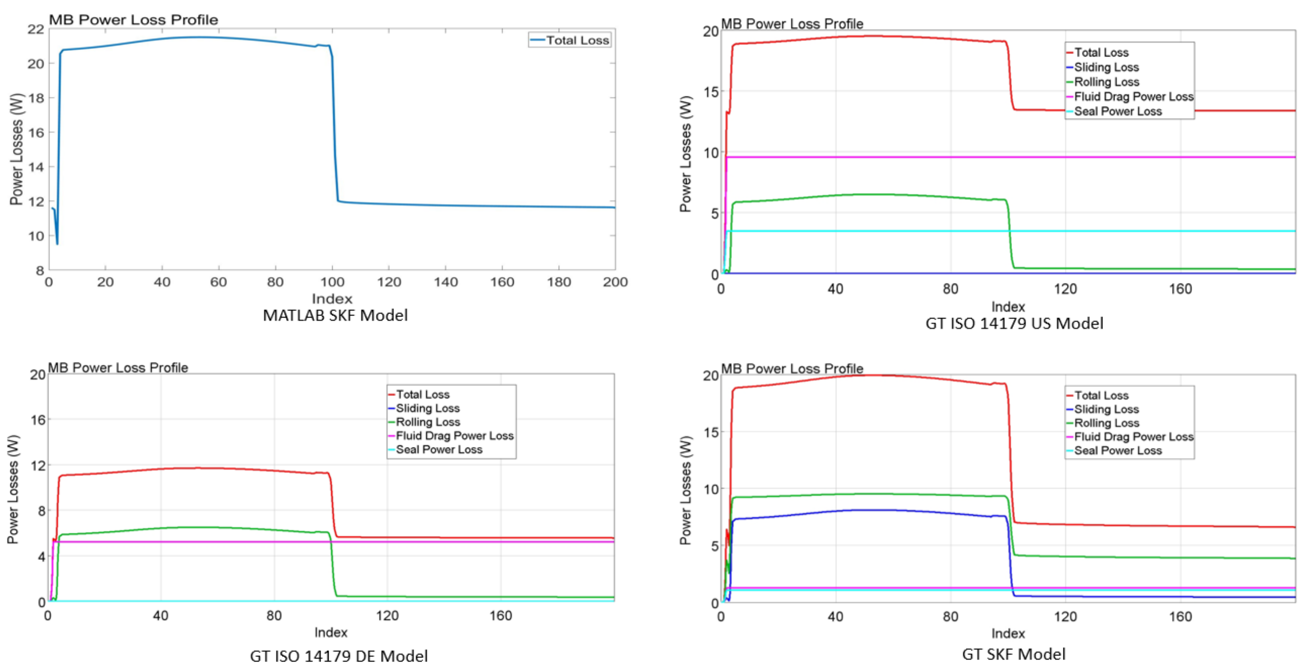

Figure 31: Tapered Roller Main Bearing Power Loss Comparison.

Figure 32: Koyo 32918JR Main Bearing.

Seal friction was found to be a significant factor in roller follower performance. Identifying roller followers without seals maybe beneficial for future designs in the VDLM. All bearings show agreement in the power loss analysis for the MATLAB and GT – SUITE models.

7. Summary

The goal of the project was to create tribological models for bearing and cam interfaces in a variable displacement linkage motor (VDLM). The first phase of the project involved the proposal and included the initial literature review for bearing friction torque and power loss. The second phase involved the development of film thickness and power loss models in MATLAB. The final phase of this project was to compare the MATLAB power loss models with simulations developed in GT – SUITE. The most recent force, angular velocity, and geometric properties from the VDLM design were incorporated at this stage.

The film thickness models were based on Grubin and Dawson’s dimensional analysis of elastohydrodynamic films [17, 18]. These results showed both elastohydrodynamic and hydrodynamic lubrication regimes depending upon the instantaneous load conditions. Film thicknesses were in the normal range for bearings and cam followers, with the cam follower film thickness being significantly lower than that of the bearings.

The power loss model was based on SKF and ISO empirical models. Speed, load, and viscosity were the major inputs. MATLAB single bearing models were compared with GT – SUITE simulation software. The methodologies displayed good agreement with each other. Bearings with low rotational frequencies, such as the rocker ground and rocker moving bearings, had low power losses. The heavy loaded cam roller follower and the large diameter tapered roller bearing had high power losses. Seal friction was found to be a significant factor in the power loss equation. The total average power loss during the motoring and pumping cycle for the single cylinder prototype was 71 watts. In the multi cylinder version of the VDLM, the total average power loss was 331 watts. The roller follower was the largest contributor to power losses in both cases.

The power loss for bearings and cam follower in the single cylinder prototype averaged 71 watts, which amounts to 0.6% of the peak output power of the motor. The power loss for bearings and cam follower in the multi cylinder prototype averaged 331 watts, which amounts to 0.7% of the peak output power of the motor.

A single cylinder prototype motor will be installed at the Fluid Power Institute in the Spring of 2020. Motor torque will be measured under varying conditions of pressure and speed. The difference between the actual torque output and theoretical torque output is, by definition, friction loss. Hence, bearing friction can be derived from the difference. These results will be used to refine the friction and kinematic models used for optimizing the multi cylinder VDLM prototype.

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Appendices

Appendix A: Harris and Kotzalas Bearing Power Loss

<u>MATLAB Script</u>

The script requires the code for Fluid Properties [Appendix E].

```
       % DESCRIPTION
       % Running this program will calculate the Frictional Torque and Power
       % Losses in all the bearings using inputs such as
       % pitch diameter, kinematic viscosity from 'FluidProp' function, rotational
       % frequency, and radial load. It also calculates the total power loss for
       % the bearings. This script is created according to the Harris and Kotzalas
model [1].

       % [1] Harris, Tedric A. and Kotzalas, Michael N., 2007.
       %     Rolling Bearing Analysis: Essential Concepts of Bearing Technology.
Taylor and Francis Group,
       %     5ed, p-176, 185-186, 190.

       % ONE-LINE DESCRIPTION
       % Calculates friction torque and power loss for all bearings.

       % REQUIRED ARGUMENTS
       % dm                                     Pitch Diameter [mm]
       % n                                      Angular velocity [rad/s converted to
rpm in formula]
       % Fr                                     Radial Load [N]
       % nu                                     Kinematic Viscosity [cSt]
       % ft                                     Equivalent thrust load factor (Figure
10.2)
       % K                                      Bearing factor (Basic dynamic radial
load rating/basic dynamic thrust load rating)
       % D                                      Mean roller diameter [mm]
       % Z                                      Number of rollers
       % l                                      Roller race contact length [mm]
       % alpha                                  Half cup angle [degree]

       % OPTIONAL ARGUMENTS
       % [No additional arguments]

       % OUTPUT VARIABLES
       % T                                      [N.mm]
       % H                                      [Watts]

       % REQUIRED FUNCTIONS (USER-DEFINED)
       % FluidParameters.m
       % FldPropModel.m
```

```
    % Roller_H.m
    % Tapered_Roller_H.m


    % PROGRAM HISTORY
    %{
    Created      : 12/26/2018, Jordan Saikia, MSOE
    Last Modified: 02/01/2019, Jordan Saikia, MSOE

    CHANGE LOG
    ===================================================================
    -12/27/2018 Updated function to one function for all needle rollers
    -02/01/2019 Included import of matlab data instead of excel file
    ===================================================================

    ===================== 12/26/2018 ==================================
    -Initial release.
    ===================================================================
    %}

    %% (ROCKER GROUND, RG)
    % INPUTS

    % Fluid properties
    % Operating parameters
    OP.FluidNum = 1;                                    % Type 1, 2, 3, 4, or 5
respectively for fluids GRP1, HVI, TMP, PAO, or PAG
    OP.T = 50;                                          % Temperature of the fluid
[C]
    OP.p = convpres(3000,'psi','pa');                   % Pressure of the fluid
[pa]
    % Get all the fixed parameters of the fluids
    [FldPar,Fld] = FluidParameters(OP.FluidNum);
    % Call FluidPropModel() function to determine fluid properties
    [FldPro] = FldPropModel(Fld,OP.T,OP.p);
    nu_0 = FldPro.nu;                                   % Inlet dynamic Viscosity
[Pa.s]

    % Rocker Ground (RG) Bearing
    dm1 = 19;                                           % Pitch Diameter [mm]

    % Kinematics and Kinetics
    load('Full Disp 200RPM 3000PSI.mat');
    Fr1 = (LD.F14r)/2;                                  % Radial Load of RG [N]
    n1 = LK.om_RG;                                      % Velocity of RG [rad/s to
rpm converted in function]
    t = linspace(1,100,100);

    % OUTPUTS
    % Friction Torque and Power Loss
```

```
	[T1,H1] = Roller_H(dm1,nu_0,n1,Fr1);

	% Plots
	figure('Name','Rocker Ground');
	suptitle('Rocker Ground')
	subplot(2,2,3)
	plot(t,T1)
	title('Friction Torque profile');
	xlabel('Time (s)')
	ylabel('Torque (N.mm)')

	subplot(2,2,4)
	plot(t,H1)
	title('Power Loss profile');
	xlabel('Time (s)')
	ylabel('Power Loss (W)')

	subplot(2,2,1)
	plot(t,Fr1)
	title('Radial Load profile');
	xlabel('Time (s)')
	ylabel('Radial Load (N)')

	subplot(2,2,2)
	plot(t,n1)
	title('Angular Velocity profile');
	xlabel('Time (s)')
	ylabel('Angular Velocity (rad/s)')

	%% (ROCKER MOVING, RM)
	% INPUTS
	dm2 = 19;                                               % Pitch Diameter [mm]

	Fr2 = (LD.F43r)/2;                                      % Radial Load of RM [N]
	n2 = LK.om_RM;                                          % Velocity of RM [rad/s to
rpm converted in function]

	% OUTPUTS
	[T2,H2] = Roller_H(dm2,nu_0,n2,Fr2);

	% Plots
	figure('Name','Rocker Moving')
	suptitle('Rocker Moving')
	subplot(2,2,3)
	plot(t,T2)
	title('Friction Torque profile');
	xlabel('Time (s)')
	ylabel('Torque (N.mm)')

	subplot(2,2,4)
```

```
        plot(t,H2)
        title('Power Loss profile');
        xlabel('Time (s)')
        ylabel('Power Loss (W)')

        subplot(2,2,1)
        plot(t,Fr2)
        title('Radial Load profile');
        xlabel('Time (s)')
        ylabel('Radial Load (N)')

        subplot(2,2,2)
        plot(t,n2)
        title('Angular Velocity profile');
        xlabel('Time (s)')
        ylabel('Angular Velocity (rad/s)')

        %% (WRIST PIN, WP)
        % INPUTS
        dm3 = 15.0815;                                          % Pitch Diameter [mm]

        Fr3 = LD.F32r;                                          % Radial Load of WP [N]
        n3 = LK.om_WP;                                          % Velocity of WP [rad/s to
rpm converted in function]

        % OUTPUTS
        [T3,H3] = Roller_H(dm3,nu_0,n3,Fr3);

        % Plots
        figure('Name','Wrist Pin')
        suptitle('Wrist Pin')
        subplot(2,2,3)
        plot(t,T3)
        title('Friction Torque profile');
        xlabel('Time (s)')
        ylabel('Torque (N.mm)')

        subplot(2,2,4)
        plot(t,H3)
        title('Power Loss profile');
        xlabel('Time (s)')
        ylabel('Power Loss (W)')

        subplot(2,2,1)
        plot(t,Fr3)
        title('Radial Load profile');
        xlabel('Time (s)')
        ylabel('Radial Load (N)')

        subplot(2,2,2)
```

```
        plot(t,n3)
        title('Angular Velocity profile');
        xlabel('Time (s)')
        ylabel('Angular Velocity (rad/s)')


        %% (ROLLER FOLLOWER, RF)
        % INPUTS
        dm4 = 9.5;                                              % Pitch Diameter [mm]

        Fr4 = LD.FC3r;                                          % Radial Load of WP [N]
        n4 = LK.om_RF;                                          % Velocity of WP [rad/s to
rpm converted in function]


        % OUTPUTS
        [T4,H4] = Roller_H(dm4,nu_0,n4,Fr4);

        % Plots
        figure('Name','Roller Follower')
        suptitle('Roller Follower')
        subplot(2,2,3)
        plot(t,T4)
        title('Friction Torque profile');
        xlabel('Time (s)')
        ylabel('Torque (N.mm)')

        subplot(2,2,4)
        plot(t,H4)
        title('Power Loss profile');
        xlabel('Time (s)')
        ylabel('Power Loss (W)')


        subplot(2,2,1)
        plot(t,Fr4)
        title('Radial Load profile');
        xlabel('Time (s)')
        ylabel('Radial Load (N)')

        subplot(2,2,2)
        plot(t,n4)
        title('Angular Velocity profile');
        xlabel('Time (s)')
        ylabel('Angular Velocity (rad/s)')


        %% (MAIN BEARING, MB)
        % INPUTS
        dm5 = 61;                                               % Pitch Diameter [mm]
```

```
      ft = 0.11;                                         % Equivalent thrust load
factor (Figure 10.2)
      K = 1.34;                                          % Bearing factor (Basic
dynamic radial load rating/basic dynamic thrust load rating)
      D = 10;                                            % Mean roller diameter [mm]
      Z = 20;                                            % Number of rollers
      l = 15;                                            % Roller race contact
length [mm]
      alpha = 16.2;                                      % Half cup angle [degree]

      Fr5 = (LD.F1Cr)/2;                                 % Radial Load of WP [N]
      n5 = LK.om_MB;                                     % Velocity of WP [rad/s to
rpm converted in function]

      % OUTPUTS
      [T5,H5] = Tapered_Roller_H(dm5,nu_0,n5,Fr5,ft,K,D,Z,l,alpha);

      % Plots
      figure('Name','Main Bearing')
      suptitle('Main Bearing')
      subplot(2,2,3)
      plot(t,T5)
      title('Friction Torque profile');
      xlabel('Time (s)')
      ylabel('Torque (N.mm)')

      subplot(2,2,4)
      plot(t,H5)
      title('Power Loss profile');
      xlabel('Time (s)')
      ylabel('Power Loss (W)')

      subplot(2,2,1)
      plot(t,Fr5)
      title('Radial Load profile');
      xlabel('Time (s)')
      ylabel('Radial Load (N)')

      subplot(2,2,2)
      plot(t,n5*ones(size(t)))
      title('Angular Velocity profile');
      xlabel('Time (s)')
      ylabel('Angular Velocity (rad/s)')

      % Combined bearing power losses for 4 lobe 7 pistons

      H_1 = 14*H1;
      H_2 = 14*H2;
      H_3 = 7*H3;
      H_4 = 7*H4;
      H_5 = 2*H5;
```

```
        % All losses of all bearings
        for i = 1:length(t)
            Htotal = H_1+H_2+H_3+H_4+H_5;
        end

        figure(6)
        plot(t,H_1,t,H_2,t,H_3,t,H_4,t,H_5,t,Htotal)
title('Combined Power losses profile');
xlabel('Time (s)')
ylabel('Power Losses (W)')
legend('Rocker Ground','Rocker Moving','Wrist Pin','Roller Follower','Main
Bearing','Total Loss')
```

MATLAB Functions

- Needle roller bearing

```
        % SYNTAX
        function [T,H] = Roller_H(dm,nu_0,n,Fr)

        % DESCRIPTION
        % Calling this function will calculate the frictional torque and power
        % losses in Radial needle roller bearing using inputs such as
        % pitch diameter, kinematic viscosity from 'FluidProperty' model, rotational
        % frequency, and radial load.

        % ONE-LINE DESCRIPTION
        % Calculates friction torque and power loss for radial needle roller bearing.

        % REQUIRED ARGUMENTS
        % dm                            Pitch Diameter [mm]
        % nu                            Viscosity [cSt] at 3000psi and 50C for fluid 1
        % n                             Angular velocity [rad/s converted to rpm in
formula]
        % Fr                            Radial Load [N]

        % OPTIONAL ARGUMENTS
        % [No additional arguments]

        % OUTPUT VARIABLES
        % T                             Frictional Torque [N.mm]
        % H                             Power loss [Watts]

        % REQUIRED FUNCTIONS (USER-DEFINED)
        % [No required functions]

        % FUNCTION HISTORY
        %{
```

```
	Created      : 12/26/2018, Jordan Saikia, MSOE
	Last Modified: 12/26/2018, Jordan Saikia, MSOE

	CHANGE LOG
	==========================================================
	-12/27/2018 Updated function to one function for all needle rollers
	===========================================================

	===================== 12/26/2018 ==========================
	-Initial release.
	===========================================================
	%}

	% CALCULATIONS
	if n>=0
	    T = dm.*(((4.5e-
7).*(nu_0.^0.3).*(abs(convangvel(n,'rad/s','rpm')).^0.6))+(0.12.*((Fr).^0.41)));
% Frictional Torque [N.mm]
	else
	    T = sign(n).*dm.*(((4.5e-
7).*(nu_0^0.3).*(abs(convangvel(n,'rad/s','rpm')).^0.6))+(0.12.*(abs(Fr).^0.41)));
% Frictional Torque [N.mm]
	end
	H = (1.047e-4).*(abs(T)).*(abs(convangvel(n,'rad/s','rpm')));
% Power Loss [Watts]
	end
```

- Tapered roller bearing

```
	% SYNTAX
	function [T,H] = Tapered_Roller_H(dm,nu_0,n,Fr,ft,K,D,Z,l,alpha)

	% DESCRIPTION
	% Calling this function will calculate the frictional torque and power
	% losses in Tapered Roller Bearing using inputs such as
	% pitch diameter, kinematic viscosity from 'FluidProperty' model, rotational
	% frequency, and radial load.

	% ONE-LINE DESCRIPTION
	% Calculates friction torque and power loss for tapered roller bearing.

	% REQUIRED ARGUMENTS
	% dm                                        Pitch Diameter [mm]
	% nu                                        Viscosity [cSt] at 3000psi and
50C for fluid 1
	% n                                         Angular velocity [rad/s
converted to rpm in formula]
	% Fr                                        Radial Load [N]
	% ft                                        Equivalent thrust load factor
(Figure 10.2)
```

```
	% K							Bearing factor (Basic dynamic radial load rating/basic dynamic thrust load rating)
	% D							Mean roller diameter [mm]
	% Z							Number of rollers
	% l							Roller race contact length [mm]
	% alpha						Half cup angle [degree]

	% OPTIONAL ARGUMENTS
	% [No additional arguments]

	% OUTPUT VARIABLES
	% T							Frictional Torque [N.mm]
	% H							Power loss [Watts]

	% REQUIRED FUNCTIONS (USER-DEFINED)
	% [No required functions]

	% FUNCTION HISTORY
	%{
	Created      : 12/26/2018, Jordan Saikia, MSOE
	Last Modified: 12/26/2018, Jordan Saikia, MSOE

	CHANGE LOG
	==========================================================

	===========================================================

	==================== 12/26/2018 ==========================
	-Initial release.
	===========================================================
	%}

	% CALCULATIONS
	G = (dm^(3/2))*(D^(1/6))*((Z*l)^(2/3))*((sind(alpha))^(-1/3));		% Bearing geometry factor
	T = (3.76e-6)*G*((n*nu_0)^(0.5)).*(((ft.*Fr)./K).^(1/3));		% Friction torque [N.mm]
	H = (1.047e-4).*(abs(T)).*n;							% Power loss [Watts]
	end
```

Appendix B: SKF Bearing Power Loss

MATLAB Functions

The script requires the code for Fluid Properties (see Appendix E). The script file follows a similar pattern as in the Harris and Kotzalas model.

- Needle roller bearing

```
% SYNTAX
function [T_rr, T_sl, T_drag, T, H] =
Roller_S(dm,nu_0,n_,Fr,R1,S2,mu_bl,mu_ehl,D,d,B,Vm,Kz,Kl)

% DESCRIPTION
% Calling this function will calculate the frictional torque and power
% losses in Radial Needle Roller Bearing using inputs such as
% pitch diameter, kinematic viscosity from 'FluidProp' function, rotational
% frequency, and radial load.

% ONE-LINE DESCRIPTION
% Calculates friction torque and power loss for Needle Roller bearing.

% REQUIRED ARGUMENTS
% dm1                        Pitch Diameter [mm]
% nu_0                       Viscosity [cSt] at at 3000psi and 50C for
fluid 1
% n                          Angular velocity [rad/s converted to rpm
% Fr                         Radial Load [N]

% OPTIONAL ARGUMENTS
% [No additional arguments]

% OUTPUT VARIABLES
% T_rr                       Frictional torque due to rolling [N.mm]
% T_sl                       Frictional torque due to sliding [N.mm]
% T_drag                     Frictional torque due to drag    [N.mm]
% T                          Total frictional torque          [N.mm]
% H                          Power loss                      [Watts]

% REQUIRED FUNCTIONS (USER-DEFINED)
% [No required functions]

% FUNCTION HISTORY
%{
Created      : 12/26/2018, Jordan Saikia, MSOE
Last Modified: 12/26/2018, Jordan Saikia, MSOE

CHANGE LOG
==========================================================
```

```
        -12/26/2018 Included all the needle roller bearings
        ============================================================

        ===================== 12/26/2018 ==========================
        -Initial release.
        ============================================================
        %}

        % CALCULATIONS
        % Rolling Friction Torque
        phi_ish = 1./(1+((1.84e-9).*((n_.*dm).^1.28)).*(nu_0.^0.64));                 %
Inlet shear heating reduction factor
        G_rr = R1.*(dm.^2.41).*(Fr.^0.31);                                            %
Geometric and load dependent variables
        T_rr = (phi_ish).*(G_rr).*((nu_0.*n_).^0.6);                                  %
Rolling Friction Torque

        % Sliding Friction Torque
        G_sl = S2.*dm.*Fr;                                                            %
Sliding Friction variable
        phi_bl = 1./(exp((2.6e-8).*((n_.*nu_0).^1.4).*dm));                           %
Weighting factor for the sliding friction coefficient
        mu_sl = (phi_bl.*mu_bl)+((1-phi_bl).*mu_ehl);                                 %
Sliding Friction coefficient
        T_sl = (G_sl).*(mu_sl);                                                       %
Sliding Friction Torque

        % Drag Friction Torque
        ld = 5.*((Kl.*B)./dm);                                                        %
Drag loss factor
        Cw = (2.789e-10.*ld.^3)-(2.786e-4.*ld.^2)+(0.0195.*ld)+0.6439;                %
Drag loss factor
        t = 2.*acos(((0.6.*dm)-D)./(0.6.*dm));                                        %
Drag loss factor
        fa = 0.05.*((Kz.*(D+d))./(D-d));                                              %
Drag loss factor
        if t >= 0 && t <= pi                                                          %
Drag loss factor
            ft = sin(0.5.*t);
        elseif t > pi && t < 2*pi
            ft = 1;
        end
        Rs = 0.36.*(dm.^2).*(t-sin(t)).*fa;                                           %
Drag loss factor
        Kroll = ((Kl*Kz.*(d+D))./(D-d)).*1e-12;                                       %
Rolling element constant
        T_drag = (4.*Vm.*Kroll.*Cw.*B.*(dm.^4).*(n_.^2))+((1.093e-
7.*(n_.^2).*(dm.^3).*(((n_.*(dm.^2).*ft)./nu_0).^-1.379)).*Rs);    % Drag Frcition
Torque
```

```
	% Total Friction Torque and Power Loss
	T = T_rr+T_sl+T_drag;                                                        %
Friction Torque [N.mm]
	H = (1.047e-4.*(abs(T)).*(n_));                                              %
Power Loss [Watts]
	end
```

- Tapered roller bearing

```
	% SYNTAX
	function [T_rr, T_sl, T_drag, T, H] =
Tapered_Roller_S(dm,nu_0,n_,Fr,Y,R1,R2,S1,S2,K,mu_bl,mu_ehl,D,d,B,Vm,Kz,Kl)

	% DESCRIPTION
	% Calling this function will calculate the frictional torque and power
	% losses in a Tapered roller bearing using inputs such as
	% pitch diameter, kinematic viscosity from 'FluidProp' function, rotational
	% frequency, and radial load.

	% ONE-LINE DESCRIPTION
	% Calculates friction torque and power loss for Tapered roller bearing.

	% REQUIRED ARGUMENTS
	% dm1                            Pitch Diameter [mm]
	% nu_0                           Viscosity [cSt] at at 3000psi and 50C for
fluid 1
	% n                              Angular velocity [rad/s converted to rpm
	% Fr                             Radial Load [N]

	% OPTIONAL ARGUMENTS
	% [No additional arguments]

	% OUTPUT VARIABLES
	% T_rr                           Frictional torque due to rolling [N.mm]
	% T_sl                           Frictional torque due to sliding [N.mm]
	% T_drag                         Frictional torque due to drag    [N.mm]
	% T                              Total frictional torque          [N.mm]
	% H                              Power loss                      [Watts]

	% REQUIRED FUNCTIONS (USER-DEFINED)
	% [No required functions]

	% FUNCTION HISTORY
	%{
	Created      : 12/26/2018, Jordan Saikia, MSOE
	Last Modified: 12/26/2018, Jordan Saikia, MSOE

	CHANGE LOG
```

```
    ==========================================================
    -12/26/2018 Included all the tapered roller bearings
    ===========================================================

    ===================== 12/26/2018 ==========================
    -Initial release.
    ===========================================================
    %}

    % CALCULATIONS
    % Rolling Friction Torque
    phi_ish = 1./(1+((1.84e-9).*((n_.*dm).^1.28)).*(nu_0.^0.64));         %
Inlet shear heating reduction factor
    Fa = K*d*1e-3;                                                         %
Preload Axial Load [N]
    G_rr = R1.*(dm.^2.38).*((Fr+(R2*Y*Fa)).^(0.31));                      %
Geometric and load dependent variables
    T_rr = (phi_ish).*(G_rr).*((nu_0.*n_).^0.6);                          %
Rolling Friction Torque

    % Sliding Friction Torque
    G_sl = S1.*(dm.^(0.82)).*(Fr+(S2*Y*Fa));                              %
Sliding Friction variable
    phi_bl = 1./(exp((2.6e-8).*((n_.*nu_0).^1.4).*dm));                   %
Weighting factor for the sliding friction coefficient
    mu_sl = (phi_bl.*mu_bl)+((1-phi_bl).*mu_ehl);                         %
Sliding Friction coefficient
    T_sl = (G_sl).*(mu_sl);                                               %
Sliding Friction Torque

    % Drag Friction Torque
    ld = 5.*((Kl.*B)./dm);                                                %
Drag loss factor
    Cw = (2.789e-10.*ld.^3)-(2.786e-4.*ld.^2)+(0.0195.*ld)+0.6439;        %
Drag loss factor
    t = 2.*acos(((0.6.*dm)-D)./(0.6.*dm));                                %
Drag loss factor
    fa = 0.05.*((Kz.*(D+d))./(D-d));                                      %
Drag loss factor
    if t >= 0 && t <= pi                                                  %
Drag loss factor
        ft = sin(0.5.*t);
    elseif t > pi && t < 2*pi
        ft = 1;
    end
    Rs = 0.36.*(dm.^2).*(t-sin(t)).*fa;                                   %
Drag loss factor
    Kroll = ((Kl*Kz.*(d+D))./(D-d)).*1e-12;                               %
Rolling element constant
```

```
      T_drag = (4.*Vm.*Kroll.*Cw.*B.*(dm.^4).*(n_.^2))+((1.093e-
7.*(n_.^2).*(dm.^3).*(((n_.*(dm.^2).*ft)./nu_0).^-1.379)).*Rs);   % Drag Frcition
Torque

      % Total Friction Torque and Power Loss
      T = T_rr+T_sl+T_drag;                                                          %
Friction Torque [N.mm]
      H = (1.047e-4.*(abs(T)).*(n_));                                                %
Power Loss [Watts]
      end
```

Appendix C: Bearing Film Thickness

MATLAB script

```
% (ROCKER GROUND, RG)
% Film Thickness in a Radial Needle Roller Bearing
% Created by: Jordan Saikia 07/10/2018
% Article Reference: Assessment of transient film thickness in a roller bearing,
Kilundu et al, pg:2457-2469
clc; clear all;
format shortG;

% Rocker Ground Bearing (Input)
eeta1 = 0.0260240;                  % Dynamic Viscosity [Pa.s] at 5000psi and 50C for
fluid 1
R1 = 0.003;                         % Radius of Roller [m]
B1 = 0.012;                         % Length of Roller [m]
E_eff1 = 2.1978*10^11;              % Elastic modulus [N/m2]
alpha1 = 20*10^-9;                  % Piezoelectric coefficient [m2/N]or[Pa^-1]
nu1 = 0.3;                          % Poissons ratio

numData1 = xlsread('Loads.xlsx');
W1 = (numData1(20,:))*10^-3;         % Radial Load [kN]
n1 = (abs(numData1(29,:)))*0.001;    % Angular velocity [rad/s converted to m/s in
excel]
t = linspace(0,1,length(W1));        % Time step [seconds]

%Film thickness (Output)

for i = 1:length(W1)
    W1_cap = W1/(E_eff1*R1*B1);                              % Dimensionless load
    n1_cap = (eeta1*n1)/(E_eff1*R1);                         % Dimensionless velocity
    G1_cap = alpha1*E_eff1;                                  % Dimensionless modulus
    g_v = (((W1_cap).^1.5)*G1_cap)./((n1_cap).^0.5);         % Viscosity number
    g_e = (W1_cap)./((n1_cap).^0.5);                         % Elasticity number
    g_h = 2.65*((g_v).^0.54).*((g_e).^0.06);                 % Film thickness number
end
for i = 1:length(W1)
    [h0(1,i)] =
Film_thickness_RG_fn(eeta1,R1,B1,E_eff1,alpha1,n1(i),W1(i),W1_cap,n1_cap,G1_cap,g_v,g_
e,g_h);  % Minimum film thickness steady state full EHL regime
end

Results1 = table(W1',n1',h0(1,:)',t','VariableNames',{'RadialLoad1' 'AngVelocity1'
'Filmthickness' 'time'})

figure('Name','Rocker Ground');
suptitle('Rocker Ground')
subplot(3,1,1)
plot(t,h0(1,:))
```

```
title('Film thickness profile');
xlabel('Time (s)')
ylabel('Film thickness (m)')
ylim([0 0.000005]);                             % Scaling axis for better view of
elastohydrodynamic lubrication
xlim([0.12 0.35]);                              % Scaling axis for better view of
elastohydrodynamic lubrication


subplot(3,1,2)
plot(t,n1)
title('Velocity profile');
xlabel('Time (s)')
ylabel('Angular Velocity (m/s)')
ylim([0 0.02]);                                 % Scaling axis for better view of
elastohydrodynamic lubrication
xlim([0.12 0.35]);                              % Scaling axis for better view of
elastohydrodynamic lubrication

subplot(3,1,3)
plot(t,W1)
title('Load profile');
xlabel('Time (s)')
ylabel('Load (kN)')
ylim([0 10]);                                   % Scaling axis for better view of
elastohydrodynamic lubrication
xlim([0.12 0.35]);                              % Scaling axis for better view of
elastohydrodynamic lubrication



% (ROCKER MOVING, RM)
% Film Thickness in a Radial Needle Roller Bearing
% Created by: Jordan Saikia 07/10/2018
% Article Refernce: Assessment of transient film thickness in a roller bearing,
Kilundu et al, pg:2457-2469
format shortG;

% Rocker Moving Bearing (Input)
eeta2 = 0.0260240;                  % Dynamic Viscosity [Pa.s] at 5000psi and 50C for
fluid 1
R2 = 0.003;                         % Radius of Roller [m]
B2 = 0.012;                         % Length of Roller [m]
E_eff2 = 2.1978*10^11;              % Elastic modulus [N/m2]
alpha2 = 0.015*10^-6;               % Piezoelectric coefficient [m2/N]or[Pa^-1]
nu2 = 0.3;                          % Poissons ratio

numData2 = xlsread('Loads.xlsx');
W2 = (numData2(22,:))*10^-3;              % Radial Load [kN]
```

```
n2 = (abs(numData2(30,:)))*0.001;          % Angular velocity [rad/s converted to m/s
in excel]
t = linspace(0,1,length(W2));              % Time step [seconds]

%Film Thickness (Output)


for i = 1:length(W2)
    W2_cap = W2/(E_eff2*R2*B2);                          % Dimensionless load
    n2_cap = (eeta2*n2)/(E_eff2*R2);                     % Dimensionless velocity
    G2_cap = alpha2*E_eff2;                              % Dimensionless modulus
    g_v2 = (((W2_cap).^1.5)*G2_cap)./((n2_cap).^0.5);    % Viscosity number
    g_e2 = (W2_cap)./((n2_cap).^0.5);                    % Elasticity number
    g_h2 = 2.65*((g_v2).^0.54).*((g_e2).^0.06);          % Film thickness number
end
for i = 1:length(W2)
    [h02(1,i)] =
Film_thickness_RG_fn(eeta2,R2,B2,E_eff2,alpha2,n2(i),W2(i),W2_cap,n2_cap,G2_cap,g_v2,g
_e2,g_h2);% Minimum film thickness steady state full EHL regime
end

Results2 = table(W2',n2',h02(1,:)','VariableNames',{'RadialLoad2' 'AngVelocity2'
'Filmthickness2'})

figure('Name','Rocker moving');
suptitle('Rocker Moving')
subplot(3,1,1)
plot(t,h02(1,:))
title('Film thickness profile');
xlabel('Time (s)')
ylabel('Film thickness (m)')
%ylim([0 0.000005]);                                  % Scaling axis for better view of
elastohydrodynamic lubrication
%xlim([0.12 0.35]);                                   % Scaling axis for better view of
elastohydrodynamic lubrication


subplot(3,1,2)
plot(t,n2)
title('Velocity profile');
xlabel('Time (s)')
ylabel('Velocity (m/s)')
%ylim([0 0.005]);                                     % Scaling axis for better view of
elastohydrodynamic lubrication
%xlim([0.12 0.35]);                                   % Scaling axis for better view of
elastohydrodynamic lubrication


subplot(3,1,3)
plot(t,W2)
```

```
title('Load profile');
xlabel('Time (s)')
ylabel('Load (kN)')
%ylim([0 10]);                                  % Scaling axis for better view of
elastohydrodynamic lubrication
%xlim([0.12 0.35]);                             % Scaling axis for better view of
elastohydrodynamic lubrication




% (WRIST PIN, WP)
% Film Thickness in a Radial Needle Roller Bearing
% Created by: Jordan Saikia 07/10/2018
% Article Refernce: Assessment of transient film thickness in a roller bearing,
Kilundu et al, pg:2457-2469
format shortG;

% Wrist Pin Bearing (Input)
eeta3 = 0.0260240;                        % Dynamic Viscosity [Pa.s] at 5000psi and
50C for fluid 1
R3 = 0.0023815;                           % Radius of Roller [m]
B3 = 0.01588;                             % Length of Roller [m]
E_eff3 = 2.1978*10^11;                    % Elastic modulus [N/m2]
alpha3 = 0.015*10^-6;                     % Piezoelectric coefficient [m2/N]or[Pa^-1]
nu3 = 0.3;                                % Poissons ratio

numData3 = xlsread('Loads.xlsx');
W3 = (numData3(26,:))*10^-3;              % Radial Load [kN]
n3 = (abs(numData3(31,:)))*0.001;         % Angular velocity [rad/s converted to m/s
in excel]
t = linspace(0,1,length(W3));             % Time step [seconds]

%Film thickness (Output)


for i = 1:length(W3)
    W3_cap = W3/(E_eff3*R3*B3);                             % Dimensionless load
    n3_cap = (eeta3*n3)/(E_eff3*R3);                        % Dimensionless velocity
    G3_cap = alpha3*E_eff3;                                 % Dimensionless modulus
    g_v3 = (((W3_cap).^1.5)*G3_cap)./((n3_cap).^0.5);       % Viscosity number
    g_e3 = (W3_cap)./((n3_cap).^0.5);                       % Elasticity number
    g_h3 = 2.65*((g_v3).^0.54).*((g_e3).^0.06);             % Film thickness number
end
for i = 1:length(W3)
    [h03(1,i)] =
Film_thickness_WP_fn(eeta3,R3,B3,E_eff3,alpha3,n3(i),W3(i),W3_cap,n3_cap,G3_cap,g_v3,g
_e3,g_h3);% Minimum film thickness steady state full EHL regime
end
```

```
Results3 = table(W3',n3',h03(1,:)','VariableNames',{'RadialLoad3' 'AngVelocity3'
'Filmthickness3'})

figure('Name','Wrist Pin');
suptitle('Wrist Pin')
subplot(3,1,1)
plot(t,h03(1,:))
title('Film thickness profile');
xlabel('Time (s)')
ylabel('Film thickness (m)')
%ylim([0 0.000005]);                        % Scaling axis for better view of
elastohydrodynamic lubrication
%xlim([0.12 0.35]);                         % Scaling axis for better view of
elastohydrodynamic lubrication


subplot(3,1,2)
plot(t,n3)
title('Velocity profile');
xlabel('Time (s)')
ylabel('Velocity (m/s)')
%ylim([0 0.012]);                           % Scaling axis for better view of
elastohydrodynamic lubrication
%xlim([0.12 0.35]);                         % Scaling axis for better view of
elastohydrodynamic lubrication


subplot(3,1,3)
plot(t,W3)
title('Load profile');
xlabel('Time (s)')
ylabel('Load (kN)')
%ylim([0 12]);                              % Scaling axis for better view of
elastohydrodynamic lubrication
%xlim([0.12 0.35]);                         % Scaling axis for better view of
elastohydrodynamic lubrication
```

MATLAB Function

```
function [h0] =
Film_thickness_RG_fn(eeta1,R1,B1,E_eff1,alpha1,n1,W1,W1_cap,n1_cap,G1_cap,g_v,g_e,g_h)
% DESCRIPTION
%Calling this function will calculate the Film Thickness
%Film Thickness in Radial Roller Bearing (Rocker Ground).

% INPUT
```

```
% eeta1 = 0.0260240;              % Dynamic Viscosity [Pa.s] at 5000psi and 50C for
fluid 1
% R1 = 0.003;                     % Radius of Roller [m]
% B1 = 0.012;                     % Length of Roller [m]
% E_eff1 = 2.1978*10^11;          % Elastic modulus [N/m2]
% alpha1 = 20*10^-9;              % Piezoelectric coefficient [m2/N]or[Pa^-1]
% nu1 = 0.3;                      % Poissons ratio
% n1 =  Export from excel sheet   % Angular velocity [rad/s converted to rpm in
formula]
% W1 = Export from excel sheet    % Radial Load [kN]

% OUTPUT
% Film thickness
% h0 = (R1*g_h*n1_cap)/((W1_cap)^(4/3));               [m]

% CALCULATION

for i = 1:length(W1)
    W1_cap = W1/(E_eff1*R1*B1);                        % Dimensionless load
    n1_cap = (eeta1*n1)/(E_eff1*R1);                   % Dimensionless velocity
    G1_cap = alpha1*E_eff1;                            % Dimensionless modulus
    g_v = (((W1_cap)^1.5)*G1_cap)/((n1_cap)^0.5);      % Viscosity number
    g_e = (W1_cap)/((n1_cap)^0.5);                     % Elasticity number
    g_h = 2.65*((g_v)^0.54)*((g_e)^0.06);              % Film thickness number
    h0 = (R1*g_h*n1_cap)/((W1_cap)^(4/3));             % Minimum film thickness steady
state full EHL regime
end
h0 = (R1*g_h*n1_cap)/((W1_cap)^(4/3));                 % Minimum film thickness steady
state full EHL regime
```

Appendix D: Cam Follower Film Thickness

MATLAB script

```
% (Cam Follower)
% Film Thickness in Cam Follower Mechanism incorporating Fluid properties
% Created by: Jordan Saikia 09/16/2018
% Article Reference: Applied Tribology: Bearing Design and lubrication, Khonsari et
al, pg:530-569
clc; clear all;
format shortG;
[K, rho, cp, k, nu, mu] =
FluidProp(11.331,8.898,0.005744,0.0006870,3.528,0.1452,0.3595,11.882,46,6.7,30,20,50,2
0.8);

% Cam Follower (Input)
mu_ = mu*10^-3;                       % Dynamic Viscosity [Pa.s] at 5000psi and 50C for
fluid 1
R1 = 9.75e-3;                           % Radius of Roller Follower [m]
R2 = 0.1011;                            % Radius of Cam [m]51.685e-3

L = 18e-3;                              % Length of Roller [m]
%E_eff = 2.1978*10^11;                   % Elastic modulus [N/m2]
pois1 =  0.3;                                        % Poisson Ratio of roller
pois2 = 0.3;                                         % Poisson Ratio of cam
E1 = 200e9;                                          % Elastic modulus of roller [Pa]
E2 = 200e9;                                          % Elastic modulus of cam[Pa]
alpha = 20*10^-9;                       % Piezoelectric coefficient [m2/N]

numData1 = xlsread('VDLMSingleCycle.xlsx');
Fr = (numData1(25,:))*10^-3;                % Radial Load [kN]
n1 = 324.7464*10^-3;                        % Cam Angular velocity [rad/s converted to
mm/s in excel then to m/s here]
n2 = (numData1(31,:))*10^-3;                % Follower Angular velocity [rad/s converted
to mm/s in excel then to m/s here]
t = linspace(0,1,length(Fr));               % Time step [seconds]

%Film thickness (Output)
[h,n] = Cam_Film_fn(R1,R2,n1,n2,mu_,L,E1,E2,pois1,pois2,Fr,alpha);


%Results1 = table(w',n',h',h0',t','VariableNames',{'RadialLoad1' 'AngVelocity1'
'Filmthickness' 'Min Filmthickness' 'time'})%real(h01)'

figure('Name','Cam Follower');
suptitle('Cam Follower')
plot(t,h(1,:)*10^6)
hold on
% plot(t,h0(1,:)*10^6)
title('Film thickness profile');
```

```
%legend({'Film thickness','Minimum film thickness'},'Location','northeast')
xlabel('Time (s)')
ylabel('Film thickness (um)')
```

MATLAB Function

```
function [h,n] = Cam_Film_fn(R1,R2,n1,n2,mu_,L,E1,E2,pois1,pois2,Fr,alpha)
% DESCRIPTION
R_eff = 1/((1/R1)+(1/R2));                    % Radius Effective [m]
n = (n1+n2)/2;
E_eff = 2*(((1-(pois1)^2)/E1)+((1-(pois2)^2)/E2))^(-1);       % Effective Elactic
Modulus [Pa]
h = (R_eff*1.657).*(((n.*mu_.*alpha)./R_eff).^0.7273).*((Fr./(L*E_eff*R_eff)).^-
0.0909);
%h0 =
(R_eff*3.63).*(((n.*mu_)./(E_eff*R_eff)).^0.68).*((alpha*E_eff)^0.49).*((w./(E_eff*(R_
eff^2))).^-0.073);  % Minimum film thickness

%function [h0,n] = Film_thickness_fn(mu,n2,n1,L,R,E_eff,w,G)
```

Appendix E: Fluid Properties

MATLAB Script

```
clear all; clc; close all;
    % DESCRIPTION
    % This is the main program which will call the program and sub program to
    % conduct complete analysis to determine fluid properties. First, it
    % will call FluidParameters() function to acquire fixed parameters of
    % the fluids and then it will call FluidPropModel() function to
    % determine fluid properties such as density, effective bulk modulus,
    % and so on at any temperature and pressure of the fluid.
    %
    % REQUIRED ARGUMENTS
    % No required argument.
    %
    % OPTIONAL ARGUMENTS
    % No optional argument.
    %
    % OUTPUT VARIABLES (in structure format)
    % No output variables
    %
    % REQUIRED FUNCTIONS (USER-DEFINED)
    % FluidParameters.m
    % FldPropModel.m

    %{
    Created      : 08/07/2018, Pawan Panwar, MSOE
    Last Modified: 10/19/2018, Pawan Panwar, MSOE

    CHANGE LOG
     ===============   1/24/2019   ===============
    -Changed kinematic viscosity (nu) to mm2/s
     mm2/s is the same as centistokes
     centistokes are used in empirical models
    -Corrected dynamic viscosity (mu)conversion factor
     units are still Pascal sec
    -Corrected an erroneous coefficeint for Fluid 3

    ==============================================

    ===============   10/19/2018   ===============
```

```
    -Formated as per the Guidelines.
    ==============================================

    ===============   08/07/2018   ===============
    -Initial release.
    ==============================================
    %}
```

```
%% Operating Parameter
```

```
OP.FluidNum = 1;                        % Type 1, 2, 3, 4, or 5 respectively for fluids
GRP1, HVI, TMP, PAO, or PAG
OP.T = 100;                              % Temperature of the fluid [C]
OP.p = convpres(5000,'psi','pa');    % Pressure of the fluid [pa]
```

```
%% Get all the fixed parameters of the fluids
```

```
       [FldPar,Fld] = FluidParameters(OP.FluidNum);
```

```
%% Call FluidPropModel() function to determine fluid properties
```

```
       [FldPro] = FldPropModel(Fld,OP.T,OP.p);

FldPro.nu;
```

MATLAB Functions

Fluid Parameters

```
function [FldPar,Fld] = FluidParameters(FluidNum)
```

```
          % [FldPar,Fld] = FluidParameters(FluidNum)
    %
    % DESCRIPTION
    % Returns fixed parameters of all the fluids and of a specified fluid
    %
    % REQUIRED ARGUMENTS
    % FluidNum : Numerical value 1, 2, 3, 4, or 5 respectively for fluids
    %            GRP1, HVI, TMP, PAO, or PAG
    %
    % OPTIONAL ARGUMENTS
    % No optional argument.
    %
```

```
    % OUTPUT VARIABLES (in structure format)
    % FldPar   : Fluid properties parameters of all the fluids
    % Fld      : Fluid properties parameters of a specified fluid
    %
    % REQUIRED FUNCTIONS (USER-DEFINED)
    % No required function

    %{
    Created      : 08/07/2018, Pawan Panwar, MSOE
    Last Modified: 10/19/2018, Pawan Panwar, MSOE

    CHANGE LOG
    ===============   10/19/2018   ===============
    -Formated as per the Guidelines.
    ==============================================

    ===============   08/07/2018   ===============
    -Initial release.
    ==============================================
    %}
```

```
%% Fluid Properties parameters (Fld)
```

```
% Parameters for Bulk modulus and Relative volume calculations
```

```
      FldPar.K0dash = [11.331 10.823 10.169 10.427 10.856];% Pressure rate of change
of isothermal bulk modulus at p = 0 [-]
FldPar.K00 = [8.898 8.718 9.283 8.032 8.405]*1E9;    % K0 at zero absolute temperature
[Pa]
FldPar.BK = [5.744 5.808 5.665 5.716 5.753]*1E-3;    % Temperature coefficient of K0
{1/K]
FldPar.av = [6.870 7.227 7.307 7.221 7.668]*1E-4;    % Thermal expansivity defined for
volume linear with temperature [1/K]

% Parameters for dynamic viscosity calculation using Tait Equation
FldPar.gamma = [3.528 3.751 3.528 3.403 3.543];       % thermodynamic interaction
parameter [-]
FldPar.mu_inf = [1.452 3.108 2.442 1.143  4.712]*1E-4;% Viscosity extrapolated to
infinite temperature [Pa.S]
FldPar.phi_inf = [0.3595 0.2820 0.2671 0.2429 0.3735];% scaling parameter for
unbounded viscosity [-]
FldPar.BF = [11.882 14.948 16.760 17.521 9.195];      % fragility parameter [-]
```

```
% Parameters for kinematic viscosity calculation using Walther Equation
FldPar.v40C = [46.0 45.0 47.5 17.0 42.0];          % Measured kinematic viscosity of
the fluid at 40C [cSt]
FldPar.v100C = [6.7 10.24 9.60 3.93 8.27];         % Measured kinematic viscosity of
the fluid at 100C [cSt]
FldPar.APIGravity = [30 34.9 22.1 34.2 30.7];      % Inverse measure of a petroleum
liquid's density relative to
                                                   % that of water (specific gravity)
[-]

% Selecting parameters of the specified fluid for the analysis
if FluidNum == 1
    Fld.K0dash = FldPar.K0dash(1); Fld.K00 = FldPar.K00(1); Fld.BK = FldPar.BK(1);
Fld.av = FldPar.av(1);
    Fld.gamma = FldPar.gamma(1); Fld.mu_inf = FldPar.mu_inf(1); Fld.phi_inf =
FldPar.phi_inf(1); Fld.BF = FldPar.BF(1);
    Fld.v40C = FldPar.v40C(1); Fld.v100C = FldPar.v100C(1); Fld.APIGravity =
FldPar.APIGravity(1);
elseif FluidNum == 2
    Fld.K0dash = FldPar.K0dash(2); Fld.K00 = FldPar.K00(2); Fld.BK = FldPar.BK(2);
Fld.av = FldPar.av(2);
    Fld.gamma = FldPar.gamma(2); Fld.mu_inf = FldPar.mu_inf(2); Fld.phi_inf =
FldPar.phi_inf(2); Fld.BF = FldPar.BF(2);
    Fld.v40C = FldPar.v40C(2); Fld.v100C = FldPar.v100C(2); Fld.APIGravity =
FldPar.APIGravity(2);
elseif FluidNum == 3
    Fld.K0dash = FldPar.K0dash(3); Fld.K00 = FldPar.K00(3); Fld.BK = FldPar.BK(3);
Fld.av = FldPar.av(3);
    Fld.gamma = FldPar.gamma(3); Fld.mu_inf = FldPar.mu_inf(3); Fld.phi_inf =
FldPar.phi_inf(3); Fld.BF = FldPar.BF(3);
    Fld.v40C = FldPar.v40C(3); Fld.v100C = FldPar.v100C(3); Fld.APIGravity =
FldPar.APIGravity(3);
elseif FluidNum == 4
    Fld.K0dash = FldPar.K0dash(4); Fld.K00 = FldPar.K00(4); Fld.BK = FldPar.BK(4);
Fld.av = FldPar.av(4);
    Fld.gamma = FldPar.gamma(4); Fld.mu_inf = FldPar.mu_inf(4); Fld.phi_inf =
FldPar.phi_inf(4); Fld.BF = FldPar.BF(4);
    Fld.v40C = FldPar.v40C(4); Fld.v100C = FldPar.v100C(4); Fld.APIGravity =
FldPar.APIGravity(4);
end

% Reference conditions
```

```
Fld.TR = 20;                                    % Reference temperature [C]
Fld.pR = 0;                                     % Reference pressure [Pa]
Fld.entAir = .005;                              % Fraction of entrained air [Vair/Vfld]

End
```

Fluid Property Model

```
       function [FldPro] = FldPropModel(Fld,T,p)

           % [FldPro] = FldPropModel(Fld,T,p)
    %
    % DESCRIPTION
    % This function to calculate Density, Bulk modulus, and viscosity of
    % the fluid at any instance.
    %
    % REQUIRED ARGUMENTS
    % T          : Temperature of the fluid at any instance [C]
    % p          : Pressure of the fluid at any instance [Pa]
    % Fld        : Fluid properties parameters of a specified fluid. This is
    %              in a structure format and contains the following parameters:
        % K0dash     : Pressure rate of change of isothermal bulk modulus at p = 0 [-]
        % K00        : K0 at zero absolute temperature [Pa]
        % BK         : Temperature coefficient of K0 {1/K]
        % av         : Thermal expansivity defined for volume linear with temperature [1/C]
        % gamma      : Thermodynamic interaction parameter [-]
        % mu_inf     : Viscosity extrapolated to infinite temperature [Pa.S]
        % phi_inf    : scaling parameter for unbounded viscosity [-]
        % BF         : fragility parameter [-]
        % pR         : Reference pressure [Pa]
        % TR         : Reference temperature [C]
        % v40C       : Measured kinematic viscosity of the fluid at 40C [cSt]
        % v100C      : Measured kinematic viscosity of the fluid at 100C [cSt]
        % APIGravity: Inverse measure of a petroleum liquid's density relative to
        %              that of water (specific gravity) [-]
        % entAir     : Fraction of entrained air [Vair/Vfld]
    %
    % OPTIONAL ARGUMENTS
    % No optional argument.
    %
    % OUTPUT VARIABLES
    % K          : Bulk modulus of the fluid [Pa]
    % rho        : Density of the fluid [kg/m^3]
    % cp         : Specific heat of the fluid [J/kg.K]
    % k          : Thermal Conductivity of the fluid [W/m.K]
    % nu         : Kinematic viscosity of the fluid [m^2/s]
    % mu         : Dynamic viscosity of the fluid [Pa.s].
    %
    % REQUIRED FUNCTIONS (USER-DEFINED)
    % No required function
```

```
    %{
    Created      : 08/07/2018, Pawan Panwar, MSOE
    Last Modified: 10/19/2018, Pawan Panwar, MSOE

    CHANGE LOG
    ===============   10/19/2018   ===============
    -Formated as per the Guidelines.
    -Removed constant parameters from required argument.
    ==============================================

    ===============   08/07/2018   ===============
    -Initial release.
    ==============================================
    %}

%% Calculation

% Bulk Modulus using the temperature modified Tait equation

K0 = Fld.K00*exp(-Fld.BK*convtemp(T,'C','K'));      % Isothermal bulk modulus K at p = 0 Pa [Pa]
Fld.K = ((1-(1/(1+Fld.K0dash))*log(1+p*(1+Fld.K0dash)/K0))*(K0+p*(1+Fld.K0dash))); % Bulk modulus
K at any p & T [Pa]
FldPro.Keff = Fld.K*(1+Fld.entAir )/(1+((Fld.pR/p)*Fld.entAir*Fld.K/p));   % Considered entrapped
air in the fluid [Pa]

% Density using the equation from Dr. Monika's Book
Fld.SG = (141.5/(Fld.APIGravity+131.5))+(60-convtemp(Fld.TR,'C','F'))*4E-4;% Specific gravity at
temperature T [-]
FldPro.rho = Fld.SG*(1+((p-Fld.pR)/Fld.K)-Fld.av*(T-Fld.TR))*1000;         % Density of fluid
[kg/m^3]

% Specific heat and Thermal Conductivity
SG1 = 141.5/(Fld.APIGravity+131.5);                       % Specific gravity at 15.6C or 60F [-]
FldPro.cp = (1/sqrt(SG1))*(1.63+0.0034*T)*1000;           % Specific heat [J/kg.K]
FldPro.k = (0.12/SG1)*(1-T*1.667E-4);                     % Thermal Conductivity [W/m.K]

% Kinematic Viscosity using Walther's Equation
A = (log10(log10(Fld.v100C+0.7))-(log10(log10(Fld.v40C+0.7))))/(log10((40+273.15)/(100+273.15)));
B = log10(log10(Fld.v100C+0.7)) + A*log10(100+273.15);
FldPro.nu = (10^(10^(B-A*log10(T+273.15)))-0.7);  % Kinematic viscosity [mm^2/s]

% Dynamic Viscosity
if p<=150E6           % If pressure p<=150 MPa (Walther's equation) - Hydrodynamic Regime
    FldPro.mu = FldPro.nu*FldPro.rho*1E-6;                  % Dynamic viscosity [Pa.S]

else p>150E6        % At pressure p>150 MPa - Elastrohydrodynamic Regime
    %  Relative volume V0/VR using Murnaghan equation [-]
    VbyVR = (1+Fld.av*(T-Fld.TR))*(1-(1/(1+Fld.K0dash))*log(1+p*(1+Fld.K0dash)/K0))
    phi = (convtemp(T,'C','K')/convtemp(Fld.TR,'C','K'))*VbyVR^Fld.gamma;
    FldPro.mu = Fld.mu_inf*exp(Fld.BF*Fld.phi_inf/(phi-Fld.phi_inf));% Dynamic viscosity [Pa.S]
end
```

```
end
```

Appendix F: Cam Power Loss

MATLAB Script

The script requires the code for Fluid Properties (see Appendix E).

```
% Copy paste sections due to mu clearing to nu. New data load is doing this
% DESCRIPTION
% Running this program will calculate the power losses in the cam follower
% mechanism using inputs such as dynamic viscosity, rotational frequency, radial load, and cam
specs.
% This script is created according to the Alakhramsing [1] and Khonsari [2] model.

% [1] Alakhramsing, Shivam. et al,2018, Lubrication and frictional analysis
% of cam-roller follower mechanism, Journal of Engineering Tribology, Vol. 232 (3), p-347-363.
% [2] Khonsari, M., Masjedi, M.,2015,An engineering approach for rapid
% evaluation of traction coefficient and wear in mixed EHL, Tribology
% International, Vol 92, p-184-190

% ONE-LINE DESCRIPTION
% Calculates power losses for cam roller follower (rolling and sliding).

% REQUIRED ARGUMENTS
% mu_0                                Inlet dynamic viscosity [Pa.s]
% n_RF                                Roller angular velocity [rad/s converted to m/s]
% n_MB                                Cam angular velocity [rad/s converted to m/s]
% Fr                                  Radial load [N]
% R1                                  Radius of roller [m]
% R2                                  Radius of cam [m] (Average of minor and major radius)
% L                                   Length of roller [m]
% pois1                               Poisson Ratio of roller [-]
% pois2                               Poisson Ratio of cam [-]
% E1                                  Elastic modulus of roller [Pa]
% E2                                  Elastic modulus of cam[Pa]
% k                                   Thermal Conductivity [W/m.K]
% rho                                 Density [kg/m^3];
% sigma                               Specific Heat [J/kgK]
% fc                                  Asperity friction coefficient [-]
% alpha                               Piezoelectric/pressure-viscosity coefficient
[m2/N]or[Pa^-1]
% sr                                  Surface roughness [m]
% v                                   Vickers hardness [Pa]
% lambda                              Limiting shear stress coefficient [-]
% Kt                                  Temperature-viscosity coefficient [K^-1]
% Z                                   Viscosity- pressure index [-]

% OPTIONAL ARGUMENTS
% [No additional arguments]

% OUTPUT VARIABLES
% O1                                  Peclet Number, Maximum Flash Temperature, Effective
Radius and Elasticity, Rolling and Sliding Speed, Half Width Contact
```

```
% O2                                    Friction and Power loss due to Sliding and Rolling, Total
losses, Film thickness

% REQUIRED FUNCTIONS (USER-DEFINED)
% Cam_power.m
% Temp.m
% FluidPropModel.m
% FluidParameters.m
% setCR.m
% setFldPro.m
% setLD.m
% setLK.m

% PROGRAM HISTORY
%{
Created      : 12/27/2018, Jordan Saikia, MSOE
Last Modified: 05/17/2019, Jordan Saikia, MSOE

CHANGE LOG
===================================================================
-01/04/2019 Updated comments
-01/25/2019 Included updated Fluid Prop model
-05/17/2019 Structure format of the code
===================================================================

===================== 12/27/2018 ==================================
-Initial release.
===================================================================
```

```
%Inputs
```

```
clc; clear all;

% Fluid properties
% Operating parameters
OP.FluidNum = 1;                                   % Type 1, 2, 3, 4, or 5 respectively for
fluids GRP1, HVI, TMP, PAO, or PAG
OP.T = 50;                                         % Temperature of the fluid [C]
OP.p = convpres(3000,'psi','pa');                  % Pressure of the fluid [pa]
% Get all the fixed parameters of the fluids
[FldPar,Fld] = FluidParameters(OP.FluidNum);
% Call FluidPropModel() function to determine fluid properties
[FldPro] = FldPropModel(Fld,OP.T,OP.p);
FldPro = setFldPro(FldPro.mu);                     % Inlet dynamic Viscosity [Pa.s]

% Cam Specs R1=0.0175,L=0.019,
Cam_Spec = setCR(0.007477,0.1011,0.018962,0.3,0.3,200e9,200e9,46.7,7800,460,0.05,20e-9,1.76022e-
6,7.875e9,0.05,0.03,0.65,50,0.05);

% Kinematics and Kinetics
load('VDLM Single Cyl Rev 1a.mat');
LD = setLD(VDLM.A5.LD.FC3r);                       % Radial load [N]
LK = setLK(((VDLM.A5.LK.om_RF)*(Cam_Spec.RBR)),((VDLM.A5.LK.om_MB)*(Cam_Spec.CR)));        %
```

```
Angular frequency [rad/s converted to m/s]
t = linspace(1,200,200);

% Outputs Peclet Number, Flash Temperature, and Conjunction Temperature

 [O1.Pec1,O1.Tf_max1,O1.Pec2,O1.Tf_max2,O1.R_eff,O1.nr,O1.ns,O1.E_eff,O1.a] =
Temp(Cam_Spec,LD,LK);

O1.Tf_max_conj = abs(((1./O1.Tf_max1)+(1./O1.Tf_max2)).^(-1));                  %
Conjunction Temperature [deg C]
O1.delT = (Cam_Spec.IT) + (O1.Tf_max_conj);         % Temperature Rise [deg C]

% Friction, Power Loss, and Film Thickness
[O2.F_sliding,O2.F_rolling,O2.H_sliding,O2.H_rolling,O2.Htot,O2.Hc] =
Cam_power(Cam_Spec,LD,LK,FldPro,O1.R_eff,O1.nr,O1.E_eff,O1.a,O1.delT);

HH = O2.Hc*O1.R_eff;                                % Actual Film Thickness [m]

% Plots
% % Power Loss
% figure('Name','Total power losses profile');
% subplot(3,1,1)
% plot(t,O2.Htot)
% title('Total power losses profile');
% xlabel('Fraction of Cycle')
% ylabel('Total power loss (W)')
% ylim([0 10]);
%
% subplot(3,1,2)
% plot(t,O2.H_sliding)
% title('Sliding power loss profile');
% xlabel('Fraction of cycle')
% ylabel(' Sliding power loss (W)')
% ylim([0 3]);
%
% subplot(3,1,3)
% plot(t,O2.H_rolling)
% title('Rolling power loss profile');
% xlabel('Fraction of cycle')
% ylabel('Rolling power loss (W)')
% ylim([0 6]);

% % Film Thickness
% figure('Name','Cam Follower');
% suptitle('Cam Follower')
% subplot(3,1,1)
% plot(t,HH)
% title('Film thickness profile');
% xlabel('Fraction of Cycle')
% ylabel('Film thickness (m)')
%
% subplot(3,1,2)
% plot(t,LD.Fr_RF)
% title('Load profile');
```

```
% xlabel('Fraction of Cycle')
% ylabel('Load (N)')
%
% subplot(3,1,3)
% plot(t,O1.nr)
% title('Velocity profile');
% xlabel('Fraction of Cycle')
% ylabel('Velocity (m/s)')

% Power Loss
figure('Name','Total power losses profile');
subplot(3,1,1)
plot(t,O2.Htot)
title('Total power losses profile');
xlabel('Fraction of Cycle')
ylabel('Total power loss (W)')
ylim([0 6]);

%figure('Name','Sliding power losses profile');
subplot(3,1,2)
plot(t,O2.H_sliding)
title('Sliding power loss profile');
xlabel('Fraction of cycle')
ylabel(' Sliding power loss (W)')
ylim([0 6]);

%figure('Name','Rolling power losses profile');
subplot(3,1,3)
plot(t,O2.H_rolling)
title('Rolling power loss profile');
xlabel('Fraction of cycle')
ylabel('Rolling power loss (W)')
ylim([0 6]);
```

MATLAB Functions

Structured Cam Roller Follower Specifications

```
function Cam_Spec =
setCR(R1,R2,L,pois1,pois2,E1,E2,k,rho,sigma,fc,alpha,sr,v,lambda,Kt,Z,T_0,coeff)

Cam_Spec.RBR = R1;                      % Radius of Roller Bearing [m]
Cam_Spec.CR = R2;                       % Average Radius of Cam from major and minor radii [m]
Cam_Spec.RBL = L;                       % Half length of Roller Bearing [m]
Cam_Spec.PR1 = pois1;                   % Poisson Ratio of roller
Cam_Spec.PR2 = pois2;                   % Poisson Ratio of cam
Cam_Spec.EMR = E1;                      % Elastic modulus of roller [Pa]
Cam_Spec.EMC = E2;                      % Elastic modulus of cam[Pa]
```

```
Cam_Spec.TC = k;                    % Thermal Conductivity [W/m.K]
Cam_Spec.Den = rho;                 % Density [kg/m^3]
Cam_Spec.SH = sigma;                % Specific Heat [J/kgK]
Cam_Spec.AFC = fc;                  % Asperity friction coefficient
Cam_Spec.PVC = alpha;               % Piezoelectric/pressure-viscosity coefficient
[m2/N]or[Pa^-1]
Cam_Spec.SR = sr;                   % Surface roughness [m]
Cam_Spec.VH = v;                    % Vickers hardness [Pa] 803 HV
Cam_Spec.SSC = lambda;              % Shear Stress coefficient [-]
Cam_Spec.TVC = Kt;                  % Temperature-viscosity coefficient [K^-1]
Cam_Spec.VPI = Z;                   % Viscosity- pressure index
Cam_Spec.IT = T_0;                  % Inlet Temperature [deg C]
Cam_Spec.COF = coeff;               % Coefficient of friction

end
```

Structured Fluid Properties Input

```
% SYNTAX
function FldPro = setFldPro(mu_0)

FldPro.mu = mu_0;        % Kinematic Viscosity [cSt]

end
```

Structured Force Input

```
function LD = setLD(Fr_RF)

LD.Fr_RF = Fr_RF;        % Roller Follower Radial Load [N]

end
```

Structured Angular Velocity Input

```
function LK = setLK(n_RF,n_MB)

LK.n_RF = n_RF;        % Roller Follower Angular Velocity [rad/s]
LK.n_MB = n_MB;        % Main Bearing Angular Velocity [rad/s]

end
```

Structured Flash Temperature

```
% SYNTAX
function [Pec1,Tf_max1,Pec2,Tf_max2,R_eff,nr,ns,E_eff,a] = Temp(Cam_Spec,LD,LK)
```

```
        % DESCRIPTION
% Running this function will calculate the maximum flash temperature
% in the cam follower mechanism using inputs such as
% rotational frequency, radial load, and cam specs. This script is created according to the
Stachowiach model.


% ONE-LINE DESCRIPTION
% Calculates maximum flash temperature for cam and follower interface.

% REQUIRED ARGUMENTS
% n_RF                                  Roller angular velocity [rad/s converted to m/s]
% n_MB                                  Cam angular velocity [rad/s converted to m/s]
% Fr                                    Radial load [N]
% R1                                    Radius of roller [m]
% R2                                    Radius of cam [m]
% L                                     Length of roller [m]
% pois1                                 Poisson Ratio of roller [-]
% pois2                                 Poisson Ratio of cam [-]
% E1                                    Elastic modulus of roller [Pa]
% E2                                    Elastic modulus of cam[Pa]
% k                                     Thermal Conductivity [W/m.K]
% rho                                   Density [kg/m^3];
% sigma                                 Specific Heat [J/kgK]

% OPTIONAL ARGUMENTS
% [No additional arguments]

% OUTPUT VARIABLES
% Pec                                    Peclet Number [-]
% Tf_max                                 Maximum Flash Temperature[°C]
% R_eff                                  Effective radius [m]
% n_r                                    Rolling Velocity [m/s]
% n_s                                    Sliding Velocity [m/s]
% E_eff                                  Effective Elasticity [Pa]
% a                                      Contact half width [m]

% REQUIRED FUNCTIONS (USER-DEFINED)
% None

% FUNCTION HISTORY
%{
Created      : 12/27/2018, Jordan Saikia, MSOE
Last Modified: 05/17/2019, Jordan Saikia, MSOE

CHANGE LOG
===================================================================
-01/04/2019 Updated comments
-04/03/2019 Updated comments
-05/17/2019 Structre format
```

```
===================================================================

===================== 12/27/2018 ==================================
-Initial release.
===================================================================
%}
```

Cam Specs Unpacking

```
        R1 = Cam_Spec.RBR;                      % Radius of Roller Bearing [m]
R2 = Cam_Spec.CR;                     % Average Radius of Cam from major and minor radii [m]
L = Cam_Spec.RBL;                     % Half length of Roller Bearing [m]
pois1 = Cam_Spec.PR1;                 % Poisson Ratio of roller
pois2 = Cam_Spec.PR2;                 % Poisson Ratio of cam
E1 = Cam_Spec.EMR;                    % Elastic modulus of roller [Pa]
E2 = Cam_Spec.EMC;                    % Elastic modulus of cam[Pa]
k = Cam_Spec.TC;                      % Thermal Conductivity [W/m.K]
rho = Cam_Spec.Den;                   % Density [kg/m^3]
sigma = Cam_Spec.SH;                  % Specific Heat [J/kgK]
coeff = Cam_Spec.COF;                 % Coefficient of friction

% Load (Unpacking)
Fr_RF = LD.Fr_RF;                     % Roller Follower Radial Load [N]

% Angular Vel (Unpacking)
n_RF = LK.n_RF;                       % Roller Follower Angular Velocity [rad/s]
n_MB = LK.n_MB;                       % Main Bearing Angular Velocity [rad/s]


% CALCULATIONS
R_eff = 1/((1/R1)+(1/R2));                                % Effective Radius [m]
E_eff = 2*(((1-(pois1)^2)/E1)+((1-(pois2)^2)/E2))^(-1);   % Effective Elactic Modulus [Pa]
a = abs((4*Fr_RF*R_eff)/(pi*L*E_eff)).^0.5;               % Half Width of Contact [m]

nr = (n_RF+n_MB)/2;                                       % Rolling Speed [m/s]
ns = abs(n_RF-n_MB);                                      % Sliding Speed [m/s]

chi = k/(rho*sigma);                                      % Thermal Diffusivity [m2/s]

Pec1 = (abs((n_RF.*a)/(2*chi)));                          % Peclet number [-]
Tf_max1 = 0.318*((coeff*Fr_RF.*abs(n_RF-n_MB))/(k*L)).*(chi./(n_RF.*a)).*((-
2.303*Pec1).*(log10(Pec1))+(1.116*Pec1)); % Flash Temperature [˚C]

Pec2 = (abs((n_MB.*a)/(2*chi)));                          % Peclet number [-]
Tf_max2 = 0.318*((coeff*Fr_RF.*abs(n_MB-n_RF))/(k*L)).*(chi./(n_MB.*a)).*((-
2.303*Pec2).*(log10(Pec2))+(1.116*Pec2)); % Flash Temperature [˚C]
```

```
        end
```

Appendix G: Structured SKF Bearing Power Loss

MATLAB Script: The script requires the code for Fluid Properties (see Appendix E).

```
% DESCRIPTION
% Running this program will calculate the frictional torque and power
% losses in all the bearings using inputs such as
% bearing specs, kinematic viscosity, rotational
% frequency, and radial load.
% This script is created according to the SKF model [1].

% [1] SKF, 2018.
%     The SKF model for calculating the frictional moment,
%     p-1-15.

% ONE-LINE DESCRIPTION
% Calculates friction torque and power loss for all bearings.

% REQUIRED ARGUMENTS
% dm                                                  Pitch Diameter [mm]
% n                                                   Angular velocity [rad/s converted to rpm in
formula]
% Fr                                                  Radial Load [N]
% nu                                                  Kinematic Viscosity [cSt]
% R1,R2                                               Geometric constants for rolling frictional
moments for Full complement bearings
% S1,S2                                               Geometric constants for sliding frictional
moments for Full complement bearings
% mu_bl                                               Constant depending on movement
% mu_ehl                                              Sliding friction coefficient in full film
conditions
% D                                                   Outer diameter[mm]
% d                                                   Inner diameter[mm]
% B                                                   Width of the bearing [mm]
% Vm                                                  Drag loss factor
% Kz                                                  Bearing type related geometric constant
% Kl                                                  Roller bearing type related geometric
constant
% Y                                                   Axial preload factor
% K                                                   Preload spring constant

% OPTIONAL ARGUMENTS
% [No additional arguments]

% OUTPUT VARIABLES
% T_RG,RM,WP,RF,MB                                    Total Torque [N.mm]
% H_RG,RM,WP,RF,MB                                    Power loss [Watts]
% Ttotal                                              Total Torque [N.mm]
% Htotal                                              Power loss [Watts]

% REQUIRED FUNCTIONS (USER-DEFINED)
% FluidParameters.m
```

```
% FldPropModel.m
% setFldPro.m
% setLK.m
% setLD.m
% setBR_RG.m
% setBR_RM.m
% setBR_WP.m
% setBR_RF.m
% setBR_MB.m
% Roller_S.m
% Tapered_Roller_S.m

% PROGRAM HISTORY
%{
Created      : 12/26/2018, Jordan Saikia, MSOE
Last Modified: 04/15/2019, Jordan Saikia, MSOE

CHANGE LOG
===================================================================
-12/26/2018 Included one function for radial needle and tapered roller bearing
-02/01/2019 Included import of matlab data instead of excel file
-04/15/2019 Structured the code according to UMN guidelines
===================================================================

===================== 12/26/2018 ==================================
-Initial release.
===================================================================
%}


clc; clear all;
format longG;
```

```
% Inputs
```

```
% Fluid properties
% Operating parameters
OP.FluidNum = 1;                                  % Type 1, 2, 3, 4, or 5 respectively for
fluids GRP1, HVI, TMP, PAO, or PAG
OP.T = 50;                                        % Temperature of the fluid [C]
OP.p = convpres(3000,'psi','pa');                 % Pressure of the fluid [pa]
% Get all the fixed parameters of the fluids
[FldPar,Fld] = FluidParameters(OP.FluidNum);
% Call FluidPropModel() function to determine fluid properties
[FldPro] = FldPropModel(Fld,OP.T,OP.p);
FldPro = setFldPro(FldPro.nu);                    % Inlet Kinematic Viscosity [m^2/s]

% Linkage Kinematics and Dynamics
%load('VDLM Data 3000 PSI 3-200 RPM.mat');
load('VDLM Single Cyl Rev 1a.mat'); % a=200 RPM, b=1RPM, c=10RPM, d=100RPM
LD =
setLD((VDLM.A5.LD.F14r),(VDLM.A5.LD.F43r),(VDLM.A5.LD.F32r),(VDLM.A5.LD.FC3r),(VDLM.A5.LD.F1Cr));
% Radial load [N]
LK =
```

```
setLK((VDLM.A5.LK.om_RG),(VDLM.A5.LK.om_RM),(VDLM.A5.LK.om_WP),(VDLM.A5.LK.om_RF),(VDLM.A5.LK.om_
MB));         % Angular frequency [rad/s converted to rpm in function]

% Bearing Specifications
BR.BR_RG = setBR_RG(19.995,2.13e-
6,0.0015,0.12,0.05,29.988,9.912,15.036,0.00105,6.2,0.7,1);%14...26.5,2.13e-
6,0.0015,0.12,0.05,30,23,15,0.00105,6.2,0.7,1
BR.BR_RM = setBR_RM(18.122,2.13e-
6,0.0015,0.12,0.05,21.014,15.23,15.904,0.00105,6.2,0.7,1);%28...18,2.13e-
6,0.0015,0.12,0.05,21,15,16,0.00105,6.2,0.7,1
BR.BR_WP = setBR_WP(21.804,2.13e-
6,0.0015,0.12,0.05,24.048,19.56,1.818,0.00105,6.2,0.7,1);%14...21,2.13e-
6,0.0015,0.12,0.05,24,18,16,0.00105,6.2,0.7,1
BR.BR_RF = setBR_RF(28.464,2.13e-
6,0.0015,0.12,0.05,41.974,14.954,18.962,0.00105,6.2,0.7,1);%7...36,2.13e-
6,0.0015,0.12,0.05,42,20,19,0.00105,6.2,0.7,1
BR.BR_MB = setBR_MB(107.529,1.7,0.02,2.31e-
6,10.9,0.009,2,0.12,0.002,125.074,89.984,24.836,0.0011,6,0.7,1);%2...107.5,1.7,0.02,2.31e-
6,10.9,0.009,2,0.12,0.002,125,90,23,0.0011,6,0.7,1

t = linspace(1,200,200);

% Outputs

% Friction Torque and Power Loss of each bearings
[Output.T_RG,Output.H_RG,Output.T_RM,Output.H_RM,Output.T_WP,Output.H_WP,Output.T_RF,Output.H_RF]
= Roller_S(BR.BR_RG,BR.BR_RM,BR.BR_WP,BR.BR_RF,LD,LK,FldPro);   % RG,RM,WP,RF
[Output.T_MB,Output.H_MB] = Tapered_Roller_S(BR.BR_MB,LD,LK,FldPro);
% MB

% Total Friction Torque and Power Loss
Output.Ttotal = Output.T_RG+Output.T_RM+Output.T_WP+Output.T_RF+Output.T_MB;
Output.Htotal = Output.H_RG+Output.H_RM+Output.H_WP+Output.H_RF+Output.H_MB;

% Plots
figure ('Name','Total Friction Torque')
plot(t,Output.T_RG,t,Output.T_RM,t,Output.T_WP,t,Output.T_RF,t,Output.T_MB,t,Output.Ttotal)
title('Combined Friction Torque profile');
xlabel('Time (s)')
ylabel('Friction Torque (N.mm)')
legend('Rocker Ground','Rocker Moving','Wrist Pin','Roller Follower','Main Bearing','Total Loss')
legend boxoff

figure ('Name','Total Power Loss')
plot(t,Output.H_RG,t,Output.H_RM,t,Output.H_WP,t,Output.H_RF,t,Output.H_MB,t,Output.Htotal)
title('Power Loss Profile');
xlabel('Index')
%ylim([0 80]);
ylabel('Power Losses (W)')
legend('Rocker Ground','Rocker Moving','Wrist Pin','Roller Follower','Main Bearing','Total Loss')
legend boxoff
```

MATLAB Functions

Needle Roller Bearings

```
% SYNTAX
function [T_RG, H_RG,T_RM, H_RM,T_WP, H_WP,T_RF,H_RF] =
Roller_S(BR_RG,BR_RM,BR_WP,BR_RF,LD,LK,FldPro)

% DESCRIPTION
% Calling this function will calculate the frictional torque and power
% losses in a Full Complement Roller Bearing using inputs such as
% bearing specs like pitch diameter, kinematic viscosity from 'FluidProp' function, rotational
% frequency, and radial load.

% ONE-LINE DESCRIPTION
% Calculates friction torque and power loss for Full Complement Roller Bearing.

% REQUIRED ARGUMENTS
% dm                                      Pitch Diameter [mm]
% nu_0                                    Viscosity [cSt] at at 3000psi and 50C for fluid 1
% n                                       Angular velocity [rad/s converted to rpm]
% Fr                                      Radial Load [N]
% R1                                      Geometric constants for rolling frictional moments for
Full complement bearings [-]
% S2                                      Geometric constants for sliding frictional moments for
Full complement bearings [-]
% mu_bl                                   Constant depending on movement [-]
% mu_ehl                                  Sliding friction coefficient in full film conditions [-]
% D                                       Outer diameter[mm]
% d                                       Inner diameter[mm]
% B                                       Width of the bearing [mm]
% Vm                                      Drag loss factor [-]
% Kz                                      Bearing type related geometric constant [-]
% Kl                                      Roller bearing type related geometric constant [-]

% OPTIONAL ARGUMENTS
% [No additional arguments]

% OUTPUT VARIABLES
% T_RG,RM,WP,RF                           Friction Torque for RG,RM,WP,RF [N.mm]
% H_RG,RM,WP,RF                           Power Loss for RG,RM,WP,RF [Watts]


% REQUIRED FUNCTIONS (USER-DEFINED)
% [No required functions]

% FUNCTION HISTORY
%{
Created      : 12/26/2018, Jordan Saikia, MSOE
Last Modified: 04/15/2019, Jordan Saikia, MSOE
```

```
CHANGE LOG
============================================================
-12/26/2018 Included all the Full Complement Roller Bearings
-04/15/2019 Structured the code according to UMN guidelines
============================================================

===================== 12/26/2018 ============================
-Initial release.
============================================================
%}

% Fluid Properties
nu_0=FldPro.nu;

% RG

Bearing specs (Unpacking)

dm_RG = BR_RG.RG_PD;
R1_RG = BR_RG.RG_GCR1;
S2_RG = BR_RG.RG_GCS2;
mu_bl_RG = BR_RG.RG_MC;
mu_ehl_RG = BR_RG.RG_SFC;
D_RG = BR_RG.RG_OD;
d__RG = BR_RG.RG_ID;
B_RG = BR_RG.RG_W;
Vm_RG = BR_RG.RG_DLF;
Kz_RG = BR_RG.RG_BGC;
Kl_RG = BR_RG.RG_RBGC;
N_RG = BR_RG.RG_N;

% Load (Unpacking)
Fr_RG = (LD.Fr_RG)/N_RG;
% Angular Vel (Unpacking)
n_RG = LK.n_RG;
n_1 = (abs(convangvel(n_RG,'rad/s','rpm')));

% CALCULATIONS
% Rolling Friction Torque
phi_ish_RG = 1./(1+((1.84e-9).*((n_1.*dm_RG).^1.28)).*(nu_0.^0.64));                % Inlet
shear heating reduction factor
G_rr_RG = R1_RG.*(dm_RG.^2.41).*(Fr_RG.^0.31);                                      %
Geometric and load dependent variables
T_rr_RG = (phi_ish_RG).*(G_rr_RG).*((nu_0.*n_1).^0.6);                              %
Rolling Friction Torque

% Sliding Friction Torque
G_sl_RG = S2_RG.*dm_RG.*Fr_RG;                                                       %
Sliding Friction variable
phi_bl_RG = 1./(exp((2.6e-8).*((n_1.*nu_0).^1.4).*dm_RG));                          %
Weighting factor for the sliding friction coefficient
mu_sl_RG = (phi_bl_RG.*mu_bl_RG)+((1-phi_bl_RG).*mu_ehl_RG);                        %
Sliding Friction coefficient
T_sl_RG = (G_sl_RG).*(mu_sl_RG);                                                     %
```

```
Sliding Friction Torque

% Drag Friction Torque
ld_RG = 5.*((Kl_RG.*B_RG)./dm_RG);                                        % Drag
loss factor
Cw_RG = (2.789e-10.*ld_RG.^3)-(2.786e-4.*ld_RG.^2)+(0.0195.*ld_RG)+0.6439;        % Drag
loss factor
t_RG = abs(2.*acos(((0.6.*dm_RG)-D_RG)./(0.6.*dm_RG)));                            %
Drag loss factor
fa_RG = 0.05.*((Kz_RG.*(D_RG+d__RG))./(D_RG-d__RG));                          % Drag
loss factor
if t_RG >= 0 && t_RG <= pi                                                   % Drag
loss factor
    ft_RG = abs(sin(0.5.*t_RG));
else
%elseif t_RG > pi && t_RG < 2*pi
    ft_RG = 1;
end
Rs_RG = 0.36.*(dm_RG.^2).*(t_RG-sin(t_RG)).*fa_RG;                              % Drag
loss factor
Kroll_RG = ((Kl_RG*Kz_RG.*(d__RG+D_RG))./(D_RG-d__RG)).*1e-12;                  %
Rolling element constant
T_drag_RG = (4.*Vm_RG.*Kroll_RG.*Cw_RG.*B_RG.*(dm_RG.^4).*(n_1.^2))+((1.093e-
7.*(n_1.^2).*(dm_RG.^3).*(((n_1.*(dm_RG.^2).*ft_RG)./nu_0).^-1.379)).*Rs_RG);   % Drag Frcition
Torque

% Total Friction Torque and Power Loss
T_RG = (T_rr_RG+T_sl_RG+T_drag_RG)*N_RG;                                     % RG
Friction Torque [N.mm]
H_RG = (1.047e-4.*(abs(T_RG)).*(n_1));                                       % RG
Power Loss [Watts]

% RM

Bearing specs (Unpacking)

dm_RM = BR_RM.RM_PD;
R1_RM = BR_RM.RM_GCR1;
S2_RM = BR_RM.RM_GCS2;
mu_bl_RM = BR_RM.RM_MC;
mu_ehl_RM = BR_RM.RM_SFC;
D_RM = BR_RM.RM_OD;
d__RM = BR_RM.RM_ID;
B_RM = BR_RM.RM_W;
Vm_RM = BR_RM.RM_DLF;
Kz_RM = BR_RM.RM_BGC;
Kl_RM = BR_RM.RM_RBGC;
N_RM = BR_RM.RM_N;
% Load (Unpacking)
Fr_RM = (LD.Fr_RM)/N_RM;
% Angular Vel (Unpacking)
n_RM = LK.n_RM;
n_2 = (abs(convangvel(n_RM,'rad/s','rpm')));
```

```
% CALCULATIONS
% Rolling Friction Torque
phi_ish_RM = 1./(1+((1.84e-9).*((n_2.*dm_RM).^1.28)).*(nu_0.^0.64));                    % Inlet
shear heating reduction factor
G_rr_RM = R1_RM.*(dm_RM.^2.41).*(Fr_RM.^0.31);                                           %
Geometric and load dependent variables
T_rr_RM = (phi_ish_RM).*(G_rr_RM).*((nu_0.*n_2).^0.6);                                   %
Rolling Friction Torque

% Sliding Friction Torque
G_sl_RM = S2_RM.*dm_RM.*Fr_RM;                                                           %
Sliding Friction variable
phi_bl_RM = 1./(exp((2.6e-8).*((n_2.*nu_0).^1.4).*dm_RM));                               %
Weighting factor for the sliding friction coefficient
mu_sl_RM = (phi_bl_RM.*mu_bl_RM)+((1-phi_bl_RM).*mu_ehl_RM);                             %
Sliding Friction coefficient
T_sl_RM = (G_sl_RM).*(mu_sl_RM);                                                         %
Sliding Friction Torque

% Drag Friction Torque
ld_RM = 5.*((Kl_RM.*B_RM)./dm_RM);                                                       % Drag
loss factor
Cw_RM = (2.789e-10.*ld_RM.^3)-(2.786e-4.*ld_RM.^2)+(0.0195.*ld_RM)+0.6439;               % Drag
loss factor
t_RM = 2.*acos(((0.6.*dm_RM)-D_RM)./(0.6.*dm_RM));                                       % Drag
loss factor
fa_RM = 0.05.*((Kz_RM.*(D_RM+d__RM))./(D_RM-d__RM));                                     % Drag
loss factor
if t_RM >= 0 && t_RM <= pi                                                               % Drag
loss factor
    ft_RM = sin(0.5.*t_RM);
elseif t_RM > pi && t_RM < 2*pi
    ft_RM = 1;
end
Rs_RM = 0.36.*(dm_RM.^2).*(t_RM-sin(t_RM)).*fa_RM;                                       % Drag
loss factor
Kroll_RM = ((Kl_RM*Kz_RM.*(d__RM+D_RM))./(D_RM-d__RM)).*1e-12;                           %
Rolling element constant
T_drag_RM = (4.*Vm_RM.*Kroll_RM.*Cw_RM.*B_RM.*(dm_RM.^4).*(n_2.^2))+((1.093e-
7.*(n_2.^2).*(dm_RM.^3).*(((n_2.*(dm_RM.^2).*ft_RM)./nu_0).^-1.379)).*Rs_RM);    % Drag Frcition
Torque

% Total Friction Torque and Power Loss
T_RM = (T_rr_RM+T_sl_RM+T_drag_RM)*N_RM ;                                                % RM
Friction Torque [N.mm]
H_RM = (1.047e-4.*(abs(T_RM)).*(n_2));                                                   % RM
Power Loss [Watts]

% WP

Bearing specs (Unpacking)

dm_WP = BR_WP.WP_PD;
R1_WP = BR_WP.WP_GCR1;
```

```
S2_WP = BR_WP.WP_GCS2;
mu_bl_WP = BR_WP.WP_MC;
mu_ehl_WP = BR_WP.WP_SFC;
D_WP = BR_WP.WP_OD;
d__WP = BR_WP.WP_ID;
B_WP = BR_WP.WP_W;
Vm_WP = BR_WP.WP_DLF;
Kz_WP = BR_WP.WP_BGC;
Kl_WP = BR_WP.WP_RBGC;
N_WP = BR_WP.WP_N;
% Load (Unpacking)
Fr_WP = (LD.Fr_WP)/N_WP;
% Angular Vel (Unpacking)
n_WP = LK.n_WP;
n_3 = (abs(convangvel(n_WP,'rad/s','rpm')));

% CALCULATIONS
% Rolling Friction Torque
phi_ish_WP = 1./(1+((1.84e-9).*((n_3.*dm_WP).^1.28)).*(nu_0.^0.64));                % Inlet
shear heating reduction factor
G_rr_WP = R1_WP.*(dm_WP.^2.41).*(Fr_WP.^0.31);                                      %
Geometric and load dependent variables
T_rr_WP = (phi_ish_WP).*(G_rr_WP).*((nu_0.*n_3).^0.6);                              %
Rolling Friction Torque

% Sliding Friction Torque
G_sl_WP = S2_WP.*dm_WP.*Fr_WP;                                                      %
Sliding Friction variable
phi_bl_WP = 1./(exp((2.6e-8).*((n_3.*nu_0).^1.4).*dm_WP));                          %
Weighting factor for the sliding friction coefficient
mu_sl_WP = (phi_bl_WP.*mu_bl_WP)+((1-phi_bl_WP).*mu_ehl_WP);                        %
Sliding Friction coefficient
T_sl_WP = (G_sl_WP).*(mu_sl_WP);                                                    %
Sliding Friction Torque

% Drag Friction Torque
ld_WP = 5.*((Kl_WP.*B_WP)./dm_WP);                                                  % Drag
loss factor
Cw_WP = (2.789e-10.*ld_WP.^3)-(2.786e-4.*ld_WP.^2)+(0.0195.*ld_WP)+0.6439;          % Drag
loss factor
t_WP = 2.*acos(((0.6.*dm_WP)-D_WP)./(0.6.*dm_WP));                                  % Drag
loss factor
fa_WP = 0.05.*((Kz_WP.*(D_WP+d__WP))./(D_WP-d__WP));                                % Drag
loss factor
if t_WP >= 0 && t_WP <= pi                                                          % Drag
loss factor
    ft_WP = sin(0.5.*t_WP);
elseif t_WP > pi && t_WP < 2*pi
    ft_WP = 1;
end
Rs_WP = 0.36.*(dm_WP.^2).*(t_WP-sin(t_WP)).*fa_WP;                                  % Drag
loss factor
Kroll_WP = ((Kl_WP*Kz_WP.*(d__WP+D_WP))./(D_WP-d__WP)).*1e-12;                      %
Rolling element constant
```

```
T_drag_WP = (4.*Vm_WP.*Kroll_WP.*Cw_WP.*B_WP.*(dm_WP.^4).*(n_3.^2))+((1.093e-
7.*(n_3.^2).*(dm_WP.^3).*(((n_3.*(dm_WP.^2).*ft_WP)./nu_0).^-1.379)).*Rs_WP);   % Drag Frcition
Torque

% Total Friction Torque and Power Loss
T_WP = (T_rr_WP+T_sl_WP+T_drag_WP)*N_WP;                                            % WP
Friction Torque [N.mm]
H_WP = (1.047e-4.*(abs(T_WP)).*(n_3));                                              % WP
Power Loss [Watts]

% RF

%Bearing Specs (Unpacking)
dm_RF = BR_RF.RF_PD;
R1_RF = BR_RF.RF_GCR1;
S2_RF = BR_RF.RF_GCS2;
mu_bl_RF = BR_RF.RF_MC;
mu_ehl_RF = BR_RF.RF_SFC;
D_RF = BR_RF.RF_OD;
d__RF = BR_RF.RF_ID;
B_RF = BR_RF.RF_W;
Vm_RF = BR_RF.RF_DLF;
Kz_RF = BR_RF.RF_BGC;
Kl_RF = BR_RF.RF_RBGC;
N_RF = BR_RF.RF_N;
% Load (Unpacking)
Fr_RF = (LD.Fr_RF)/N_RF;
% Angular Vel (Unpacking)
n_RF = LK.n_RF;
n_4 = (abs(convangvel(n_RF,'rad/s','rpm')));

% CALCULATIONS
% Rolling Friction Torque
phi_ish_RF = 1./(1+((1.84e-9).*((n_4.*dm_RF).^1.28)).*(nu_0.^0.64));               % Inlet
shear heating reduction factor
G_rr_RF = R1_RF.*(dm_RF.^2.41).*(Fr_RF.^0.31);                                      %
Geometric and load dependent variables
T_rr_RF = (phi_ish_RF).*(G_rr_RF).*((nu_0.*n_4).^0.6);                              %
Rolling Friction Torque

% Sliding Friction Torque
G_sl_RF = S2_RF.*dm_RF.*Fr_RF;                                                      %
Sliding Friction variable
phi_bl_RF = 1./(exp((2.6e-8).*((n_4.*nu_0).^1.4).*dm_RF));                          %
Weighting factor for the sliding friction coefficient
mu_sl_RF = (phi_bl_RF.*mu_bl_RF)+((1-phi_bl_RF).*mu_ehl_RF);                        %
Sliding Friction coefficient
T_sl_RF = (G_sl_RF).*(mu_sl_RF);                                                    %
Sliding Friction Torque

% Drag Friction Torque
ld_RF = 5.*((Kl_RF.*B_RF)./dm_RF);                                                  % Drag
loss factor
Cw_RF = (2.789e-10.*ld_RF.^3)-(2.786e-4.*ld_RF.^2)+(0.0195.*ld_RF)+0.6439;          % Drag
```

```
loss factor
t_RF = 2.*acos(((0.6.*dm_RF)-D_RF)./(0.6.*dm_RF));                              % Drag
loss factor
fa_RF = 0.05.*((Kz_RF.*(D_RF+d__RF))./(D_RF-d__RF));                             % Drag
loss factor
for i = 1:length(t_RF)
if t_RF(i) >= 0 && t_RF(i) <= pi                                                      %
Drag loss factor
    ft_RF = sin(0.5.*t_RF(i));
else
    %elseif t_RF(i) > pi && t_RF(i) < 2*pi
    ft_RF = 1;
end
end
Rs_RF = 0.36.*(dm_RF.^2).*(t_RF-sin(t_RF)).*fa_RF;                                % Drag
loss factor
Kroll_RF = ((Kl_RF*Kz_RF.*(d__RF+D_RF))./(D_RF-d__RF)).*1e-12;                    %
Rolling element constant
T_drag_RF = (4.*Vm_RF.*Kroll_RF.*Cw_RF.*B_RF.*(dm_RF.^4).*(n_4.^2))+((1.093e-
7.*(n_4.^2).*(dm_RF.^3).*(((n_4.*(dm_RF.^2).*ft_RF)./nu_0).^-1.379)).*Rs_RF);   % Drag Frcition
Torque

% Total Friction Torque and Power Loss
T_RF = (T_rr_RF+T_sl_RF+T_drag_RF)*N_RF;                                          % RF
Friction Torque [N.mm]
H_RF = (1.047e-4.*(abs(T_RF)).*(n_4));                                            % RF
Power Loss [Watts]

       end
```

Tapered Roller Bearings

```
% SYNTAX
function[T_MB, H_MB] = Tapered_Roller_S(BR_MB,LD,LK,FldPro)

       % DESCRIPTION
% Calling this function will calculate the frictional torque and power
% losses in a Full Complement Roller Bearing using inputs such as
% bearing specs like pitch diameter, kinematic viscosity from 'FluidProp' function, rotational
% frequency, and radial load.

% ONE-LINE DESCRIPTION
% Calculates friction torque and power loss for Full Complement Roller Bearing.

% REQUIRED ARGUMENTS
% dm                                        Pitch Diameter [mm]
% nu_0                                      Viscosity [cSt] at at 3000psi and 50C for fluid 1
% n                                         Angular velocity [rad/s converted to rpm]
% Fr                                        Radial Load [N]
% R1,R2                                     Geometric constants for rolling frictional moments for
Full complement bearings [-]
```

```
% S1,S2                                   Geometric constants for sliding frictional moments for
Full complement bearings [-]
% mu_bl                                   Constant depending on movement [-]
% mu_ehl                                  Sliding friction coefficient in full film conditions [-]
% D                                       Outer diameter[mm]
% d                                       Inner diameter[mm]
% B                                       Width of the bearing [mm]
% Vm                                      Drag loss factor [-]
% Kz                                      Bearing type related geometric constant [-]
% Kl                                      Roller bearing type related geometric constant [-]
% Y                                       Axial preload factor [-]
% K                                       Preload spring constant [-]

% OPTIONAL ARGUMENTS
% [No additional arguments]

% OUTPUT VARIABLES
% T_MB                                    Friction Torque for RG,RM,WP,RF [N.mm]
% H_MB                                    Power Loss for RG,RM,WP,RF [Watts]

% REQUIRED FUNCTIONS (USER-DEFINED)
% [No required functions]

% FUNCTION HISTORY
%{
Created      : 12/26/2018, Jordan Saikia, MSOE
Last Modified: 04/15/2019, Jordan Saikia, MSOE

CHANGE LOG
============================================================
-12/26/2018 Included all the tapered roller bearings
-04/15/2019 Structured the code according to UMN guidelines
============================================================

===================== 12/26/2018 ==========================
-Initial release.
===========================================================
%}

% Fluid Properties

nu_0=FldPro.nu;

% MB

Bearing specs (Unpacking)

dm_MB = BR_MB.MB_PD;
Y_MB = BR_MB.MB_APF;
K_MB = BR_MB.MB_PSC;
R1_MB = BR_MB.MB_GCR1;
R2_MB = BR_MB.MB_GCR2;
S1_MB = BR_MB.MB_GCS1;
S2_MB = BR_MB.MB_GCS2;
```

```
mu_bl_MB = BR_MB.MB_MC;
mu_ehl_MB = BR_MB.MB_SFC;
D_MB = BR_MB.MB_OD;
d_MB = BR_MB.MB_ID;
B_MB = BR_MB.MB_W;
Vm_MB = BR_MB.MB_DLF;
Kz_MB = BR_MB.MB_BGC;
Kl_MB = BR_MB.MB_RBGC;
N_MB = BR_MB.MB_N;

% Load (Unpacking)
Fr_MB = (LD.Fr_MB)/N_MB;
% Angular Vel (Unpacking)
n_MB = LK.n_MB;
n_5 = (abs(convangvel(n_MB,'rad/s','rpm')));

% CALCULATIONS
% Rolling Friction Torque
phi_ish_MB = 1./(1+((1.84e-9).*((n_5.*dm_MB).^1.28)).*(nu_0.^0.64));                 % Inlet
shear heating reduction factor
Fa_MB = K_MB*d_MB*1e-3;                                                              %
Preload Axial Load [N]
G_rr_MB = R1_MB.*(dm_MB.^2.38).*((Fr_MB+(R2_MB*Y_MB*Fa_MB)).^(0.31));                %
Geometric and load dependent variables
T_rr_MB = (phi_ish_MB).*(G_rr_MB).*((nu_0.*n_5).^0.6);                               %
Rolling Friction Torque

% Sliding Friction Torque
G_sl_MB = S1_MB.*(dm_MB.^(0.82)).*(Fr_MB+(S2_MB*Y_MB*Fa_MB));                        %
Sliding Friction variable
phi_bl_MB = 1./(exp((2.6e-8).*((n_5.*nu_0).^1.4).*dm_MB));                           %
Weighting factor for the sliding friction coefficient
mu_sl_MB = (phi_bl_MB.*mu_bl_MB)+((1-phi_bl_MB).*mu_ehl_MB);                         %
Sliding Friction coefficient
T_sl_MB = (G_sl_MB).*(mu_sl_MB);                                                     %
Sliding Friction Torque

% Drag Friction Torque
ld_MB = 5.*((Kl_MB.*B_MB)./dm_MB);                                                   % Drag
loss factor
Cw_MB = (2.789e-10.*ld_MB.^3)-(2.786e-4.*ld_MB.^2)+(0.0195.*ld_MB)+0.6439;           % Drag
loss factor
t_MB = 2.*acos(((0.6.*dm_MB)-D_MB)./(0.6.*dm_MB));                                   % Drag
loss factor
fa_MB = 0.05.*((Kz_MB.*(D_MB+d_MB))./(D_MB-d_MB));                                   % Drag
loss factor
if t_MB >= 0 && t_MB <= pi                                                           % Drag
loss factor
    ft_MB = sin(0.5.*t_MB);
elseif t_MB > pi && t_MB < 2*pi
    ft_MB = 1;
end
Rs_MB = 0.36.*(dm_MB.^2).*(t_MB-sin(t_MB)).*fa_MB;                                   % Drag
loss factor
```

```
Kroll_MB = ((Kl_MB*Kz_MB.*(d_MB+D_MB))./(D_MB-d_MB)).*1e-12;                           %
Rolling element constant
T_drag_MB = (4.*Vm_MB.*Kroll_MB.*Cw_MB.*B_MB.*(dm_MB.^4).*(n_5.^2))+((1.093e-
7.*(n_5.^2).*(dm_MB.^3).*(((n_5.*(dm_MB.^2).*ft_MB)./nu_0).^-1.379)).*Rs_MB);   % Drag Frcition
Torque

% Total Friction Torque and Power Loss
T_MB = (T_rr_MB+T_sl_MB+T_drag_MB)*N_MB;                                                %
Friction Torque [N.mm]
H_MB = (1.047e-4.*(abs(T_MB)).*(n_5));                                                  % Power
Loss [Watts]

       end
```

Structure Main Bearing Specification Input

```
       function BR_MB =
setBR_MB(dm_MB,Y_MB,K_MB,R1_MB,R2_MB,S1_MB,S2_MB,mu_bl_MB,mu_ehl_MB,D_MB,d_MB,B_MB,Vm_MB,Kz_MB,Kl
_MB,N_MB)

BR_MB.MB_PD = dm_MB;                 % Pitch Diameter [mm]
BR_MB.MB_APF = Y_MB;                 % Axial preload factor
BR_MB.MB_PSC = K_MB;                 % Preload spring constant
BR_MB.MB_GCR1 = R1_MB;               % Geometric constants for rolling frictional moments for Full
complement bearings [-]
BR_MB.MB_GCR2 = R2_MB;               % Second geometric constants for rolling frictional moments
for Full complement bearings [-]
BR_MB.MB_GCS1 = S1_MB;               % Geometric constants for sliding frictional moments for Full
complement bearings [-]
BR_MB.MB_GCS2 = S2_MB;               % Second geometric constants for sliding frictional moments
for Full complement bearings [-]
BR_MB.MB_MC = mu_bl_MB;              % Constant depending on movement [-]
BR_MB.MB_SFC = mu_ehl_MB;            % Sliding friction coefficient in full film conditions [-]
BR_MB.MB_OD = D_MB;                  % Outer diameter[mm]
BR_MB.MB_ID = d_MB;                  % Inner diameter[mm]
BR_MB.MB_W = B_MB;                   % Width of the bearing [mm]
BR_MB.MB_DLF = Vm_MB;                % Drag loss factor [-]
BR_MB.MB_BGC = Kz_MB;                % Bearing type related geometric constant[-]
BR_MB.MB_RBGC = Kl_MB;               % Roller bearing type related geometric constant [-]
BR_MB.MB_N = N_MB;                   % Number of Bearings [-]

end
```

Structure Roller Follower Specification Input

```
       function BR_RF =
setBR_RF(dm_RF,R1_RF,S2_RF,mu_bl_RF,mu_ehl_RF,D_RF,d_RF,B_RF,Vm_RF,Kz_RF,Kl_RF,N_RF)

BR_RF.RF_PD = dm_RF;                 % Pitch Diameter [mm]
```

```
BR_RF.RF_GCR1 = R1_RF;           % Geometric constants for rolling frictional moments for Full
complement bearings [-]
BR_RF.RF_GCS2 = S2_RF;           % Second geometric constants for sliding frictional moments
for Full complement bearings [-]
BR_RF.RF_MC = mu_bl_RF;          % Constant depending on movement [-]
BR_RF.RF_SFC = mu_ehl_RF;        % Sliding friction coefficient in full film conditions [-]
BR_RF.RF_OD = D_RF;              % Outer diameter[mm]
BR_RF.RF_ID = d_RF;              % Inner diameter[mm]
BR_RF.RF_W = B_RF;               % Width of the bearing [mm]
BR_RF.RF_DLF = Vm_RF;            % Drag loss factor [-]
BR_RF.RF_BGC = Kz_RF;            % Bearing type related geometric constant[-]
BR_RF.RF_RBGC = Kl_RF;           % Roller bearing type related geometric constant [-]
BR_RF.RF_N = N_RF;               % Number of Bearings [-]

end
```

Structure Rocker Ground Specification Input

```
        function BR_RG =
setBR_RG(dm_RG,R1_RG,S2_RG,mu_bl_RG,mu_ehl_RG,D_RG,d_RG,B_RG,Vm_RG,Kz_RG,Kl_RG,N_RG)

BR_RG.RG_PD = dm_RG;             % Pitch Diameter [mm]
BR_RG.RG_GCR1 = R1_RG;           % Geometric constants for rolling frictional moments for Full
complement bearings [-]
BR_RG.RG_GCS2 = S2_RG;           % Second geometric constants for sliding frictional moments
for Full complement bearings [-]
BR_RG.RG_MC = mu_bl_RG;          % Constant depending on movement [-]
BR_RG.RG_SFC = mu_ehl_RG;        % Sliding friction coefficient in full film conditions [-]
BR_RG.RG_OD = D_RG;              % Outer diameter[mm]
BR_RG.RG_ID = d_RG;              % Inner diameter[mm]
BR_RG.RG_W = B_RG;               % Width of the bearing [mm]
BR_RG.RG_DLF = Vm_RG;            % Drag loss factor [-]
BR_RG.RG_BGC = Kz_RG;            % Bearing type related geometric constant[-]
BR_RG.RG_RBGC = Kl_RG;           % Roller bearing type related geometric constant [-]
BR_RG.RG_N = N_RG;               % Number of Bearings [-]

end
```

Structure Rocker Moving Specification Input

```
        function BR_RM =
setBR_RM(dm_RM,R1_RM,S2_RM,mu_bl_RM,mu_ehl_RM,D_RM,d_RM,B_RM,Vm_RM,Kz_RM,Kl_RM,N_RM)

BR_RM.RM_PD = dm_RM;             % Pitch Diameter [mm]
BR_RM.RM_GCR1 = R1_RM;           % Geometric constants for rolling frictional moments for Full
complement bearings [-]
BR_RM.RM_GCS2 = S2_RM;           % Second geometric constants for sliding frictional moments
for Full complement bearings [-]
BR_RM.RM_MC = mu_bl_RM;          % Constant depending on movement [-]
```

```
BR_RM.RM_SFC = mu_ehl_RM;          % Sliding friction coefficient in full film conditions [-]
BR_RM.RM_OD = D_RM;                % Outer diameter[mm]
BR_RM.RM_ID = d_RM;                % Inner diameter[mm]
BR_RM.RM_W = B_RM;                 % Width of the bearing [mm]
BR_RM.RM_DLF = Vm_RM;              % Drag loss factor [-]
BR_RM.RM_BGC = Kz_RM;              % Bearing type related geometric constant[-]
BR_RM.RM_RBGC = Kl_RM;             % Roller bearing type related geometric constant [-]
BR_RM.RM_N = N_RM;                 % Number of Bearings [-]

end
```

Structure Wrist Pin Specification Input

```
       function BR_WP =
setBR_WP(dm_WP,R1_WP,S2_WP,mu_bl_WP,mu_ehl_WP,D_WP,d_WP,B_WP,Vm_WP,Kz_WP,Kl_WP,N_WP)

BR_WP.WP_PD = dm_WP;               % Pitch Diameter [mm]
BR_WP.WP_GCR1 = R1_WP;             % Geometric constants for rolling frictional moments for Full
complement bearings [-]
BR_WP.WP_GCS2 = S2_WP;             % Second geometric constants for sliding frictional moments
for Full complement bearings [-]
BR_WP.WP_MC = mu_bl_WP;            % Constant depending on movement [-]
BR_WP.WP_SFC = mu_ehl_WP;          % Sliding friction coefficient in full film conditions [-]
BR_WP.WP_OD = D_WP;                % Outer diameter[mm]
BR_WP.WP_ID = d_WP;                % Inner diameter[mm]
BR_WP.WP_W = B_WP;                 % Width of the bearing [mm]
BR_WP.WP_DLF = Vm_WP;              % Drag loss factor [-]
BR_WP.WP_BGC = Kz_WP;              % Bearing type related geometric constant[-]
BR_WP.WP_RBGC = Kl_WP;             % Roller bearing type related geometric constant [-]
BR_WP.WP_N = N_WP;                 % Number of Bearings [-]

end
```

Force Input

```
       function LD = setLD(Fr_RG,Fr_RM,Fr_WP,Fr_RF,Fr_MB)

LD.Fr_RG = Fr_RG;       % Rocker Ground Radial Load [N]
LD.Fr_RM = Fr_RM;       % Rocker Moving Radial Load [N]
LD.Fr_WP = Fr_WP;       % Wrist Pin Radial Load [N]
LD.Fr_RF = Fr_RF;       % Roller Follower Radial Load [N]
LD.Fr_MB = Fr_MB;       % Main Bearing Radial Load [N]

end
```

Angular Velocity Input

```
        function LK = setLK(n_RG,n_RM,n_WP,n_RF,n_MB)

LK.n_RG = n_RG;        % Rocker Ground Angular Velocity [rad/s]
LK.n_RM = n_RM;        % Rocker Moving Angular Velocity [rad/s]
LK.n_WP = n_WP;        % Wrist Pin Angular Velocity [rad/s]
LK.n_RF = n_RF;        % Roller Follower Angular Velocity [rad/s]
LK.n_MB = n_MB;        % Main Bearing Angular Velocity [rad/s]

end
```

Fluid Viscosity Input

```
        % SYNTAX
function FldPro = setFldPro(nu_0)

FldPro.nu = nu_0;        % Kinematic Viscosity [cSt]

end
```

Appendix H: Force and Angular Velocity Input from UMN

Table H-1: Force and Angular Velocity Input.

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
383.5084	20.94395102	383.5084	267.7576773	821.6439	0.001419844	822.073	-0.000197624	971.258	0.001222221
353.9991		353.9991	267.4943188	760.7309	0.422610721	761.1994	-0.058809892	902.988	0.363800829
41.8314		41.8314	267.2519749	12.02387	0.850507457	29.34868	-0.11828304	11.9367	0.732224417
6281.006		6281.006	267.030434	12184.33	1.285720026	12184.35	-0.178625786	14579.5	1.10709424
6486.645		6486.645	266.8305823	12504.21	1.726722226	12504.19	-0.239546294	15034.4	1.487175931
6500.445		6500.445	266.6525506	12451.6	2.17301586	12451.54	-0.300893022	15045.3	1.872122838
6512.072		6512.072	266.4961655	12392.35	2.624600438	12392.24	-0.362580655	15049.8	2.262019782
6523.278		6523.278	266.3611436	12329.94	3.081714224	12329.77	-0.424553804	15052	2.65716042
6534.967		6534.967	266.2478645	12265.49	3.542895342	12265.25	-0.486519994	15053.1	3.056375349
6546.753		6546.753	266.156025	12199.36	4.007818336	12199.03	-0.548343858	15053.5	3.459474478
6559.372		6559.372	266.0853877	12131.89	4.476020237	12131.47	-0.609868631	15053.5	3.866151605
6572.231		6572.231	266.0355164	12063.2	4.947182699	12062.67	-0.670958485	15053.1	4.276224214

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
6586.03	20.94395102	6586.03	266.0060141	11993.52	5.420989278	11992.87	-0.731475362	15052.4	4.689513915
6600.499		6600.499	265.9967285	11922.96	5.895252034	11922.18	-0.791032181	15051.6	5.104219853
6615.542		6615.542	266.0068777	11851.64	6.369850229	11850.72	-0.849524573	15050.6	5.520325656
6631.21		6631.21	266.0356603	11779.72	6.844942121	11778.64	-0.906885503	15049.3	5.938056618
6647.774		6647.774	266.0824883	11707.33	7.319000759	11706.09	-0.962826067	15048	6.356174692
6664.761		6664.761	266.146489	11634.59	7.791390192	11633.18	-1.017183125	15046.5	6.774207067
6682.635		6682.635	266.2267924	11561.67	8.261302783	11560.08	-1.06977378	15045	7.191529003
6701.171		6701.171	266.3224332	11488.7	8.726413689	11486.92	-1.120229212	15043.3	7.606184477
6720.072		6720.072	266.4322469	11415.85	9.186653175	11413.86	-1.168482546	15041.5	8.018170629
6739.67		6739.67	266.5551266	11343.29	9.641365548	11341.1	-1.21439552	15039.6	8.426970028
6759.82		6759.82	266.6898335	11271.25	10.08854922	11268.84	-1.25767213	15037.8	8.830877091
6780.515		6780.515	266.8350275	11199.95	10.52659437	11197.33	-1.298081534	15036	9.228512831
6801.517		6801.517	266.9893652	11129.61	10.95457949	11126.77	-1.335493246	15034.3	9.619086241
6822.821		6822.821	267.1515351	11060.4	11.37211908	11057.32	-1.369852483	15032.7	10.0022666

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
6844.79	20.94395102	6844.79	267.3199314	10992.44	11.77641942	10989.14	-1.400827113	15031.3	10.37559231
6866.687		6866.687	267.4930883	10926.09	12.16557209	10922.56	-1.428219829	15030.1	10.73735227
6888.545		6888.545	267.6699804	10861.63	12.53982664	10857.87	-1.452100889	15029	11.08772576
6910.826		6910.826	267.8488193	10799.12	12.89603727	10795.14	-1.472159499	15028.3	11.42387777
6932.67		6932.67	268.0281392	10738.87	13.23211717	10734.68	-1.488241715	15027.8	11.74387545
6954.232		6954.232	268.2070582	10681.09	13.54820993	10676.69	-1.500460376	15027.5	12.04774956
6975.74		6975.74	268.3841989	10625.98	13.84136453	10621.38	-1.508599922	15027.6	12.33276461
6996.389		6996.389	268.5585207	10573.74	14.11039845	10568.95	-1.512671153	15027.9	12.5977273
7016.34		7016.34	268.7296464	10524.38	14.35658516	10519.42	-1.512955729	15028.6	12.84362943
7035.871		7035.871	268.8968521	10477.89	14.57861094	10472.77	-1.509464393	15029.6	13.06914655
7054.221		7054.221	269.0599808	10434.39	14.77703547	10429.13	-1.5024194	15030.9	13.27461607
7072.206		7072.206	269.2189654	10393.78	14.95181672	10388.39	-1.491976659	15032.6	13.45984006
7089.1		7089.1	269.3737404	10356.14	15.10217308	10350.64	-1.478232212	15034.6	13.62394086
7105.051		7105.051	269.5249414	10321.46	15.22979773	10315.86	-1.461524652	15036.9	13.76827308

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
7120.459	20.94395102	7120.459	269.6728153	10289.65	15.33356146	10283.98	-1.441915588	15039.7	13.89164587
7134.529		7134.529	269.8180689	10260.82	15.41400884	10255.09	-1.419637897	15042.8	13.99437094
7146.624		7146.624	269.9615517	10232.79	15.47154428	10227.01	-1.39490122	15043.3	14.07664306
7155.851		7155.851	270.1041393	10205.4	15.50606722	10199.59	-1.367875196	15040.7	14.13819203
7164.131		7164.131	270.2470535	10180.85	15.51916917	10175.03	-1.338870464	15038.5	14.1802987
7171.697		7171.697	270.391295	10159.07	15.50974147	10153.26	-1.307957558	15036.7	14.20178391
7177.71		7177.71	270.5381983	10140.17	15.47952153	10134.38	-1.275453717	15035.1	14.20406781
7183.521		7183.521	270.6890271	10123.86	15.42791677	10118.11	-1.241463165	15034	14.18645361
7187.566		7187.566	270.8451335	10110.39	15.35539879	10104.69	-1.206189934	15033.1	14.14920885
7191.099		7191.099	271.0079123	10099.47	15.26398155	10093.84	-1.169935596	15032.7	14.09404596
7193.757		7193.757	271.1788389	10091.13	15.1516442	10085.58	-1.132689733	15032.5	14.01895446
7194.77		7194.77	271.3591378	10085.44	15.02178111	10079.99	-1.094849487	15032.6	13.92693162
7195.853		7195.853	271.5502984	10082.04	14.87354327	10076.69	-1.05646907	15033.2	13.8170742
7195.116		7195.116	271.7537102	10081.22	14.70707586	10076	-1.017690852	15033.9	13.68938501

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
7193.955	20.94395102	7193.955	271.9703365	10082.67	14.52499378	10077.57	-0.978802338	15035	13.54619145
7191.954		7191.954	272.2018446	10086.39	14.32554761	10081.43	-0.939789349	15036.4	13.38575826
7188.625		7188.625	272.4488842	10092.41	14.1114306	10087.6	-0.900929342	15038	13.21050126
7185.083		7185.083	272.7126533	10100.46	13.88263276	10095.81	-0.862300248	15039.9	13.02033252
7180.353		7180.353	272.9944514	10110.66	13.63873904	10106.17	-0.82396066	15042	12.81477838
7174.664		7174.664	273.2941345	10122.9	13.38305425	10118.57	-0.786179063	15044.3	12.59687518
7168.824		7168.824	273.6131481	10136.94	13.11429282	10132.79	-0.748932215	15046.9	12.3653606
7161.811		7161.811	273.9524353	10152.93	12.83245312	10148.95	-0.71228081	15049.6	12.12017231
7153.972		7153.972	274.3111146	10170.71	12.54094293	10166.92	-0.676457339	15052.5	11.86448559
7145.924		7145.924	274.6903062	10190.14	12.23885435	10186.52	-0.641440389	15055.6	11.59741396
7137.041		7137.041	275.0911229	10211.29	11.925456	10207.86	-0.607225315	15058.8	11.31823068
7127.293		7127.293	275.5126586	10234.14	11.60316857	10230.9	-0.57396039	15062.1	11.02920818
7117.241		7117.241	275.9553486	10258.57	11.27199142	10255.5	-0.541656251	15065.5	10.73033517
7106.624		7106.624	276.4205722	10284.58	10.93061525	10281.7	-0.510262814	15069.1	10.42035244

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
7095.053	20.94395102	7095.053	276.9073227	10312.21	10.58089629	10309.51	-0.479876883	15072.6	10.10101941
7083.118		7083.118	277.4150165	10341.33	10.22384773	10338.82	-0.450539686	15076.3	9.773308045
7070.791		7070.791	277.9451319	10371.92	9.857946512	10369.58	-0.422177751	15080	9.43576876
7057.581		7057.581	278.4968944	10404.01	9.4842648	10401.85	-0.394831227	15083.6	9.089433573
7044.002		7044.002	279.0693462	10437.5	9.103895287	10435.51	-0.368528204	15087.3	8.735367083
7029.654		7029.654	279.6616843	10472.41	8.717545126	10470.58	-0.343274959	15090.9	8.374270167
7015.199		7015.199	280.2741933	10508.6	8.32482426	10506.93	-0.319026727	15094.6	8.005797533
7000.151		7000.151	280.9078948	10546.12	7.92466678	10544.61	-0.295715938	15098.2	7.628950842
6984.348		6984.348	281.5601338	10584.95	7.519301102	10583.59	-0.27339418	15101.6	7.245906922
6968.228		6968.228	282.2292942	10624.99	7.109812972	10623.78	-0.25205814	15105	6.857754833
6951.837		6951.837	282.916221	10666.21	6.695150209	10665.14	-0.231625253	15108.3	6.463524956
6934.775		6934.775	283.6193898	10708.62	6.276188374	10707.68	-0.21208016	15111.4	6.064108214
6917.741		6917.741	284.3388389	10752.09	5.85255917	10751.28	-0.193358789	15114.5	5.659200381
6899.898		6899.898	285.0733786	10796.68	5.424756205	10795.98	-0.175426433	15117.4	5.249329772

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
6881.906	20.94395102	6881.906	285.8203689	10842.27	4.994300716	10841.68	-0.158273856	15120	4.83602686
6863.637		6863.637	286.5803124	10888.84	4.560359715	10888.35	-0.141814092	15122.5	4.418545624
6844.939		6844.939	287.3513409	10936.37	4.123763538	10935.98	-0.126011142	15124.8	3.997752396
6826.028		6826.028	288.1314999	10984.83	3.685337065	10984.51	-0.110824452	15126.9	3.574512612
6807.009		6807.009	288.9206723	11034.16	3.244653867	11033.92	-0.096173637	15128.8	3.148480231
6787.702		6787.702	289.7176447	11084.36	2.801994751	11084.19	-0.081998817	15130.5	2.719995933
6768.363		6768.363	290.5212624	11135.4	2.35759959	11135.27	-0.068236766	15132	2.289362824
6748.708		6748.708	291.3294904	11187.25	1.912198972	11187.17	-0.054837254	15133.3	1.857361718
6729.184		6729.184	292.1410726	11239.86	1.466048049	11239.82	-0.041734356	15134.5	1.424313693
6709.482		6709.482	292.9552803	11293.23	1.019055652	11293.21	-0.028852053	15135.5	0.9902036
6689.758		6689.758	293.769582	11347.29	0.572180668	11347.29	-0.016143205	15136.3	0.556037463
6670.196		6670.196	294.5841972	11401.95	0.124804667	11401.96	-0.003515752	15136.9	0.121288915
6764.088		6764.088	295.3965209	11617.28	-0.322132118	11617.33	0.009078454	15357.9	-0.313053665
6744.534		6744.534	296.2039143	11674.58	-0.767670037	11674.62	0.021686883	15358.7	-0.745983154

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
6726.503	20.94395102	6726.503	297.0064786	11734.81	-1.212360142	11734.82	0.034398989	15362.5	-1.177961153
6714.32		6714.32	297.8005788	11805.833	-1.654760581	11805.82	0.047247	15379.7	-1.607513581
6736.133		6736.133	298.5839801	11937.34	-2.094174291	11937.29	0.060282038	15474.4	-2.033892253
6117.709		6117.709	299.3536926	10919.65	-2.529552542	10919.55	0.073542963	14086.1	-2.456009579
1688.409		1688.409	300.1090926	2983.768	-2.961011181	2983.696	0.087098709	3834.95	-2.873912473
500.4985		500.4985	300.8488898	840.7326	-3.388361079	840.8865	0.101008161	1079.5	-3.287352918
486.1793		486.1793	301.5687608	822.3189	-3.809863263	822.3972	0.11527936	1050.23	-3.694583903
478.8807		478.8807	302.2693966	816.2963	-4.226249582	816.2878	0.129990389	1036.93	-4.096259193
473.7878		473.7878	302.9475386	814.7554	-4.636331254	814.6439	0.145158376	1029.34	-4.491172878
470.1682		470.1682	303.6014527	815.3709	-5.039603338	815.1541	0.160814616	1024.57	-4.878788722
467.0877		467.0877	304.2312075	817.1547	-5.436437034	816.8227	0.177015694	1021.3	-5.25942134
464.3937		464.3937	304.8336462	819.6362	-5.825638565	819.1825	0.193762707	1018.95	-5.631875858
462.0004		462.0004	305.407426	822.5631	-6.206851569	821.9827	0.2110778	1017.2	-5.995773769
459.666		459.666	305.9499775	825.7647	-6.579140582	825.0506	0.228958011	1015.83	-6.350182572

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
457.7527	20.94395102	457.7527	306.4617156	829.1941	-6.942892517	828.3473	0.247439045	1014.82	-6.695453472
455.7738		455.7738	306.9424864	832.7405	-7.298242758	831.7542	0.266546859	1013.99	-7.031695898
453.9598		453.9598	307.3898444	836.3915	-7.644230793	835.2641	0.286261112	1013.36	-7.357969681
452.2005		452.2005	307.8031538	840.1087	-7.980705318	838.8375	0.306585359	1012.87	-7.674119959
450.3971		450.3971	308.1795805	843.8574	-8.306504314	842.4393	0.327478102	1012.49	-7.979026212
448.8981		448.8981	308.519551	847.6805	-8.621872327	846.1187	0.348944226	1012.29	-8.2729281
447.4629		447.4629	308.8249007	851.5341	-8.927679563	849.8278	0.371013746	1012.2	-8.556665817
445.9831		445.9831	309.0934527	855.3803	-9.222992057	853.5281	0.393639359	1012.18	-8.829352698
444.5193		444.5193	309.3228164	859.1969	-9.506745844	857.1998	0.416758512	1012.22	-9.089987332
443.1437		443.1437	309.5127858	862.9501	-9.778804929	860.8111	0.440340815	1012.28	-9.338464114
441.8603		441.8603	309.6646052	866.6169	-10.03964645	864.3392	0.464380151	1012.33	-9.575266296
440.6013		440.6013	309.7789455	870.1804	-10.28948073	867.7667	0.488857159	1012.37	-9.800623568
439.1861		439.1861	309.853495	873.5935	-10.5271987	871.0454	0.513685617	1012.34	-10.01351309
437.8805		437.8805	309.8871915	876.9072	-10.75217801	874.2303	0.538791925	1012.3	-10.21338608

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
436.7568	20.94395102	436.7568	309.8832399	880.1406	-10.96563662	877.3408	0.564191514	1012.28	-10.40144511
435.3486		435.3486	309.8399449	883.1769	-11.16667746	880.2562	0.589793731	1012.16	-10.57688373
434.1474		434.1474	309.7561906	886.1278	-11.35456976	883.094	0.615504441	1012.05	-10.73906532
432.9569		432.9569	309.6345596	888.9456	-11.53019735	885.8045	0.641314943	1011.92	-10.8888824
431.7546		431.7546	309.4749882	891.6203	-11.69327543	888.378	0.667151939	1011.75	-11.02612349
430.4474		430.4474	309.2760824	894.1213	-11.84292169	890.784	0.692902486	1011.53	-11.1500192
429.3897		429.3897	309.0398382	896.5474	-11.97969485	893.1239	0.718530506	1011.33	-11.26116434
428.3145		428.3145	308.7692774	898.8313	-12.10463738	895.328	0.744034362	1011.09	-11.36060302
427.1486		427.1486	308.4632842	900.9497	-12.21696877	897.3733	0.769301358	1010.81	-11.44766741
426.1704		426.1704	308.1234456	902.9857	-12.31704545	899.3446	0.79428391	1010.53	-11.52276154
425.3551		425.3551	307.7544259	904.9416	-12.40660077	901.2428	0.819028727	1010.27	-11.58757204
424.1047		424.1047	307.3531857	906.6313	-12.4840117	902.8805	0.843367542	1009.88	-11.64064415
423.3252		423.3252	306.9208325	908.3335	-12.54935713	904.54	0.867231048	1009.57	-11.68212609
422.5772		422.5772	306.4639487	909.9247	-12.60523689	906.0946	0.890735171	1009.24	-11.71450172

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
421.4713	20.94395102	421.4713	305.9790078	911.2742	-12.64977328	907.4137	0.913688157	1008.81	-11.73608512
420.7548		420.7548	305.4671581	912.6236	-12.68305911	908.741	0.936024985	1008.44	-11.74703412
419.997		419.997	304.9331754	913.8418	-12.7068994	909.9431	0.957816131	1008.03	-11.74908327
419.1208		419.1208	304.3757589	914.9002	-12.72035959	910.9922	0.978931331	1007.58	-11.74142826
418.3091		418.3091	303.7950456	915.8597	-12.72310238	911.9493	0.99927885	1007.13	-11.72382353
417.789		417.789	303.1960122	916.8069	-12.71700821	912.9	1.018943299	1006.71	-11.69806491
416.91		416.91	302.5784564	917.5175	-12.70166852	913.6203	1.037838933	1006.21	-11.66382959
416.1778		416.1778	301.9407768	918.1633	-12.67592106	914.2824	1.055808408	1005.71	-11.62011266
415.5692		415.5692	301.2869609	918.7431	-12.64121633	914.8842	1.072911337	1005.22	-11.568305
414.7827		414.7827	300.6171562	919.15	-12.59725419	915.3193	1.089067389	1004.67	-11.5081868
414.2435		414.2435	299.9330612	919.5384	-12.54441417	915.7411	1.10425138	1004.17	-11.44016279
413.5892		413.5892	299.236943	919.7816	-12.48341286	916.023	1.118475248	1003.62	-11.36493761
412.7729		412.7729	298.5265738	919.8573	-12.41278014	916.1438	1.131554513	1003.03	-11.28122563
412.4084		412.4084	297.8051042	920.1758	-12.3335779	916.5115	1.143528116	1002.73	-11.19004978

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
411.8826	20.94395102	411.8826	297.0750167	920.4038	-12.24665794	916.7939	1.154428345	1002.466	-11.09222959
411.4014		411.4014	296.3355434	920.545	-12.1512323	916.9942	1.164132973	1002.2	-10.98709933
410.8529		410.8529	295.5874148	920.5568	-12.0472434	917.0702	1.172587554	1001.89	-10.87465585
410.4938		410.4938	294.8329362	920.5409	-11.93541094	917.1219	1.179813942	1001.62	-10.755597
409.9391		409.9391	294.0729342	920.3496	-11.81577064	917.0032	1.185773749	1001.29	-10.62999689
409.431		409.431	293.3068924	920.0744	-11.68761038	916.8048	1.190350493	1000.95	-10.49725989
409.0687		409.0687	292.5378076	919.7582	-11.55204186	916.5682	1.193613841	1000.63	-10.35842802
408.5696		408.5696	291.7666618	919.2917	-11.4091944	916.1853	1.195540558	1000.27	-10.21365384
407.9992		407.9992	290.9923704	918.6975	-11.25805484	915.6789	1.19598586	999.879	-10.06206899
407.5659		407.5659	290.2166736	918.0588	-11.09907925	915.1304	1.194957616	999.504	-9.904121634
407.1996		407.1996	289.4425152	917.3521	-10.93345025	914.516	1.192549154	999.136	-9.740901094
406.4897		406.4897	288.6681329	916.4125	-10.75979802	913.674	1.188580901	998.67	-9.571217115
406.0281		406.0281	287.8937937	915.4713	-10.57773711	912.8318	1.182972292	998.251	-9.394764814
405.7282		405.7282	287.1243102	914.5014	-10.38955203	911.9612	1.175948367	997.862	-9.213603664

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
405.0478	20.94395102	405.0478	286.3584512	913.2881	-10.19417981	910.8521	1.167367055	997.37	-9.026812758
404.4698		404.4698	285.5947755	912.0178	-9.990266332	909.6879	1.157043253	996.889	-8.83322308
404.1446		404.1446	284.8374798	910.7557	-9.779809967	908.5307	1.145180946	996.461	-8.634629021
403.6307		403.6307	284.0878764	909.3314	-9.563245939	907.2139	1.131814689	995.98	-8.431431249
403.0057		403.0057	283.3442397	907.7727	-9.339090598	905.7649	1.116750996	995.462	-8.222339602
402.5089		402.5089	282.6076996	906.1736	-9.107545531	904.2755	1.099993326	994.965	-8.007552205
402.0245		402.0245	281.8802864	904.4924	-8.869454815	902.704	1.081628376	994.465	-7.78782644
401.6143		401.6143	281.1634676	902.7548	-8.625365192	901.0753	1.061712484	993.977	-7.563652708
401.015		401.015	280.4570153	900.8597	-8.374752773	899.291	1.040174436	993.437	-7.334578337
400.4371		400.4371	279.7604352	898.8867	-8.116819524	897.4286	1.016906272	992.894	-7.099913252
399.974		399.974	279.075583	896.8734	-7.852366147	895.5239	0.992001301	992.375	-6.860364847
399.423		399.423	278.4033306	894.7448	-7.581603259	893.5042	0.965484734	991.83	-6.616118525
398.9897		398.9897	277.7445571	892.5788	-7.304747564	891.4443	0.937384827	991.311	-6.367362737
398.4172		398.4172	277.0996726	890.2811	-7.021734189	889.2531	0.907697865	990.755	-6.114036323

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
397.8291	20.94395102	397.8291	276.4682206	887.8975	-6.731793359	886.9746	0.876329121	990.191	-5.855464238
397.3228		397.3228	275.851278	885.4675	-6.43530674	884.6469	0.84333452	989.648	-5.59197222
396.815		396.815	275.2497792	882.9847	-6.132660807	882.2644	0.808775416	989.124	-5.323885391
396.3334		396.3334	274.6645209	880.4453	-5.823838104	879.8227	0.772663374	988.647	-5.051174729
395.8071		395.8071	274.0952149	877.8608	-5.508523996	877.3343	0.734972959	988.203	-4.773551037
395.4758		395.4758	273.5431153	875.3022	-5.187270157	874.8661	0.695796265	987.852	-4.491473891
394.906		394.906	273.008035	872.6195	-4.859608225	872.2744	0.655093489	987.481	-4.204514735
394.5031		394.5031	272.4898253	869.9436	-4.524941614	869.684	0.612807218	987.184	-3.912134396
394.1721		394.1721	271.9900326	867.2387	-4.184313756	867.0598	0.569105806	986.936	-3.61520795
393.6617		393.6617	271.5082029	864.4185	-3.836960726	864.3193	0.523916526	986.665	-3.3130442
393.4149		393.4149	271.0450282	861.6204	-3.483138216	861.5925	0.477306772	986.479	-3.005831443
392.9454		392.9454	270.6008423	858.6867	-3.122884359	858.7293	0.429319076	986.25	-2.693565283
392.6414		392.6414	270.1755965	855.7297	-2.7557464	855.8351	0.379930434	986.065	-2.375815967
392.2916		392.2916	269.7700577	852.6742	-2.382236891	852.8379	0.329254679	985.858	-2.052982212

Main		Roller Follower		RG		RM		WP	
Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)	Load (N)	Angular Vel (rad/sec)
391.9848	20.94395102	391.9848	269.3844446	849.5264	-2.002303416	849.7419	0.277331247	985.628	-1.724972169
391.4621		391.4621	269.0183614	846.1748	-1.615169054	846.4394	0.224102511	985.26	-1.391066543
391.1981		391.1981	268.6726509	842.6854	-1.221486835	842.9869	0.169711969	984.794	-1.051774866
390.4286		390.4286	268.3472383	838.7271	-0.820969012	839.0657	0.114177094	983.886	-0.706791917
389.6988		389.6988	268.0421081	834.234	-0.413210278	834.5971	0.057501897	982.412	-0.355708381
383.5083		383.5083	267.7576773	821.6438	0.001419844	822.0729	-0.000197624	971.258	0.001222221

Appendix I: Measured Bearing Specifications

Table I-1: Main Bearing (Koyo 32918JR).

Bearing bore dia	89.9		Bearing Width	24.83
	89.99			24.84
	90.02			24.82
	90.01			24.85
	90			24.84
	89.984			24.836
Bearing outside dia	125.15			
	125.07			
	125.07			
	125.03			
	125.05			
	125.074			
Length of roller	16.32			
	16.34			
	16.31			
	16.33			
	16.31			
	16.322			
Diameter of roller				
	7.5			
No. of roller	34			

Table I-2: Roller Follower (Schaeffler NUTR1542).

Bearing bore dia	14.97	Bearing Width	18.92
	14.94		18.92
	14.96		18.96
	14.95		19.01
	14.95		19
	14.954		18.962
Bearing outside dia	41.95		
	41.99		
	41.98		
	41.98		
	41.97		
	41.974		
Length of roller			
	5.5		
Diameter of roller			
	4		
No. of roller	36		

Table I-3: Rocker Ground (Schaeffler NATR10-PP).

Bearing bore dia	9.93	Bearing Width	15.04
	9.9		15.04
	9.89		15.1
	9.93		15.05
	9.91		14.95
	9.912		15.036
Bearing outside dia	30		
	29.99		
	30		
	30		
	29.95		
	29.988		
Length of roller			
	6.8		
Diameter of roller			
	2.2		
No. of roller	18		

Table I- 4: Rocker Moving (Schaeffler HN 1516).

Bearing bore dia	15.27	Bearing Width	15.9
	15.3		15.89
	15.2		15.89
	15.15		15.95
	15.23		15.89
	15.23		15.904
Bearing outside dia	21.02		
	21.01		
	21.01		
	21.02		
	21.01		
	21.014		
Length of roller	13.69		
	13.68		
	13.69		
	13.69		
	13.7		
	13.69		
Diameter of roller	1.95		
	1.94		
	1.94		
	1.95		
	1.94		
	1.944		
No. of roller	27		

Table I-5: Wrist Pin (Schaeffler HN1816).

Bearing bore dia	19.53	Bearing Width	15.81
	19.57		15.81
	19.58		15.82
	19.58		15.83
	19.54		15.82
	19.56		15.818
Bearing outside dia	24.06		
	24.05		
	24.05		
	24.04		
	24.04		
	24.048		
Length of roller	13.5		
	13.48		
	13.49		
	13.54		
	13.49		
	13.5		
Diameter of roller	2		
	2		
	2.01		
	2		
	2		
	2.002		
No. of roller	31		

Appendix J: Viscosity Input Obtained from Appendix E

Table J-1: Fluid 1 (GRP 1, ISO 46).

Temp (C)	Pressure(psi)	mu (Pa.s)		Temp	nu (m2/s)	at 3000 psi		100C and 5000Pa	
50	250	0.025551		10	267.5618				
50	500	0.025582		15	187.8195			Bulk	1.43E+09
50	750	0.025612		20	135.5102			Density	8.46E+02
50	1000	0.025641		25	1.00E+02			Sp Heat	2.10E+03
50	1250	0.02567		30	75.80382				
50	1500	0.025697		35	58.50997				
50	1750	0.025725		40	46				
50	2000	0.025751		45	36.77371				
50	2250	0.025777		50	29.84746				
50	2500	0.025802		55	24.56262				
50	2750	0.025827		60	20.46947				
50	3000	0.025851		65	17.25539				
50	3250	0.025874		70	14.69941				
50	3500	0.025897		75	12.64285				
50	3750	0.025919		80	10.97013				
50	4000	0.025941		85	9.595937				
50	4250	0.025963		90	8.456467				
50	4500	0.025984		95	7.503459				
50	4750	0.026004		100	6.7				
50	5000	0.026024							
80	250	0.009196							
80	500	0.009209							
80	750	0.009222							
80	1000	0.009235							
80	1250	0.009247							
80	1500	0.009259							
80	1750	0.00927							
80	2000	0.009281							
80	2250	0.009292							
80	2500	0.009303							
80	2750	0.009313							
80	3000	0.009323							
80	3250	0.009333							
80	3500	0.009342							
80	3750	0.009351							
80	4000	0.00936							
80	4250	0.009369							
80	4500	0.009378							
80	4750	0.009386							
80	5000	0.009394							

Table J-2: Fluid 2 (HV1).

Temp (C)	Pressure(psi)	mu (Pa.s)		Temp	nu (m2/s)	at 3000 psi		100C and 5000Pa	
50	250	0.027292		10	1.48E+02				
50	500	0.027326		15	1.18E+02			Bulk	1.37E+09
50	750	0.02736		20	94.82207			Density	8.20E+02
50	1000	0.027392		25	7.74E+01			Sp Heat	2.14E+03
50	1250	0.027424		30	63.91294				
50	1500	0.027455		35	53.36019				
50	1750	0.027486		40	45				
50	2000	0.027515		45	38.30413				
50	2250	0.027544		50	32.88647				
50	2500	0.027572		55	28.46125				
50	2750	0.0276		60	24.81452				
50	3000	0.027626		65	21.78435				
50	3250	0.027652		70	19.24691				
50	3500	0.027678		75	17.1066				
50	3750	0.027703		80	15.28893				
50	4000	0.027728		85	13.73534				
50	4250	0.027751		90	12.39947				
50	4500	0.027775		95	11.24428				
50	4750	0.027798		100	10.24				
50	5000	0.02782							
80	250	0.01241							
80	500	0.012429							
80	750	0.012447							
80	1000	0.012465							
80	1250	0.012482							
80	1500	0.012499							
80	1750	0.012515							
80	2000	0.012531							
80	2250	0.012547							
80	2500	0.012561							
80	2750	0.012576							
80	3000	0.01259							
80	3250	0.012604							
80	3500	0.012617							
80	3750	0.01263							
80	4000	0.012643							
80	4250	0.012655							
80	4500	0.012667							
80	4750	0.012679							
80	5000	0.012691							

Table J-3: Fluid 3 (TMP).

Temp (C)	Pressure(psi)	mu (Pa.s)		Temp	nu (m2/s)	at 3000 psi		100C and 5000Pa	
50	250	0.030294		10	1.78E+02				
50	500	0.030329		15	1.38E+02			Bulk	1.47E+09
50	750	0.030362		20	1.08E+02			Density	8.86E+02
50	1000	0.030395		25	8.64E+01			Sp Heat	2.05E+03
50	1250	0.030428		30	69.92693				
50	1500	0.030459		35	57.29443				
50	1750	0.03049		40	47.5				
50	2000	0.030521		45	39.80933				
50	2250	0.03055		50	33.69921				
50	2500	0.030579		55	28.79153				
50	2750	0.030608		60	24.80942				
50	3000	0.030636		65	21.5476				
50	3250	0.030663		70	18.85209				
50	3500	0.03069		75	16.60612				
50	3750	0.030716		80	14.72024				
50	4000	0.030742		85	13.12524				
50	4250	0.030767		90	11.76709				
50	4500	0.030792		95	10.60327				
50	4750	0.030816		100	9.6				
50	5000	0.03084							
80	250	0.012939							
80	500	0.012957							
80	750	0.012974							
80	1000	0.012991							
80	1250	0.013008							
80	1500	0.013024							
80	1750	0.013039							
80	2000	0.013055							
80	2250	0.01307							
80	2500	0.013084							
80	2750	0.013098							
80	3000	0.013112							
80	3250	0.013125							
80	3500	0.013139							
80	3750	0.013152							
80	4000	0.013164							
80	4250	0.013177							
80	4500	0.013189							
80	4750	0.0132							
80	5000	0.013212							

Table J-4: Fluid 4 (PAO).

Temp (C)	Pressure(psi)	mu (Pa.s)		Temp	nu (m2/s)	at 3000 psi		100C and 5000Pa	
50	250	0.010276		10	61.06972				
50	500	0.01029		15	47.36437			Bulk	1.31E+09
50	750	0.010303		20	37.42468			Density	8.24E+02
50	1000	0.010316		25	3.01E+01			Sp Heat	2.13E+03
50	1250	0.010329		30	24.53501				
50	1500	0.010341		35	20.29539				
50	1750	0.010353		40	17				
50	2000	0.010365		45	14.40285				
50	2250	0.010376		50	12.32978				
50	2500	0.010387		55	10.65555				
50	2750	0.010398		60	9.288747				
50	3000	0.010409		65	8.161735				
50	3250	0.010419		70	7.223839				
50	3500	0.010429		75	6.43663				
50	3750	0.010439		80	5.770644				
50	4000	0.010449		85	5.203055				
50	4250	0.010458		90	4.716005				
50	4500	0.010468		95	4.295396				
50	4750	0.010477		100	3.93				
50	5000	0.010485							
80	250	0.004704							
80	500	0.004712							
80	750	0.004719							
80	1000	0.004726							
80	1250	0.004733							
80	1500	0.00474							
80	1750	0.004746							
80	2000	0.004753							
80	2250	0.004759							
80	2500	0.004765							
80	2750	0.00477							
80	3000	0.004776							
80	3250	0.004781							
80	3500	0.004787							
80	3750	0.004792							
80	4000	0.004797							
80	4250	0.004802							
80	4500	0.004807							
80	4750	0.004811							
80	5000	0.004816							

Engineering

Capstone Report Approval Form

Master of Science in Engineering – MSE

Milwaukee School of Engineering

This capstone report, titled "Tribological Models for Bearing and Cam Interfaces in a Variable Displacement Linkage Motor (VDLM)," submitted by the student Jordan Saikia, has been approved by the following committee:

Faculty Co-Advisor: ______________________________ Date:___________________.

Mr. Paul Michael, B.S., M.B.A 01/28/2020

Faculty Co-Advisor: ______________________________ Date:___________________.

Dr. Subha Kumpaty, Ph.D. 01/28/2020

Faculty Co-Advisor: ______________________________ Date:___________________.

Professor Gary Shimek, M.L.I.S. 01/28/2020