Metrology Evaluation and Calibration Tool Kit for Rapid Tooling Processes

by

Vito R. Gervasi

A Report Submitted to the Faculty of the
Milwaukee School of Engineering
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

Milwaukee, Wisconsin

April, 2003

Abstract

Many rapid tooling processes are currently under development and in use for applications such as injection molding and die-casting. To evaluate and improve rapid tooling processes, several benchmarking geometries and software programs have been developed but none of these lend themselves to the evaluation and improvement of the Prototype Hard And Soft Tooling (PHAST) process. This project provides a new benchmark geometry, designed specifically for indirect rapid tooling processes such as PHAST. Also, a mathematical procedure to accompany the geometry has been developed.

The new benchmark geometry and mathematical procedure were applied to the PHAST indirect rapid tooling process. The goal was to prove that the mean and standard deviation of shrinkage could be estimated from the mean and standard deviation of shrinkage of individual process steps. By establishing a procedure where the mean and standard deviation of shrinkage for an entire process are calculated, based upon a number of independent steps, one can make process changes without reevaluating the entire process. Another use of data gathered from this procedure was to identify those steps in greatest need of improvement as well as prediction of shrinkage. Using the mathematical procedure and 2D calibration geometry developed in this project, the mean and standard deviation of shrinkage were successfully estimated. A plot was generated to identify which process steps are in greatest need of refinement. Another plot was generated to predict process shrinkage as a function of feature size.

Acknowledgments

The Author gratefully acknowledges the following organizations and individuals for their support of this project:

- Dr. Gottfried Hoffmann, advisor, for his direction and knowledge on this project.
- Tom Niemiec of Imperial Tool and Plastic for their donation of precision machined master patterns.
- Professor Larry Korta, Dr. Jay Warner and Dr. Ron Jorgensen for their reinforcement and suggestions on statistics questions.
- Dickten & Masch Mfg. Co. for access and training on the optical comparator.
- The Rapid Prototyping Center (RPC) for the production of benchmark geometries on the stereolithography apparatus.
- ME-TC Student Jay Bollman for his assistance and ideas during various stages of this project.
- MSOE Library Personnel, especially Anne Mosgaller and Gary Shimek for their assistance in tracking down publications and advising on documentation methods.

Table of Contents

List of Figures

List of Tables

Nomenclature

F	calculated F-distribution value
F_s	starting feature size
F_f	final feature size
i	ordered sample number
n, n_1, n_2	number of samples, set n, n_1, n_2
p	probability
$\overline{P}_{overall-shrinkage}$	relative shrinkage, overall process
P_s	relative shrinkage
$\overline{P}_s$	average relative shrinkage (process step)
$\overline{P}_{s_1}$	shrinkage (metal to mold-master)
$\overline{P}_{s_2}$	shrinkage (mold-master to ceramic)
$\overline{P}_{s_3}$	shrinkage (ceramic to fired ceramic)
$\overline{P}_{s_4}$	shrinkage (fired ceramic to MMC)
S	standard deviation
S_1	standard deviation (sample set 1)
S_2	standard deviation (sample set 2)
S_A	larger of two standard deviations
S_B	smaller of two standard deviations
S_o	standard deviation, overall process
S_p	standard deviation (process step)
S_{P_1}	standard deviation (metal to mold-master)
S_{P_2}	standard deviation (mold-master to ceramic)
S_{P_3}	standard deviation (ceramic to fired ceramic)
S_{P_4}	standard deviation (fired ceramic to MMC)
$S_{P_{Est.}}$	estimated shrinkage, overall process
$SD_{P_{est.}}$	estimated process standard deviation
t	student "t" test value
$\overline{X}$	average feature size
X	feature size
$\overline{X}_1$	mean value(sample set 1)
$\overline{X}_2$	mean value(sample set 2)
z	value used for normal distribution function
α	area under the upper critical value of the Student "t" test curve
v_1	degree of freedom (n_1-1) sample set 1
v_2	degree of freedom (n_2-1) sample set 2

Glossary

Cavity	The cavity features of a mold are those features which are recessed.
ChristmasTrees	Calibration geometry used for setting shrinkage compensation and laser diameter compensation for the stereolithography process.
Core	The core features of a mold are those features which protrude from the surface.
Cure-through	Undesirable addition to down-facing layers due to laser energy traveling through solid portions of a part in the stereolithography process.
Direct Rapid Tooling	Mold-making process whereby the bold geometry is additively created, layer by layer.
High Speed Machining	Computer controlled machining process which employs small cutters with very high cutting speeds.
Indirect Rapid Tooling	Pattern based mold making process consisting of at least one transfer.
KelTool	An indirect rapid tooling process which starts with a pattern of the mold and ends with a metal matrix composite mold material.
laser diameter	Laser based solid freeform fabrication processes typically compensate for laser beam diameter by moving the path of the laser to the inside of the CAD geometry for better accuracy.
layer thickness	Spacing between build layers for additive manufacturing processes.
RSP	An indirect rapid tooling process which starts with a pattern of the part and ends with a spray metal coating applied to an expendable ceramic mold.
Shut-off	A region on a mold which kisses the other side to form a through hole in the molded part.
SLS LaserForm	A material used to produce metal matrix composite inserts for injection molding via selective laser sintering, a direct rapid tooling process.
Surface Adjustments	Adjustments which take place at the surface of a casting due to surface tension of molded materials as well as interactions with the atmosphere, among other contributors.

Tool Offset	A standard adjustment used when machining to compensate for the diameter of the cutting tool.
Windowpanes	A stereolithography calibration tool used to optimize build parameters for fastest build speed.

1 Introduction

1.1 Purpose

This project report describes a new benchmark geometry and mathematical procedure to evaluate and improve indirect Rapid Tooling (RT) processes. The indirect-RT process chosen for this project was the Prototype Hard And Soft Tooling (PHAST) [1] process. The results of this research provide a useful mathematical model of the mean and standard deviation of shrinkage for the overall PHAST process based on individual process steps. The approach used enables the user of the process to change any one step of the process without the need to reevaluate the entire process. So long as the mean and standard deviation of shrinkage for the step of interest is determined, the resulting numbers can be used to accurately recalculate the mean and standard deviation of shrinkage for the overall process. In addition to this beneficial outcome, plots produced, with the mean and standard deviation of shrinkage shown, help to identify those process steps in greater need of improvement. Prediction charts are also produced using the same data, providing critical shrinkage estimates as a function of feature size.

This report provides an explanation of the concept of rapid tooling; the need for an evaluation method for indirect-RT; the benchmark design employed to evaluate the chosen indirect-RT process; the procedure used to prepare and measure samples; the mathematical procedures adopted; results of their application to the PHAST process; and discussion of results, and finally, conclusions.

This research aims to provide a useful model of the PHAST process to further the use of PHAST technology by industry and improve the ability to specify capabilities and identify and measure process improvements.

The development of this benchmark geometry and mathematical procedure was initiated due to the lack of industrial standard procedures for evaluating, and more specifically, improving indirect-RT processes.

1.2 Scope

As indirect-RT processes and materials arise and evolve, the metrological evaluation of these new combinations is important *if the greatest benefit and minimal scrap is to be realized.* The main objective of this research is to show that the mean and standard deviation of shrinkage for the PHAST process can be closely estimated by applying a mathematical procedure to the combined process steps. By establishing an accurate estimation method, one can focus on any one process step, improve it in some way, reevaluate the process step, and finally, reinsert the new mean and standard deviation of shrinkage into the overall process model. This approach eliminates the requirement to look at all process steps when only one step has been modified.

A second objective is to determine variance for each process step for process improvement purposes. The third objective is to provide critical shrinkage prediction plots as a function of feature size. A final underlying objective of this research is to reduce the cost of evaluating indirect-RT processes by using a simplified, low cost benchmark geometry, shown in section 2 of this report.

For the purpose of simplicity, this study focused on the mean and standard deviation of relative shrinkage. Factors such as surface roughness and pattern stair-stepping are not taken into account and assumed to be negligible. Table 1 outlines the influencing factors for each step of the PHAST process. As shown, shrinkage is an influencing factor for each process step. Laser compensation or tool offset errors as well as stair-stepping or machining marks are strictly pattern-based sources of error and can be separated from the evaluation of the mold master, ceramic, and PHAST Metal Matrix Composite (MMC) processing steps. Polishing of patterns and final MMC can also be separated. Fine-tuning of stereolithography and CNC machining is well established and is not part of this analysis.

Table 1. Influencing factors for the PHAST Process.

Influencing Factors	PHAST Process Steps			
Resulting Error of Process step:	Master Pattern	Mold Master	Ceramic	MMC
Shrinkage	✔	✔	✔	✔
Laser Compensation or tool offset	✔			
Stair Stepping Machining marks	✔			
Surface Roughness	✔		✔	✔
Polishing Losses	✔			✔

For this study, error due to polishing was ignored since no polishing was performed. Typically, when the master pattern is polished to remove evidence of build method (stair-stepping or machine marks) the resulting pattern is larger or smaller for cavities and cores, respectively. This may be strongly influenced by the individual polishing [2], and will vary from person to person.

When the MMC is polished to achieve a specific surface roughness, material is again removed. This material is removed in the opposite direction when compared to the initial master pattern (for the PHAST process). Again, to minimize human error, polishing was not carried out in this study. Surface roughness of the initial master pattern (shown in figure 1) was 15 micro-inches (0.38 microns), much smoother than an SFF pattern (typically 80-250 micro-inches, 2-6 Microns). A finer finish was employed to minimize the error due to surface roughness on the initial step of the process. This fine surface finish was used so that pattern surface roughness, transferred from the master pattern, would not become noise in later process steps. Parameters pertaining to the use of tooling are well established and published for numerous molded materials; therefore, the evaluation of the injection molding process was not included in this study

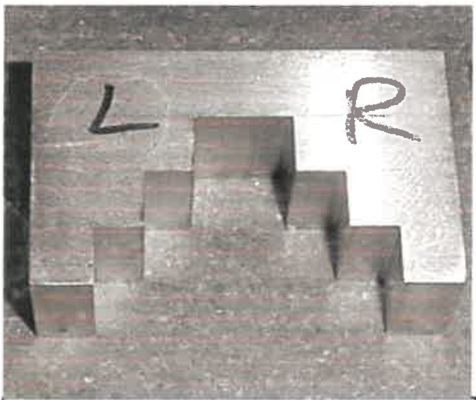


Figure 1. 2D Pyramid Geometry.

1.2.1 Relevant RP Publications, Practice and Research

The following paragraphs provide an overview of information related to this project that has either been published or provided by users and experts in the RP and RT industry.

An early publication by 3D systems describes their stereolithography calibration tools, Windowpanes™ and ChristmasTrees™ [3]. The ChristmasTrees™ calibration tool is related to this work but lacked sufficient published detail to be of great assistance. Due to the competitive nature of the RP industry, machine manufacturers -such as 3D

Systems, manufacturers of stereolithography- are reluctant to share their proprietary software.

A number of RP benchmarking parts have been proposed and studies have been presented which focus on amplifying warpage [4], detecting cure-through [5], determining thermal gradient impact on dimensions [6], and providing overall comparisons of RP process capabilities. Chrysler [7] published a study comparing the top RP processes in 1993, using a speedometer adapter-comparison part. A publication by Paul Jacobs was helpful in identifying a source of noise common with one RP process [8]. Jacobs suggests the presence of noise in all processes involving a phase change, calling it a "random noise shrinkage constant."

The majority of publications related to accuracy or calibration focus on the introduction of new material or processing parameters to users. The publications led to a number of phone and face-to-face interviews with experts in the industry. Overall, RP-related publications were not directly helpful but did provide much useful information on sources of error and benchmark design approaches.

Over the past decade a number of papers have been published related to the accuracy capability of rapid tooling processes. Unfortunately, little has been published or shared on the details of how these RT processes are evaluated and improved. The author spoke with experts on RSP [9] and KelTool [10], but without resulting in significant leads to benchmarking methods or parts. There was a common thread in all experts contacted in that they would like to have access to a benchmarking method if one is found or developed. In some cases, it is suspected that published accuracy claims are estimates, based on insufficient statistical investigations. Ideally, these claims should be based upon industry standard evaluation procedures, but nothing of the kind has been established for the RT community. Some RT users bypass the need to know accuracy capabilities of a specific RT process by creating a "steel-safe" tool. With a steel-safe condition, in critical regions, the tool is later brought into specification by conventional material removal methods. The lack of standard evaluation and improvement procedures makes the assessment of RT processes challenging and places a higher risk onto the user. This lack of standards also slows RT development and acceptance by users, and in some cases, damages the reputation of the indirect-RT providers.

1.2.2 Relevant Metal Casting Publications, Practice and Research

In parallel with seeking RP and RT-related information, publications in the metal casting industry were sought to find a test procedure that could be adopted for this project. Methods for evaluating metal casting processes and alloys for the foundry industry range from computer animations to simple cast-it and measure-it approaches. One interesting approach links a micrometer to an object floating in a metal casting to observe volumetric shrinkage through the entire cooling cycle [11]. This doesn't apply to the thrust of this research, though it did identify some of the critical causes of volumetric shrinkage in cast metal, namely liquid-metal thermal contraction, mold dilation effects, and actual solidification shrinkage or expansion. After speaking to several foundrymen and the advisor on this project, it was learned that the simplest method used to determine shrinkage is to cast a sample using a pattern of known dimensions. The shrinkage can be determined from measuring the change in dimension. From a conversation with Leonard Ceriotti [12], Wisconsin Precision Casting, the author learned there have been stepped objects used to determine shrinkage by the investment casting industry. Ceriotti also mentioned research aimed at the prediction of how metal solidifies around a mold core feature. Much is published on the prediction of porosity in castings and the computer modeling and simulation of casting processes, but the one standard procedure that would lend itself to indirect-RT was not found [13,14,15].

1.3 Background

In an effort to reduce lead times of conventional tooling approaches, a number of direct and indirect Rapid Tooling (RT) processes, utilizing Solid Freeform Fabrication (SFF)[16], have been developed. Rapid Tooling has proven very effective in reducing lead times for applications such as injection molding, die-casting, and blow molding.

1.3.1 Solid Freeform Fabrication

Unlike traditional material removal fabrication techniques, such as CNC machining, SFF is an additive process, starting with a build platform, and adding material one layer at a time, to create a 3D object (figure 2). CAD data are used to define the surface and interior of the solid 3D object, making complex part geometries as well as complex internal structures possible and commonplace. As a pattern source for indirect-RT, SFF can provide objects with very complex and accurate surfaces. No CNC programming and no fixturing is required, and patterns are produced fairly quickly.

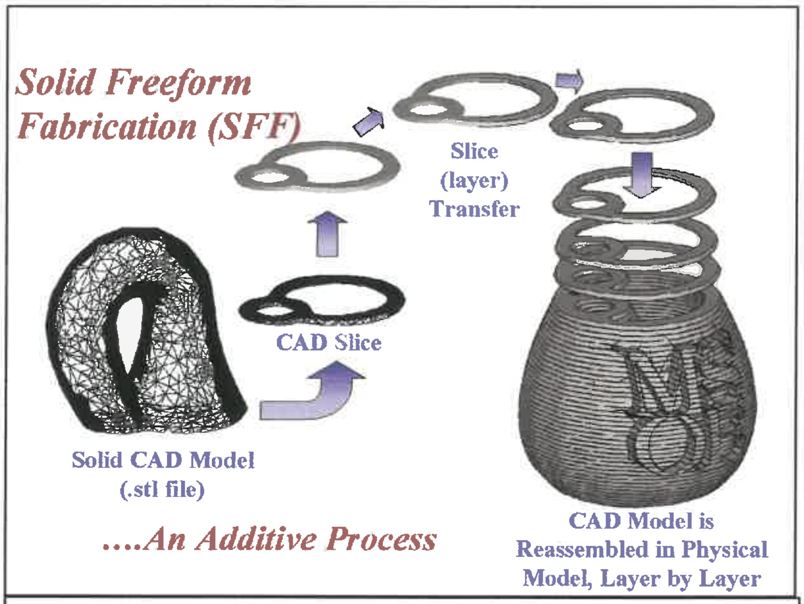


Figure 2. Solid Freeform Fabrication (SFF).

SFF is also known as additive manufacturing or Rapid Prototyping (RP). When SFF pattern sources are used, influences such as laser diameter, layer thickness, material surface roughness, surface treatments, and material shrinkage become part of accuracy losses in indirect-RT processes.

1.3.2 Rapid Tooling

Rapid tooling is a term adopted to describe those tool-making approaches that employ SFF to generate functional tools directly or indirectly. The term RT has also been used to describe tooling produced via high-speed machining, a high-end conventional tool-making approach. SFF based RT can be divided into two main groups: Direct and Indirect. In the case of direct-RT, mold geometry is additively "grown," without additional transfers, using a SFF process. The mold geometry often goes through a number of steps to improve the mechanical properties and stabilize the material, after which it is ready for use. In the case of Indirect Rapid Tooling, an SFF pattern is produced and after at least one transfer, the pattern geometry is reproduced in a durable mold material. Figure 3 illustrates the steps involved in the PHAST indirect-RT process. Starting with an SFF pattern, the process ends with a Metal Matrix Composite (MMC) mold insert which can be used for injection molding or die-casting. There are three transfers required: 1) from the pattern to the mold master, 2) from the mold master to the ceramic, and 3) from the ceramic to the metal matrix composite. Also shown in figure 3 is the transition from unfired ceramic to fired ceramic, for a total of four process steps.

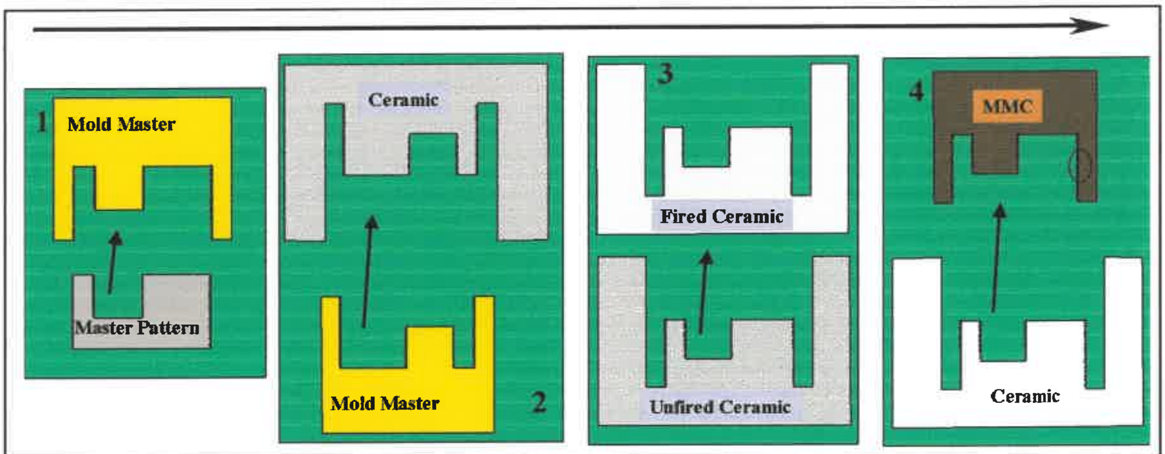


Figure 3. PHAST Process Steps.

This research is primarily interested in indirect, pattern-based, RT processes. Patterns for indirect-RT processes are typically produced via SFF. Other pattern sources include objects that are hand sculpted, CNC machined, previously manufactured, or naturally occurring.

Indirect-RT is used for prototyping or production of molded parts in materials ranging from wax to die-cast aluminum. Thermal plastics are the most common material processed in indirect-RT. Processes such as KelTool, PHAST, and RSP can be used for higher temperature molding, and in some cases, die-casting.

A number of indirect-RT processes have been developed over the years, some before SFF pattern sources were available. Following is a list of seven indirect RT processes:

- Silicone Molding [17] (prototyping, single transfer step, polyuerethanes);
- KelTool [18] (production, 2-3 transfer steps, thermal plastics);
- Spray Metal [19] (prototypes to bridge tooling, single transfer step, thermal plastics);
- Composite Tooling [20] (prototype to bridge tooling, 1-2 transfer steps, thermal plastics);
- Electro-plating [21] (prototype to production tooling, 1 transfer step, thermal plastics);
- PHAST [1] (prototype to production tooling, 3 transfer steps, thermal plastics);
- Rapid Solidification Tooling [22] (RSP, prototype to production tooling, 3 transfer steps, thermal plastics).

Each indirect-RT process has at least one transfer step and some have thermal processing or curing, all of which lead to slight changes in the dimensions of features and overall accuracy. To compensate for changes, such as shrinkage and surface roughness, pattern geometry is scaled up in the CAD design and material is sometimes added to critical regions for later "fine-tuning."

1.3.3 Rapid Tooling Evaluation and Improvement Practices

Currently, indirect-RT developers and users employ a few standard geometries in combination with their own benchmark designs to evaluate and improve their process. Standardization is limited and many RT users make adjustments on the fly or make molding tools, which are "steel-safe" ("designed-in" machine stock). Several material and process characteristics, including warpage, shrinkage, surface adjustments, and resolution are key to the proper evaluation of processes and material capabilities.

Several SFF and RT calibration and evaluation geometries have been proposed and are in use, including the following:

- The 3D Systems "Christmas Tree" [3] was one of the first calibration geometries provided to the RP community. The geometry is used to calibrate the Stereolithography (SLA) process shrinkage and beam compensation. The geometry is not designed for RT evaluation but is an excellent means to calibrate the SLA capabilities.

- The SLA Users Group Eastman-Kodak Benchmark Part [23,24] was proposed by Douglas Van Putte, Eastman Kodak, in 1997. The geometry, shown in figure 4, is designed to provide several basic mold-making challenges, including several rib geometries, a shut-off, and an uneven parting surface. Several automated CMM programs were developed and shared to evaluate the geometry. The main drawback to this geometry is that it is expensive to process and doesn't provide simple mean and standard deviation of shrinkage information. Also, this looks specifically at the part produced using a tool while the goal is to look specifically at the tool-making process.

Figure 4. Kodak Benchmark.

- A newer calibration geometry [25] was recently adopted for the SLS LaserForm material, shown in figure 5. This geometry is built and measured and results are entered into a proprietary software program. The output is shrinkage and line-width compensation. The main drawbacks of this geometry are that it is designed for direct-RT and the owner is not sharing the mathematical procedure, rather selling the evaluation tool as an encrypted software package.

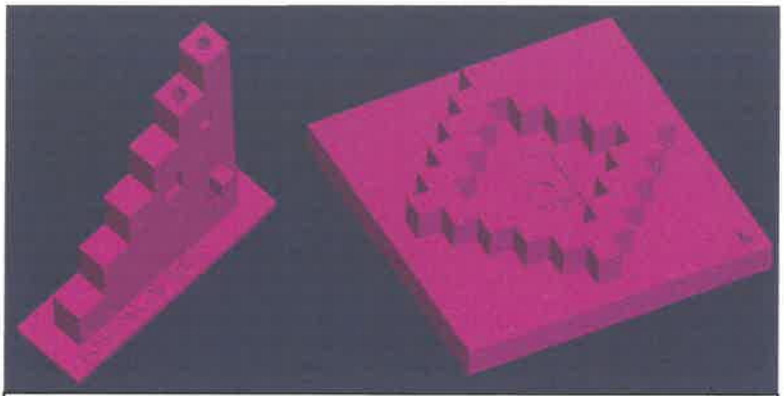


Figure 5. SLS LaserForm Calibration Geometry.

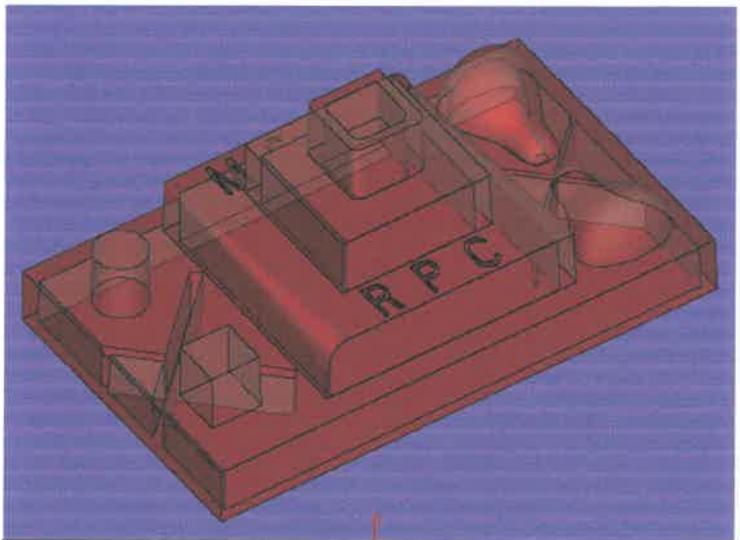


Figure 6. RPC RT Benchmark Part.

- The Rapid Tooling Committee of the Rapid Prototyping Consortium (RPC-MSOE) developed the RPC-benchmark part, shown in figure 6. This geometry provides relatively easy-to-measure features along with process-challenging features such as ribs and a shutoff. It is one step above the Eastman-Kodak geometry. The main drawback of this geometry is that it only provides process capability information rather than calibration information. Also, the drafted geometry offers challenges when measuring features with calipers.

For this project, the factors most strongly influencing the approach were simplicity and ease of measurement. By removing several factors such as surface finish, pattern production, and human variation in polishing, the evaluation becomes manageable. One factor that was included was a range of feature sizes since it was suspected that shrinkage is not uniform. Inclusion of horizontal and vertical features was suspected to be very important due to the effect of gravity, and solidification shrinkage on several process steps.

2 Benchmark Geometry

A benchmark geometry used to capture step-specific information is required and many geometries have been proposed over the years ranging from simple to complex. Within the time frame of this project, a number of geometries were conceived, two of which are described in the following paragraphs.

2.1 *Initial Benchmark Geometry*

In an effort to integrate core and cavity features, and building on the RPC benchmark part, the author developed a benchmark-geometry with a core-like and cavity-like stepped configuration. The center part shown in figure 7 illustrates this geometry. Several problems with this design were uncovered during this research project, including the

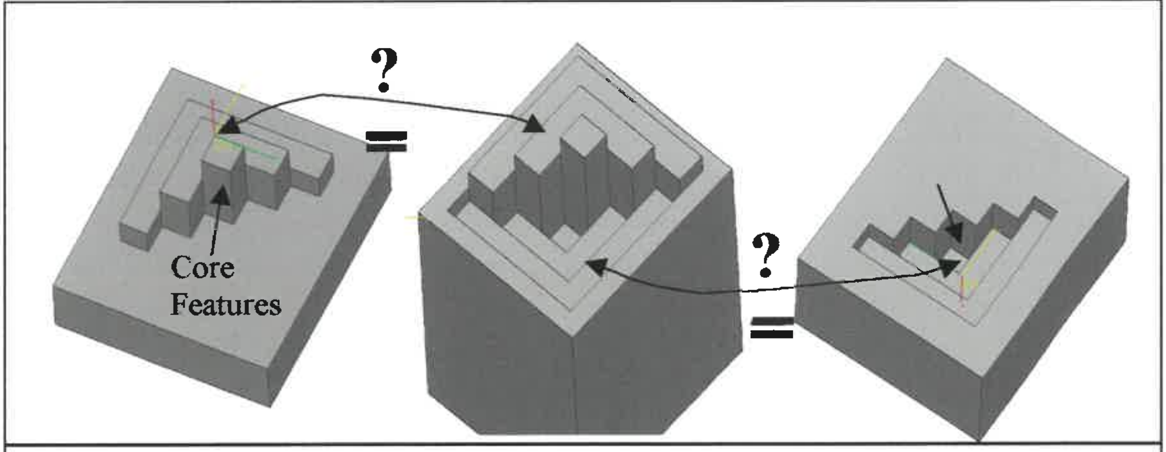


Figure 7. Pyramid Core-cavity Test Part.

undesirable interaction between core and cavity features. A study was conducted using the three geometries shown in figure 7. One geometry had core features only; one had core and cavity features; and a third had cavity features only. Assuming no interactions were present and core and cavity features were independent, final core-core dimensions and cavity-cavity dimensions on castings should have been identical. They were not and significant interactions were encountered.

Another problem with this geometry, realized soon after the core-cavity interaction was uncovered, was that this geometry provides more information than required. For cast steps of a process only one set of horizontal and vertical features are needed; this geometry provided two sets of horizontal features. For these reasons this geometry was abandoned and a new, simplified geometry was developed.

2.2 Simplified Benchmark Geometry

The revised benchmark geometry, shown in figure 8, is designed to be a simple form while still having the ability to capture critical information about shrinkage, relative to feature size and feature orientation. The form shown in figure 8 is specifically for the

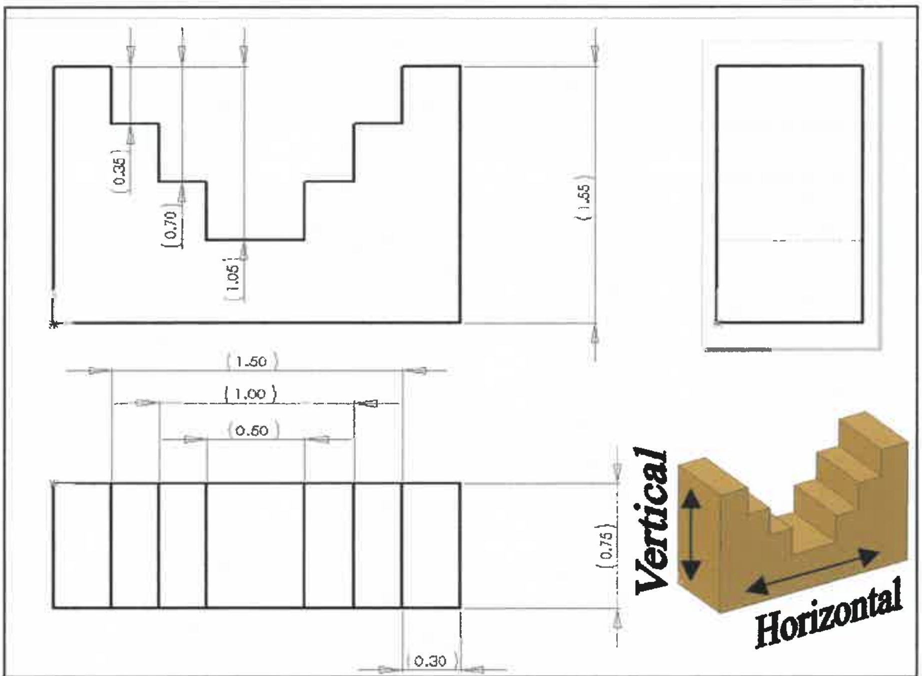


Figure 8. Benchmark Master Pattern Geometry

5.2.11 Comparison Plot

In addition to the student "t" test, we can also visually compare a plot of estimated shrinkage to measured shrinkage. Figure 10 illustrates the similarity of the estimated and measured shrinkage for vertical features; both sets of points are very close. Figure 11 illustrates the closeness of fit for horizontal feature shrinkage, estimated and measured. The slight difference between the two could be attributed to a number of factors including temperature during measurement, method of measurement, surface roughness or human error.

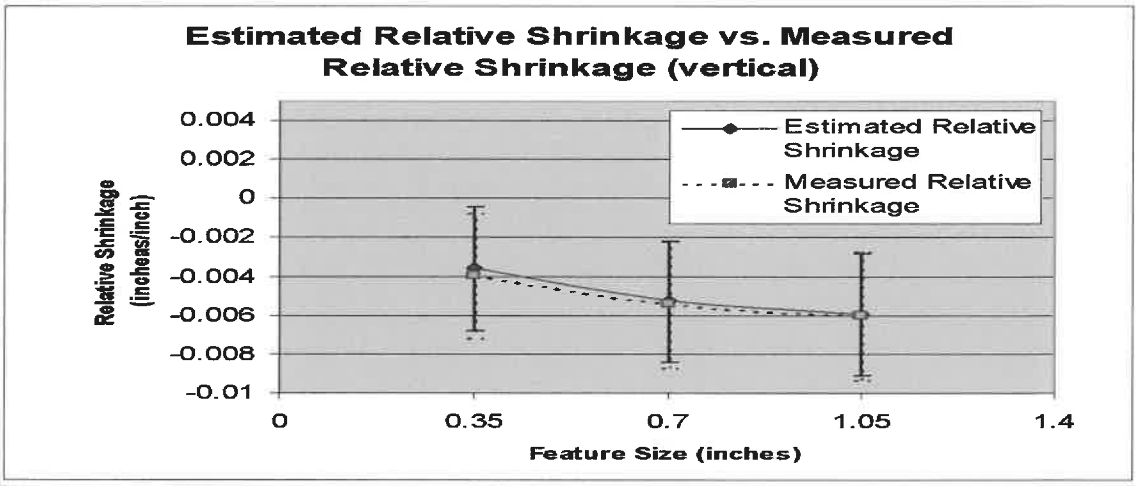


Figure 10. Comparison Plot of Vertical Shrinkage.

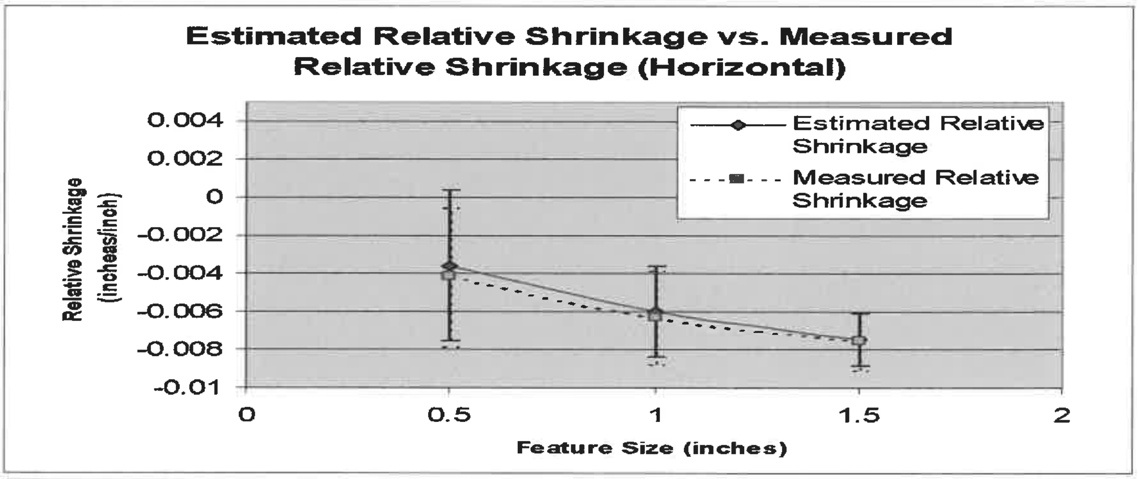


Figure 11. Comparison Plot of Horizontal Shrinkage.

evaluation of mold core features; to evaluate cavity features, the geometry is inverted. In this study three precision-machined steel benchmark patterns or "master patterns" were produced. The dimensions shown are not as critical as the accurate measurement of machined features. For this project all three master patterns were machined together so the dimensions of the features were almost identical. Each process step results in a reversal of the geometry. Master patterns are precision ground to achieve a fine surface roughness of 15 micro-inches (0.38 microns) or less. This geometry can be measured using a micrometer, calipers, optical comparator, or CMM.

3 Mathematical Approach

3.1 *Hypothesis*

Each step of the PHAST processes is independent of other process steps.

Assuming the above statement is true, each step of the process can be isolated and evaluated independently for shrinkage and standard deviation.

Furthermore, the mathematically combined mean and standard deviation of shrinkage from each process step is the same as the measured process shrinkage and standard deviation.

3.2 *Statistics*

Another body of relevant information, and perhaps most helpful, was the multitude of statistics publications and internet sites which proved to be most useful for this project. Books by Johnson [26], Wadsworth [27], Ostle [28], and Kempthorne [29] were significantly helpful. Several websites hosted by NIST [30], Rice university, and the University of Connecticut were useful starting points. Several statistical tests were employed including the "F" test, Student "t" test, and Theorem for the Standard Deviation Of the Sum of Independent Variables.

- **F Distribution Test**

  To show that the standard deviation of two sample populations are or are not from the same populations, an F distribution test was employed. The goal of the F distribution test is to verify that the standard deviations of the two sample populations are different, and therefore independent, or the same, and therefore from a common population.

- **Student "t" Test**

  The Student "t" test is used if it cannot be shown that the standard deviation of the two sample sets are from two independent populations and the sample size is small (less than 30). The Student "t" test is used to assert that two measured sample sets are significantly different, at a specific level of confidence.

- **Theorem for the Standard Deviation of the Sum of Independent Variables**

  To estimate the overall standard deviation, based on shrinkage standard deviation for each process step, we apply the theorem for the standard deviation of the sum of independent variables [31]. This method is commonly used by industry for stacked tolerance evaluations of assembled parts. Basically, if the standard deviation for each part of an assembly is summed, to determine the overall standard deviation of an assembly, this would ignore overlapping regions of the normal curves of two or more populations. This approach is very useful, and has been repeatedly proven, by experimentation, to provide an accurate standard deviation for an assembly or sequence. Relating back to this project, an assembly of steps, each with its own distribution, is being combined, adding to the overall standard deviation.

These and other more common statistical tools were used for the following mathematical procedure.

3.3 Procedure

The following mathematical procedure is applied to each horizontal and vertical feature independently. The goal and procedure of each analysis step and related equation is described below.

3.3.1 Evaluation of Raw Data Quality

The following steps are conducted on each feature size data set for all process samples. The 2D pyramid is carried through all steps as follows:

- 3 precision steel master patterns;
- 15 mold masters;
- 15 ceramics;
- 15 fired ceramics;
- 15 metal matrix composites.

Each sample consists of 6 feature sizes, 3 horizontal and 3 vertical.

Step 1. Before treating sample data as part of a normal population we must first inspect the data to see how closely it resembles a normal distribution. To inspect for normality of sample data a uniform probability plot for 15 samples is prepared using the following equation and procedure:

$$p = \frac{3i-1}{3n+1}, \qquad (1)$$

where p=probability (percent);

i =ordered sample number;

n=number of samples.

- Data is ordered smallest to largest, m1 to m15;
- Plot ith largest observation , versus the ith probability (p), for all i (table 2);
- Check for linearity.

Table 2. Uniform Probability	
mi	**(p)**
m1	4.3%
m2	10.9%
m3	17.4%
m4	23.9%
m5	30.4%
m6	37.0%
m7	43.5%
m8	50.0%
m9	56.5%
m10	63.0%
m11	69.6%
m12	76.1%
m13	82.6%
m14	89.1%
m15	95.7%

Step 2. For each sample set measured, determine average feature size,

$$\bar{X} = \frac{\sum X}{n}, \qquad (2)$$

where $\bar{X}$ =*average feature size;*

X= feature size;

n=number of samples.

Step 3. For each sample set measured, determine standard deviation of that sample set,

$$S = \sqrt{\frac{\sum (X - \bar{X})^2}{n-1}}, \qquad (3)$$

where S=standard deviation;

$\bar{X}$ =*average feature size;*

X= feature size;

n=number of samples.

Step 4. To verify that each progressive sample set does not fall within the noise of its predecessor, an F distribution test is conducted. The goal of the F distribution test is to verify that the standard deviations of the two sample sets (e.g., master pattern versus mold-master, unfired ceramic versus fired ceramic, etc.) are different and therefore independent. To accomplish this the following equation [28] is employed:

$$F = \frac{S_A^2}{S_B^2}, \qquad (4)$$

where F *=calculated F-distribution value;*

S_A*=Larger of two standard deviations;*

S_B*=Smaller of two standard deviations.*

From the table in Appendix C, for a sample size of 3 by 15, the F value is 3.29 (0.05 level of significance). For a sample size of 15 by 15 the F value is 2.40 (0.05 level of significance). Now we establish our hypothesis:

Null Hypothesis: S_A=S_B (for a 3x15 sample calculated F must is less than 3.29);

Alternative Hypothesis: $S_A \neq S_B$ (for 3x15 sample calculated F is larger than 3.29).

Ideally, the standard deviations will differ sufficiently to reject the null hypothesis outright so we can be confident that the data are from two separate populations. If the null hypothesis is true, we cannot say that these samples are significantly different and further analysis is required.

Step 5. If the null hypothesis is true in step 4, the next step towards asserting that these measured samples are significantly different, with a level of confidence, is the Student "t" test. The student "t" test is a statistical tool that can provide rejection foundation of a claim based on the probability of the outcome occurring. For example, if two means are compared, each with comparable standard deviations (passed the F test), the probability is less than 5% of an average falling within the area of the outermost tail regions of a student t distribution (a student "t" test distribution is very similar to a normal distribution in form). If a calculated value of t falls in the very small tail region, the claim could be rejected. In other words the two averages are significantly different. Following is the equation [26,27, 28] used to calculate t:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(v_1)S_1^2 + (v_2)S_2^2}{n_1 + n_2 - 2} \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \qquad (5)$$

where *t=Student "t" test;*

$\bar{X}_1$ *=mean value(sample set 1);*

$\bar{X}_2$ *=mean value(sample set 2);*

v_1*=degrees of freedom (*n_1*-1) sample set one;*

v_2*=degrese of freedom (*n_2*-1) sample set two;*

n1=number of samples in sample set 1

n2= number of samples in sample set 2

S_1*= standard deviation (sample set 1);*

S_2*= standard deviation (sample set 2).*

From Appendix C, Student "t" test Distribution table, for $\nu = n_1 + n_2 - 2 = 15+15-2 = 28$ we get a value of 2.048. We can now establish the following hypothesis:

- null hypothesis $\overline{X}_1 = \overline{X}_2$, $-2.048 \leq t \leq 2.048$;
- alternative hypothesis $\overline{X}_1 \neq \overline{X}_2$ $t>2.048$, $t<-2.048$;
- level of confidence = 95% (level of significance $\alpha/2=0.025$).

This is a two-tailed test: if "t" falls outside of ±2.048, the null hypothesis is rejected and the alternative is accepted. Basically, if the null is rejected, it is due to the low probability, less than 5%, of the calculated t value falling within those small areas of the tips of the tails. If the null hypothesis holds true, we cannot, with a significant level of confidence, claim that the sample set does not fall within the noise of its predecessor.

3.3.2 Mean and Standard Deviation of Shrinkage, and Normality for each Process Step

Now that we are comfortable with our data, we need to extract the mean and standard deviation of shrinkage occurring between each process step. For this portion of the analysis we look specifically at four process steps, for all six feature sizes, as follows:

- master pattern→mold master;
- mold master →unfired ceramic;
- unfired ceramic→fired ceramic;
- fired ceramic→metal matrix composite.

To determine mean and standard deviation of shrinkage, and normality, the following steps and equations are used:

Step 1. Calculate relative shrinkage by comparing initial feature size to final feature size within each process step. The following equation is employed.

$$P_s = \frac{F_s - F_f}{F_s}, \tag{6}$$

where P_s *=relative shrinkage;*

F_s*=starting feature size;*

F_f*=final feature size.*

Step 2. Use the normal scores plot procedure from step 1, of 3.3.1 above, to verify that shrinkage follows a normal distribution for each process step and feature size.

Step 3. Given that the shrinkage from step 2 is from a normal population, calculate relative shrinkage for each feature size and each of four process steps. Average feature shrinkage is

$$\overline{P}_s = \left(\frac{\sum P_s}{n}\right), \tag{7}$$

where P_s *=relative shrinkage;*

$\overline{P}_s$ *=average relative shrinkage (process step).*

This calculation provides average shrinkage for each feature size of each process step.

Step 4. Calculate standard deviation of average feature shrinkage for each feature size and for each of four process steps. Standard deviation is determined by

$$S_p = \sqrt{\frac{\sum (P_s - \overline{P}_s)^2}{n-1}}, \tag{8}$$

where P_s *=relative shrinkage;*

$\overline{P}_s$ *=average relative shrinkage (process step);*

S_p *=standard deviation (process step).*

3.3.3 Estimated Mean and Standard Deviation of Shrinkage for Overall Process

Based on the mean and standard deviation of relative shrinkage for each process step, simple calculations can be used to estimate "overall" process shrinkage and standard deviation.

Step 1. Calculate estimated process shrinkage (by feature size) by summing the shrinkage for each process step. Assuming shrinkage is less than one percent for each process step, estimated process shrinkage is calculated as follows:

$$S_{P_{Est.}} = \overline{P}_{s_1} + \overline{P}_{s_2} + \overline{P}_{s_3} + \overline{P}_{s_4}, \tag{9a}$$

where $S_{P_{Est.}}$ *=estimated shrinkage, overall process;*

$\overline{P}_{s_1}$ *=shrinkage (metal to mold-master);*

$\overline{P}_{s_2}$ *=shrinkage (mold-master to ceramic);*

$\overline{P}_{s_3}$ *=shrinkage (ceramic to fired ceramic);*

$\overline{P}_{s_4}$ *=shrinkage (fired ceramic to MMC).*

If shrinkage exceeds one percent the following equation can be used in place of (9a):

$$S_{P_{Est.}} = 1-((1+\overline{P}_{s_1})\cdot(1+\overline{P}_{s_2})\cdot(1+\overline{P}_{s_3})\cdot(1+\overline{P}_{s_4})), \quad (9b)$$

where $\overline{P}_{s_1}$ *=shrinkage (metal to mold-master);*

$\overline{P}_{s_2}$ *=shrinkage (mold-master to ceramic);*

$\overline{P}_{s_3}$ *=shrinkage (ceramic to fired ceramic);*

$\overline{P}_{s_4}$ *=shrinkage (fired ceramic to MMC).*

Step 2. Now to estimate the overall standard deviation, based on shrinkage standard deviation for each process step, we apply the Theorem For The Standard Deviation Of The Sum Of Independent Variables [31]:

$$SD_{P_{est.}} = \sqrt{S_{P_1}^2 + S_{P_2}^2 + S_{P_3}^2 + S_{P_4}^2}\ , \quad (10)$$

where $SD_{P_{est.}}$ *=estimated process standard deviation;*

S_{P_1} *=standard deviation (metal to mold-master);*

S_{P_2} *= standard deviation (mold-master to ceramic);*

S_{P_3} *= standard deviation (ceramic to fired ceramic);*

S_{P_4} *= standard deviation (fired ceramic to MMC).*

This method is commonly used by industry for stacked tolerance evaluations of assembled parts. This approach is very useful, and has been proven by experimentation to provide an accurate standard deviation for an assembly or sequence. A less accurate approach to determine the overall standard deviation of an assembly is to collect the standard deviation for each part of an assembly and sum them. This approach would ignore overlapping regions of the normal curves of two or more populations. A more accurate approach is the Theorem For The Standard Deviation Of The Sum Of Independent Variables, which takes into account the overlapping regions of the normal populations of each part of the assembly. Relating back to this project, an assembly of steps, each with its own distribution, is being combined, adding to the overall standard deviation.

3.3.4 Mean and Standard Deviation of Shrinkage, and Normality for Overall Process

The objective of this portion of the analysis is to take a broader look at the overall process shrinkage, ignoring the individual steps. The overall process mean and standard deviation of shrinkage are calculated using the steps described in 3.3.2. Measured results from master patterns and final metal matrix composites are used for each feature size:

- Master-pattern → metal matrix composite,

where $\overline{P}_{overall-shrinkage}$ = overall process shrinkage,

and S_o = overall standard deviation.

3.3.5 Student "t" Test

Step 1. Before using the Student "t" test we must verify that the standard deviations are not significantly different, again using the F-test for each feature size:

$$F = \frac{S^2_{P_{overall}}}{SD^2_{P_{est.}}}, \qquad (11)$$

where F *=calculated F-distribution value;*

$S_{P_{Est.}}$ *=estimated shrinkage, overall process;*

$SD_{P_{est.}}$ =*estimated process standard deviation.*

Step 2. Now the student "t" test is applied to compare estimated shrinkage to measured shrinkage:

$$t = \frac{S_{P_{Est.}} - \overline{P}_{S_{overall}}}{\sqrt{\dfrac{(n_1 - 1)SD_{P_{est.}}^{\;2} + (n_2 - 1)S_o^2}{n_1 + n_2 - 2} \cdot \left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} , \tag{12}$$

where t=*Student "t" test;*

$S_{P_{Est.}}$ =*estimated shrinkage, overall process;*

$\overline{P}_{overall-shrinkage}$ = *relative shrinkage, overall process;*

$SD_{P_{est.}}$ =*estimated process standard deviation;*

S_o = *standard deviation, overall process;*

$n_1 = n_2 = 15$.

From Appendix C, Student t Distribution table, for $v = n_1 + n_2 - 2 = 15+15-2 = 28$ we get a value of 2.048. We can now establish the following hypothesis:

- null hypothesis $S_{P_{Est.}} = \overline{P}_{overall-shrinkage}$, $-2.048 \le t \le 2.048$;
- alternative hypothesis $S_{P_{Est.}} \neq \overline{P}_{overall-shrinkage}$ $t > 2.048$, $t < -2.048$;

and level of confidence = 95% (level of significance $\alpha/2 = 0.025$).

4 Sample Preparation and Measurement

The following steps were used to prepare and carry out the production of sample benchmark parts for the PHAST process.

Step 0. CAD to Machined Master Patterns.

a. Produce three master patterns of core pyramid with 15 micro-inch (0.38 microns) surface roughness and high degree of parallelism (figure 8).

b. Coordinate Measuring Machine (CMM) to verify and gather dimensions.

Step 1. Machined Master Patterns To Reusable Mold Masters.

a. Prepare mold boxes for mold master stage.

b. Cast 15 core mold masters.

- Constants: temperature 82F, volume, processing times and methods, and batch.
- Variables: Oven position (random).

c. 30X Optical Comparator to gather dimensions.

d. Analyze data.

Step 2. Reusable Mold Masters To Unfired Ceramic.

a. Prepare mold box for ceramic casting.

b. Using three mold-master cores to prepare 15 core ceramics.

- Constants: temperature, volume, processing times and method.
- Variables: Furnace position (random).

c. CMM to verify and gather dimensions.

d. Analyze data.

Step 3. Unfired Ceramic To Fired Ceramic.

a. Fire ceramics using current proprietary schedule.

b. CMM to verify and gather dimensions.

c. Analyze data.

Step 4. Fired Ceramic To PHAST Metal Matrix Composite.

a. Prepare mold box for MMC stage.

b. Sacrifice all 15 ceramic samples to prepare 15 core MMCs.

- Constants: temperature, volume, and processing times and methods.
- Variables: furnace position (random).

c. CMM to verify and gather dimensions.

d. Analyze data.

5 Results and Discussion

5.1 Processing

Sample processing went well with only a few reruns required. Figure 9 shows the three metal masters, three of fifteen mold masters, fifteen assembled and fired ceramics and fifteen MMC samples.

Figure 9. Metal Masters, Mold Masters, Assembled and Fired Ceramics, and Final Metal Matrix Composites (MMCs).

5.2 Data Analysis

The following paragraphs will describe the mathematical procedure used as well as discussion of results for each data analysis step.

5.2.1 Uniform Probability Plots

Using (1) uniform probability plots for measurements of each sample set and each feature size for a total of 24 plots shown in Appendix B. Upon inspection, all uniform probability plots were linear in form and the assumption of normality was made. Due to the small sample size of three, the master patterns were not plotted.

5.2.1 Average feature size and standard deviation

Mean and standard deviation of feature size were calculated using (2) and (3). Raw data, average feature size, and standard deviation of feature size are shown in Appendix A.

5.2.2 F-distribution Test

Using (4), the F test was employed to determine if the standard deviations of average feature sizes were significantly different from one stage to another. Table 3 lists the results of the F test. Those cells shaded have a significantly different standard deviation while the other cells have similar standard deviations. The cell with a value of 219.6 is relatively high due to a small but acceptable standard deviation in the numerator. The next step is to look at the student "t" test on those cells with similar standard deviations.

5.2.3 Student "t" Test

For those cells with similar standard deviations from table 3, the student "t" test procedure was applied at the 0.05 level of significance. The table in Appendix C was used to determine "t" values. Equation (5) was used and results are listed in table 4. Cells that passed the null hypothesis are shaded. Cells masked out are those with dissimilar standard deviations based on the F-test. The remaining cells failed the null hypothesis and we can state, with confidence, that the average feature measurements are dissimilar. For those cells passing the null hypothesis, we cannot state with confidence that feature sizes are significantly different when metal masters are compared to mold masters.

5.2.4 Relative shrinkage and standard deviation

The mean and standard deviation of shrinkage were calculated for each process step and feature size and results are shown in tables 5 and 6. Equations (6), (7) and (8) were used to calculate shrinkages, average shrinkage, and standard deviations (relative shrinkage results are listed in Appendix D). Uniform probability plots for shrinkages for each feature size were prepared and can be found in Appendix E. Plots of average relative shrinkages as a function of feature size are located in Appendix F.

Table 3. F-distribution results.

	Feature Size (inches)	Metal-->mold master (0.0<F<3.29)	Mold master--> unfired Ceramic (0.0<F<3.29)	unfired Ceramic --> fired Ceramic (0.0<F<2.4)	fired Ceramic --> MMC (0.0<F<2.4)
Vertical feature	0.35 inches (8.89 mm)	1.6	13.9	1.3	1.4
	0.7 inches (17.78 mm)	3.7	56.4	1.4	1.6
	1.05 inches (26.67 mm)	1.7	44.4	1.2	1.5
Horizontal feature	0.5 inches (12.7 mm)	1.2	219.6	1.2	1.2
	1.0 inches (25.4 mm)	1.1	2.8	1.2	1.3
	1.5 inches (38.1 mm)	1.4	0.6	1.5	1.4

Table 4. Student "t" test results.

	Feature Size (inches)	Metal-->mold master (-2.12<t<2.12)	Mold master--> unfired Ceramic (-2.12<t<2.12)	unfired Ceramic --> fired Ceramic (-2.048<t<2.048)	fired Ceramic --> MMC (-2.048<t<2.048)
Vertical feature	0.35 inches (8.89 mm)	1.8	[redacted]	6.6	6.6
	0.7 inches (17.78 mm)	[redacted]	[redacted]	7.4	9.9
	1.05 inches (26.67 mm)	2.3	[redacted]	6.6	10.9
Horizontal feature	0.5 inches (12.7 mm)	-0.7	[redacted]	4.9	11.7
	1.0 inches (25.4 mm)	0.2	-5.3	4.9	15.5
	1.5 inches (38.1 mm)	1.0	-6.7	8.9	23.2

5.2.5 Estimated process shrinkage

To estimate process shrinkage the sum of shrinkage for each process step is calculated using (9a). Table 7 shows the resulting shrinkage for each process step and feature size.

5.2.6 Estimated process standard deviation

To estimate process shrinkage standard deviation, the square root of the sum of all standard deviations for each feature size is calculated. Equation (10) was used to calculate the results shown in table 6.

5.2.7 Overall process shrinkage

Overall process shrinkage was calculated using (7) from the initial stage (master patterns) to final stage (MMCs) for each feature size. Results are shown in table 7 for both vertical and horizontal features.

5.2.8 Overall process standard deviation

Overall process shrinkage standard deviation was calculated using (8) for each feature size. Results are shown in table 7.

5.2.9 F-distribution Test

Before conducting the student "t" test, the F values, based on estimated and actual process standard deviations, were calculated using (11). All feature size F results

Table 5. Average Shrinkage by feature size.

	Feature Size (inches)	Metal--> mold master	mold master--> unfired ceramic	unfired ceramic--> fired ceramic	fired ceramic--> MMC
Vertical feature	0.35 inches (8.89 mm)	-0.08%	0.42%	-0.32%	-0.38%
	0.7 inches (17.78 mm)	-0.08%	0.40%	-0.32%	-0.53%
	1.05 inches (26.67 mm)	-0.07%	0.41%	-0.32%	-0.62%
Horizontal feature	0.5 inches (12.7 mm)	0.03%	0.47%	-0.13%	-0.73%
	1.0 inches (25.4 mm)	-0.01%	0.35%	-0.21%	-0.74%
	1.5 inches (38.1 mm)	-0.05%	0.29%	-0.24%	-0.74%

Table 6. Standard Deviation by feature size.

	Feature Size (inches)	Metal--> mold master	mold master--> unfired ceramic	unfired ceramic--> fired ceramic	fired ceramic--> MMC
Vertical feature	0.35 inches (8.89 mm)	0.05%	0.12%	0.03%	0.08%
	0.7 inches (17.78 mm)	0.04%	0.11%	0.03%	0.10%
	1.05 inches (26.67 mm)	0.05%	0.12%	0.02%	0.08%
Horizontal feature	0.5 inches (12.7 mm)	0.06%	0.17%	0.05%	0.05%
	1.0 inches (25.4 mm)	0.05%	0.10%	0.03%	0.04%
	1.5 inches (38.1 mm)	0.04%	0.04%	0.03%	0.03%

suggested that estimated and actual standard deviation were similar. Results are shown in table 7.

5.2.10 Student "t" Test

And finally, the student "t" test was conducted to accept or reject the Null hypothesis: *estimated shrinkage is equal to actual process shrinkage*. Equation (12) was used and all six feature size shrinkage estimates were found to be significantly similar to the actual shrinkage and the null hypothesis was accepted for all, with the "t" value falling well within +/- 2.048 span. Results are shown in table 7.

Table 7. Final Results.

	Feature Size (inches)	Estimated process shrinkage	Estimated process shrinkage standard deviation	Overall Process Shrinkage	Overall Process Shrinkage Standard Deviation	F-distribution result	Student "t" test result
Vertical feature	0.35 inches (8.89 mm)	-0.36%	0.16%	-0.39%	0.16%	1.01	**0.62**
	0.7 inches (17.78 mm)	-0.53%	0.15%	-0.54%	0.16%	1.1	**0.25**
	1.05 inches (26.67 mm)	-0.60%	0.16%	-0.60%	0.17%	1.1	**0.12**
Horizontal feature	0.5 inches (12.7 mm)	-0.36%	0.20%	-0.42%	0.18%	1.15	**0.87**
	1.0 inches (25.4 mm)	-0.60%	0.12%	-0.63%	0.12%	1.06	**0.68**
	1.5 inches (38.1 mm)	-0.74%	0.07%	-0.75%	0.08%	1.22	**0.32**

5.2.11 Comparison Plot

In addition to the student "t" test, we can also visually compare a plot of estimated shrinkage to measured shrinkage. Figure 10 illustrates the similarity of the estimated and measured shrinkage for vertical features; both sets of points are very close. Figure 11 illustrates the closeness of fit for horizontal feature shrinkage, estimated and measured. The slight difference between the two could be attributed to a number of factors including temperature during measurement, method of measurement, surface roughness or human error.

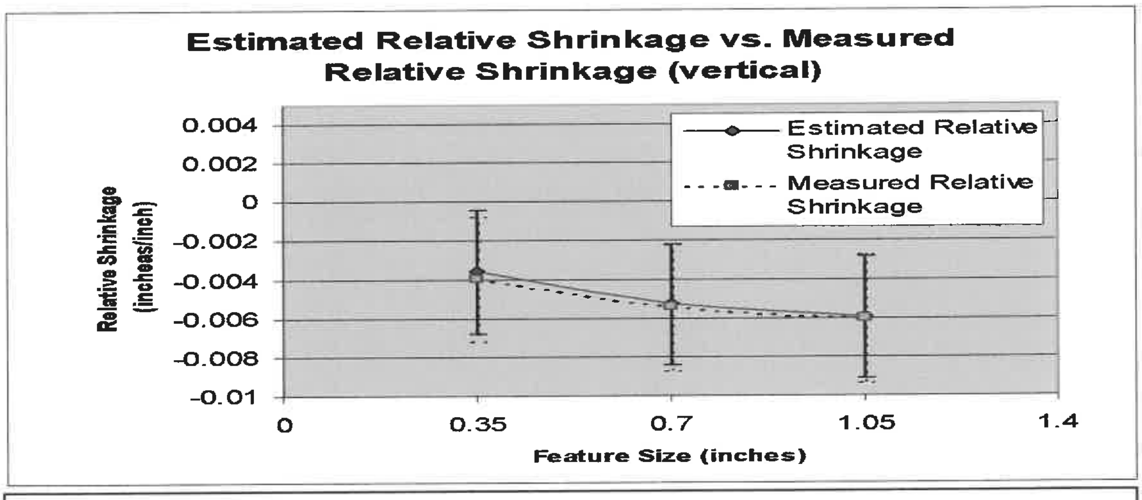


Figure 10. Comparison Plot of Vertical Shrinkage.

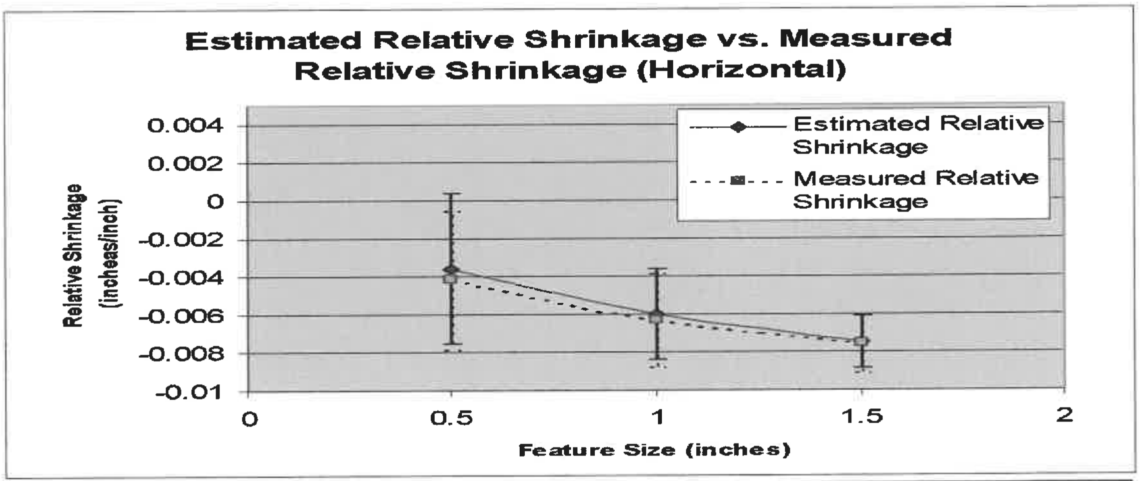


Figure 11. Comparison Plot of Horizontal Shrinkage.

5.2.12 Relative Standard Deviation of Shrinkage

Figure 12 illustrates the process standard deviation for each step and each feature size. Using this illustration it becomes relatively easy to ascertain which process step(s) adds the greatest amount of error (variance). The greatest error occurs during the transfer from the mold master to unfired ceramic, step 2. If error is reduced, by improving processing methods or material, the overall process standard deviation could be improved. The benchmark part would again be used for the focused study and improvements would be implemented. Estimated mean and standard deviation of shrinkage would be updated and recalculated for this step of the overall process.

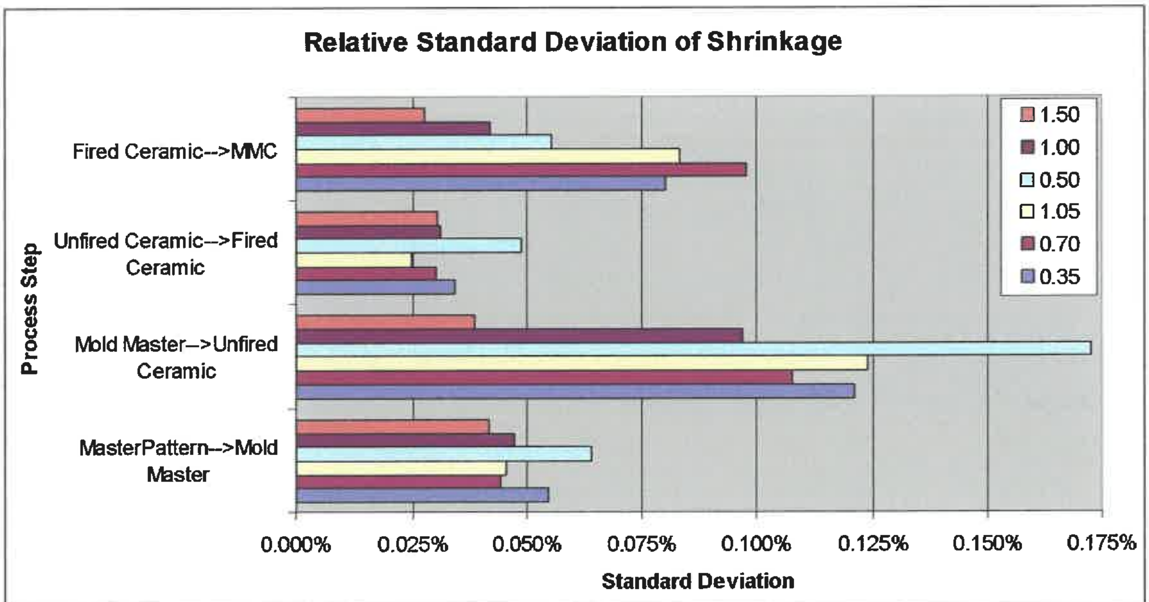


Figure 12. Relative Standard Deviation of Shrinkage

5.2.13 Shrink Factor Prediction

The estimated mean and standard deviation of shrinkage can be used to predict needed shrink factors and tolerance capabilities of future processing. Figure 13 is an example of a predictive chart that could be used for future processing. By knowing the feature size one can determine the appropriate mean and standard deviation of shrinkage for vertical and horizontal feature sizes. Extrapolation to the zero feature size does not go through the origin as expected. More experimental data is required in order to analyze if there is an effect due to geometrical correlations of the features or if there is an effect of the scattering of data.

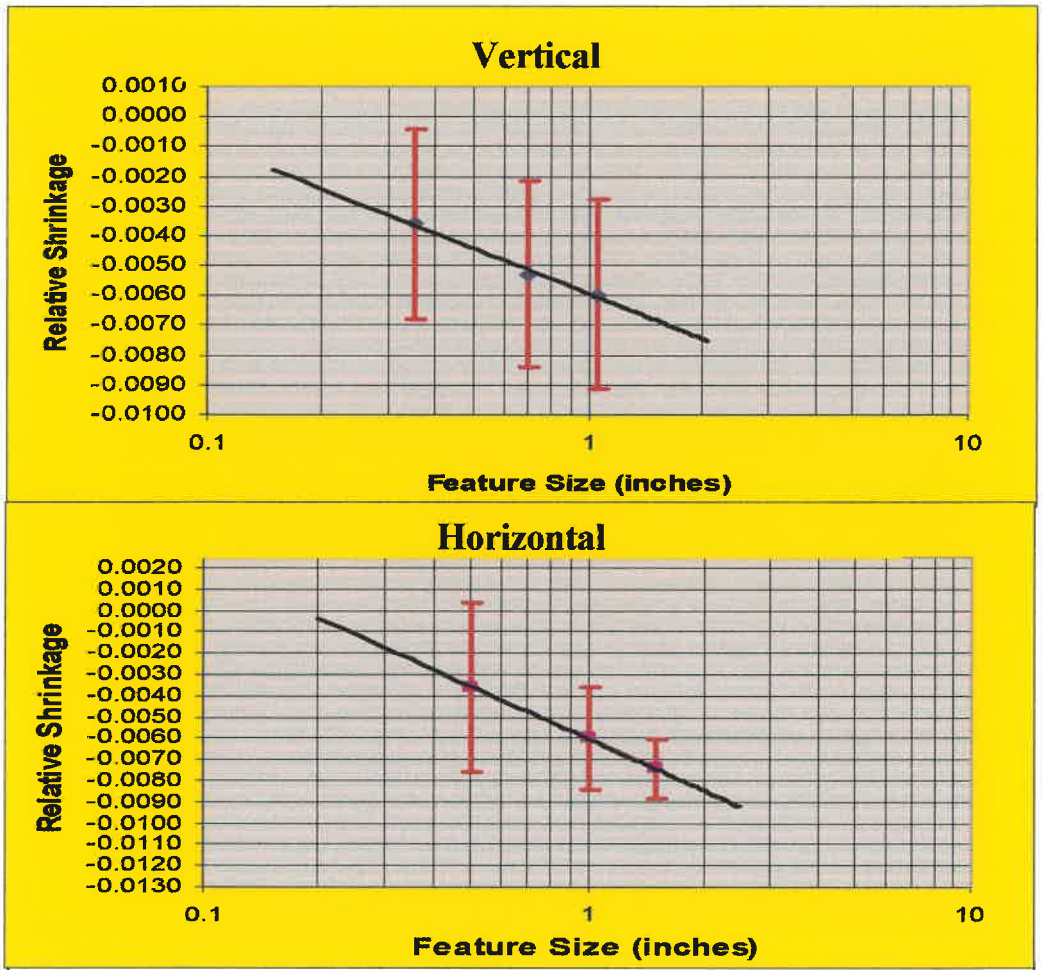


Figure 13. Shrinkage Prediction Plots.

6 Conclusions

The main objective of this research was to show that the PHAST process mean and standard deviation of shrinkage can be closely estimated by applying a mathematical procedure to the combined results of all process steps. It was shown that the estimated and measured mean and standard deviation of shrinkage are within the statistical error.

The second objective was to determine standard deviation of average shrinkage for each process step, for process improvement purposes. The variance for each process step was calculated and several steps were identified as good starting points for improvement. The casting of the ceramic has the greatest variance. Improvements in processing such as reduction in exothermal emission may be helpful. Using a different mold-master material may be helpful too.

The third objective was to provide critical shrinkage prediction plots as a function of feature size. This was achieved with two plots prepared with error bands included, providing a link to part tolerance requirements. Vertical and horizontal shrinkage follow a similar trend and are different by a small amount.

The final objective -reducing the cost of evaluating indirect-RT processes by using a simplified, low cost benchmark geometry- was also achieved. The simple geometry was easier to physically measure and required less measurements. The benchmark geometry also provides critical feature specific data points so that shrinkage curve plots could be produced.

References

[1] Tobin, James R. [inventor]. 16 April 1996. "Method of Constructing Fully Dense Metal Molds And Parts." United States Patent 5,507,336.

[2] Schmidt, L. "Benchmarking Comparison of Commercial Techniques in Rapid Prototyping." 12-15 June 1994. *The Fifth International Conference on Rapid Prototyping*. Dayton, Ohio: Society of Manufacturing Engineers, pp. 333-351.

[3] "Improving Accuracy, Diagnostic Test Parts." February 1993. *Rapid Prototyping Report* Vol. 3, no. 2, pp. 4-5.

[4] Pang, Thomas H., Michelle D. Guertin and Hop D. Nguyen. "Accuracy of Stereolithography Parts: Mechanism and Modes of Distortion for a Letter-H Diagnostic Part." In Solid Freeform Fabrication Symposium. 7-9 August 1995. *Solid Freeform Fabrication Symposium* 1995, Proceedings. Austin, Texas: University of Texas, pp.170-180.

[5] Tata, Kamesh and Dave Flynn. "Quantification of Down Facing Z-Error & Associated Problems." In North American Stereolithography Users Group Conference. 10-14 March 1996. *1996 Conference and Annual Meeting*. San Diego, California. A copy of this document is available from the author.

[6] Shen, J., J. Steinberger, J. Göpfert, R. Gerner, F. Daiber, K. Manetsberger and S. Ferstl. "Inhomogeneous Shrinkage of Polymer Materials in Selective Laser Sintering." In Solid Freeform Fabrication Symposium. 7-9 August 2000. *Solid Freeform Fabrication Symposium Proceedings, August 2000*. Austin, Texas: University of Texas, pp. 298-305.

[7] "Chrysler speedometer adapter-comparison part." August 1993. *Rapid Prototyping Report* Vol. 3, no. 8, pp. 3-6.

[8] Jacobs, Paul. "The Effects of Shrinkage Variation on Rapid Tooling Accuracy." In 3D Systems North American Stereolithography Users Group Conference. 1-5 March 1998. *1998 Conference and Annual Meeting*. San Antonio, Texas. A copy of this document is available from the author.

[9] Knirsch, James. RSP Tooling, LLC. 14 August 2002. Conversation with the author, Milwaukee, Wisconsin.

[10] Fox, Brad. President, Rapid Tooling Technologies. 19 July 2002. Conversation with the author, Milwaukee, Wisconsin.

[11] Winter, B. P., T. R. Ostrom, D. J. Hartman, P. K. Trojan and R. D. Pehlke. 1984. "Mold Dilation and Volumetric Shrinkage of White, Gray, and Ductile Cast Irons." *Transactions of the American Foundrymen's Society* Vol. 92, Paper 84-69, pp 551-560.

[12] Ceriotti, Leonard, Wisconsin Precision Foundry. 18 July 2002. Conversation with the author, Milwaukee, Wisconsin.

[13] Suri, V.K., A.J. Paul, N. El-Kaddah and J.T. Berry. 1994. "Determination of Correlation Factors for Prediction of Shrinkage in Castings—Part I: Prediction of Microporosity in Castings; A Generalized Criterion." *Transactions of the American Foundrymen's Society* Vol. 102, Paper 94-138, pp 861-867.

[14] Niyama, E., T. Uchida, M. Morikawa and S. Saito. August 1982. "A Method of Shrinkage Prediction and its Application to Steel Casting Practice." *Imono (Journal of Japanese Foundrymen's Society)* Vol. 54, No. 8, pp. 507-517.

[15] Wheeldon, J.B. 6 June 1991. "The Basic Causes of Shrinkage Defects in Ductile Iron Castings." Paper presented at the Annual Conference of the Institute of British Foundrymen. Harrogate, United Kingdom.

[16] Burns, Marshal. 1993. *Automated Fabrication: Improving Productivity in Manufacturing.* PTR Prentice Hall.

[17] Ellis, Derek. "Secondary Tooling: Part Reproduction." In North American Stereolithography Users Group Conference. 10-14 March 1996. *1996 Conference and Annual Meeting.* San Diego, California. A copy of this document is available from the author.

[18] "Keltool." June 91. *Rapid Prototyping Report* Vol. 1, no. 1, pp. 3-4.

[19] Lindblom, Brian, "Spray Metal Mold Procedure." In Rapid Prototyping and Manufacturing Conference. 2-4 May 1995. *Rapid Prototyping and Manufacturing '95.* Part 3, no. 13. Dearborn, Michigan: Society of Manufacturing Engineers.

[20] Greenwood, Douglas D. and Todd A Grimm. "Composite Tooling from SLA Patterns for Injection Molding." 2-4 May 1995. In Rapid Prototyping and Manufacturing Conference. *Rapid Prototyping and Manufacturing 1995.* Part 1, no. 13. Dearborn, Michigan: Society of Manufacturing Engineers.

[21] Willis, Steve and Sean Wise. "Net Shape Nickel Ceramic Composite Tooling from RP Models." 23-25 April 1996. In Rapid Prototyping and Manufacturing Conference. *Rapid Prototyping and Manufacturing Conference 1996.* Part 3, no. 4. Dearborn, Michigan: Society of Manufacturing Engineers

[22] McHugh, Kevin and James Knirsch. March 2001. "Producing Production Level Tooling in Prototype Timing." *Time Compression Technologies* Vol. 6, no. 2, pp. 23-24.

[23] Van Putte, Douglas A. "Proposal to Adopt a NASUG User Mold." In North American Stereilithography User's Group Conference. 16-20 February 1997. *1997 Conference and Annual Meeting.* Orlando, Florida. A copy of this document is available from the author.

[24] Van Putte, Douglas A. Consultant & Sales Agent, Cross-Bow Rapid Tool Associates, Inc. 18 July 2002. Conversation with the author, Milwaukee, Wisconsin.

[25] Dean Parra, SLS Operator, Rapid Solutions, A Division of Dickten & Masch Mfg. Co. 27 February 2002. Conversation with the author, SLS CAD files provided. Milwaukee, Wisconsin.

[26] Johnson, Richard A. 1994. *Miller and Freund's Probability and Statistics for Engineers*. Fifth ed. Englewood Cliffs, New Jersey: Prentice Hall, Inc.

[27] Wadsworth, Harrison M., Kenneth S. Stephens, and A. Blanton Godfrey. 1986. *Modern Methods for Quality Control and Improvement*. New York: Wiley.

[28] Ostle, Bernard. 1963. *Statistics in Research*. Ames: The Iowa State University Press.

[29] Kempthorne, Oscar. 1952. *The Design and Analysis of Experiments.* New York: John Wiley and Sons, Inc.

[30] Filliben, James and Alan Heckert. "Upper Critical Values of Student's t Distribution." In Carroll Croarkin and Tobias (eds.). 2001. *Engineering Statistics Handbook.* Gaithersburg, Md. And Austin, Tx: National Institute of Standards and Technology and International SEMATECH. [Internet, WWW]. *Address:* http://www.itl.nist.gov/div898/handbook/eda/section3/eda3672.htm; [accessed: 11 February 2003]. A copy of this document is available from the author.

[31] Epstein, B. 3 January 1946. "Tolerance of Assemblies." *American Machinist* Vol. 90, no. 1, pp. 119-121.

Appendix A. Raw data, average feature size, and standard deviation

Master Pattern

Vertical (0.35 inches) Master Pattern	
n	raw data
1	0.3501
2	0.35025
3	0.3505
Average=	0.3502833
n=	3
SD=	0.0002021

Horizontal (0.5 inches) Master Pattern	
n	raw data
1	0.4977
2	0.4982
3	0.4984
Average=	0.4981
n=	3
SD=	0.0003606

Vertical (0.70 inches)	
n	raw data
1	0.69985
2	0.7001
3	0.70015
Average=	0.7000333
n=	3
SD=	0.0001607

Horizontal (0.5 inches)	
n	raw data
1	0.9983
2	0.999
3	0.99756
Average=	0.9982867
n=	3
SD=	0.0007201

Vertical (1.05 inches)	
n	raw data
1	1.0503
2	1.0506
3	1.0511
Average=	1.0506667
n=	3
SD=	0.0004041

Horizontal (0.5 inches)	
n	raw data
1	1.4964
2	1.4988
3	1.499
Average=	1.4980667
n=	3
SD=	0.0014468

Mold Master

Vertical (0.35 inches) Mold Master	
n	raw data
1	0.3496
2	0.3498
3	0.3498
4	0.3498
5	0.3498
6	0.3498
7	0.3500
8	0.3500
9	0.3501
10	0.3501
11	0.3501
12	0.3502
13	0.3502
14	0.3503
15	0.3507
Average=	0.3500
n=	15
SD=	0.0003

Horizontal (0.5 inches) Mold Master	
n	raw data
1	0.4979
2	0.4979
3	0.4980
4	0.4980
5	0.4980
6	0.4980
7	0.4980
8	0.4982
9	0.4983
10	0.4984
11	0.4984
12	0.4985
13	0.4985
14	0.4987
15	0.4990
Average=	0.4983
n=	15
SD=	0.0003

Vertical (0.70 inches) Mold Master	
n	raw data
1	0.6990
2	0.6991
3	0.6992
4	0.6992
5	0.6993
6	0.6994
7	0.6994
8	0.6994
9	0.6995
10	0.6995
11	0.6996
12	0.6997
13	0.6999
14	0.6999
15	0.7000
Average=	0.6995
n=	15
SD=	0.0003

Horizontal (1.0 inches) Mold Master	
n	raw data
1	0.9969
2	0.9972
3	0.9972
4	0.9975
5	0.9979
6	0.9980
7	0.9982
8	0.9983
9	0.9984
10	0.9985
11	0.9986
12	0.9987
13	0.9988
14	0.9991
15	0.9995
Average=	0.9982
n=	15
SD=	0.0007

Vertical (1.05 inches) Mold Master	
n	raw data
1	1.0493
2	1.0494
3	1.0494
4	1.0495
5	1.0496
6	1.0496
7	1.0497
8	1.0498
9	1.0499
10	1.0500
11	1.0501
12	1.0504
13	1.0505
14	1.0508
15	1.0510
Average=	1.0499
n=	15
SD=	0.0005

Horizontal (1.5 inches) Mold Master	
n	raw data
1	1.4954
2	1.4955
3	1.4958
4	1.4959
5	1.4965
6	1.4973
7	1.4975
8	1.4975
9	1.4977
10	1.4978
11	1.4978
12	1.4981
13	1.4982
14	1.4991
15	1.4993
Average=	1.4973
n=	15
SD=	0.0012

Unfired Ceramic

Vertical (0.35 inches) Unfired Ceramic	
n	raw data
1	0.3506
2	0.3509
3	0.3509
4	0.3510
5	0.3510
6	0.3510
7	0.3513
8	0.3513
9	0.3514
10	0.3516
11	0.3516
12	0.3517
13	0.3518
14	0.3518
15	0.3521
Average=	0.3513
n=	15
SD=	0.0004

Horizontal (0.5 inches) Unfired Ceramic	
n	raw data
1	0.4991
2	0.4994
3	0.4995
4	0.4996
5	0.5000
6	0.5001
7	0.5002
8	0.5002
9	0.5002
10	0.5005
11	0.5005
12	0.5006
13	0.5015
14	0.5018
15	0.5019
Average=	0.5003
n=	15
SD=	0.0009

Vertical (0.70 inches) Unfired Ceramic	
n	raw data
1	0.7008
2	0.7011
3	0.7015
4	0.7015
5	0.7020
6	0.7020
7	0.7021
8	0.7021
9	0.7022
10	0.7022
11	0.7022
12	0.7027
13	0.7028
14	0.7034
15	0.7037
Average=	0.7021
n=	15
SD=	0.0008

Horizontal (1.0 inches) Unfired Ceramic	
n	raw data
1	1.0000
2	1.0001
3	1.0002
4	1.0007
5	1.0010
6	1.0010
7	1.0013
8	1.0014
9	1.0014
10	1.0015
11	1.0017
12	1.0018
13	1.0021
14	1.0036
15	1.0037
Average=	1.0014
n=	15
SD=	0.0011

Vertical (1.05 inches) Unfired Ceramic	
n	raw data
1	1.0524
2	1.0526
3	1.0527
4	1.0529
5	1.0530
6	1.0533
7	1.0536
8	1.0539
9	1.0543
10	1.0545
11	1.0551
12	1.0557
13	1.0559
14	1.0559
15	1.0562
Average=	1.0541
n=	15
SD=	0.0013

Horizontal (1.5 inches) Unfired Ceramic	
n	raw data
1	1.4995
2	1.5000
3	1.5003
4	1.5007
5	1.5013
6	1.5017
7	1.5018
8	1.5019
9	1.5021
10	1.5021
11	1.5021
12	1.5023
13	1.5023
14	1.5025
15	1.5027
Average=	1.5015
n=	15
SD=	0.0010

Fired Ceramic

Vertical (0.35 inches) Fired Ceramic	
n	raw data
1	0.3495
2	0.3495
3	0.3497
4	0.3497
5	0.3499
6	0.3500
7	0.3501
8	0.3502
9	0.3502
10	0.3504
11	0.3506
12	0.3506
13	0.3507
14	0.3509
15	0.3512
Average=	0.3502
n=	15
SD=	0.0005

Horizontal (0.5 inches) Fired Ceramic	
n	raw data
1	0.4984
2	0.4989
3	0.4989
4	0.4992
5	0.4992
6	0.4992
7	0.4994
8	0.4995
9	0.4995
10	0.4998
11	0.4999
12	0.5005
13	0.5007
14	0.5008
15	0.5012
Average=	0.4997
n=	15
SD=	0.0008

Vertical (0.70 inches) Fired Ceramic	
n	raw data
1	0.6985
2	0.6986
3	0.6990
4	0.6996
5	0.6997
6	0.6997
7	0.6997
8	0.6998
9	0.6999
10	0.7000
11	0.7000
12	0.7004
13	0.7006
14	0.7014
15	0.7017
Average=	0.6999
n=	15
SD=	0.0009

Horizontal (1.0 inches) Fired Ceramic	
n	raw data
1	0.9973
2	0.9978
3	0.9984
4	0.9987
5	0.9987
6	0.9988
7	0.9994
8	0.9996
9	0.9996
10	0.9996
11	0.9997
12	0.9999
13	0.9999
14	1.0016
15	1.0018
Average=	0.9994
n=	15
SD=	0.0012

Vertical (1.05 inches) Fired Ceramic	
n	raw data
1	1.0488
2	1.0490
3	1.0493
4	1.0495
5	1.0499
6	1.0501
7	1.0502
8	1.0504
9	1.0509
10	1.0510
11	1.0515
12	1.0524
13	1.0525
14	1.0529
15	1.0532
Average=	1.0508
n=	15
SD=	0.0015

Horizontal (1.5 inches) Fired Ceramic	
n	raw data
1	1.4954
2	1.4962
3	1.4966
4	1.4974
5	1.4978
6	1.4979
7	1.4979
8	1.4980
9	1.4981
10	1.4986
11	1.4990
12	1.4990
13	1.4990
14	1.4992
15	1.4996
Average=	1.4980
n=	15
SD=	0.0012

Metal Matrix Composite (MMC)

Vertical (0.35 inches) MMC	
n	raw data
1	0.3479
2	0.3482
3	0.3483
4	0.3485
5	0.3486
6	0.3486
7	0.3487
8	0.3487
9	0.3490
10	0.3492
11	0.3493
12	0.3495
13	0.3496
14	0.3497
15	0.3498
Average=	0.3489
n=	15
SD=	0.0006

Horizontal (0.5 inches) MMC	
n	raw data
1	0.4946
2	0.4948
3	0.4953
4	0.4955
5	0.4956
6	0.4957
7	0.4957
8	0.4958
9	0.4960
10	0.4960
11	0.4965
12	0.4967
13	0.4971
14	0.4974
15	0.4977
Average=	0.496012
n=	15
SD=	0.000901

Vertical (0.70 inches) MMC	
n	raw data
1	0.6946
2	0.6947
3	0.6953
4	0.6954
5	0.6954
6	0.6956
7	0.6960
8	0.6963
9	0.6964
10	0.6966
11	0.6967
12	0.6967
13	0.6977
14	0.6980
15	0.6983
Average=	0.6962
n=	15
SD=	0.0011

Horizontal (1.0 inches) Fired Ceramic	
n	raw data
1	0.9900
2	0.9903
3	0.9903
4	0.9907
5	0.9909
6	0.9915
7	0.9919
8	0.9922
9	0.9922
10	0.9926
11	0.9926
12	0.9930
13	0.9931
14	0.9941
15	0.9947
Average=	0.991992
n=	15
SD=	0.001399

Vertical (1.05 inches) MMC	
n	raw data
1	1.0416
2	1.0424
3	1.0424
4	1.0427
5	1.0428
6	1.0435
7	1.0440
8	1.0446
9	1.0447
10	1.0449
11	1.0452
12	1.0459
13	1.0459
14	1.0472
15	1.0474
Average=	1.0443
n=	15
SD=	0.0018

Horizontal (1.5 inches) MMC	
n	raw data
1	1.4836
2	1.4842
3	1.4860
4	1.4861
5	1.4862
6	1.4865
7	1.4869
8	1.4869
9	1.4875
10	1.4876
11	1.4876
12	1.4881
13	1.4881
14	1.4882
15	1.4885
Average=	1.4868
n=	15
SD=	0.0014

Appendix B. Uniform Probability Plots of Raw Data

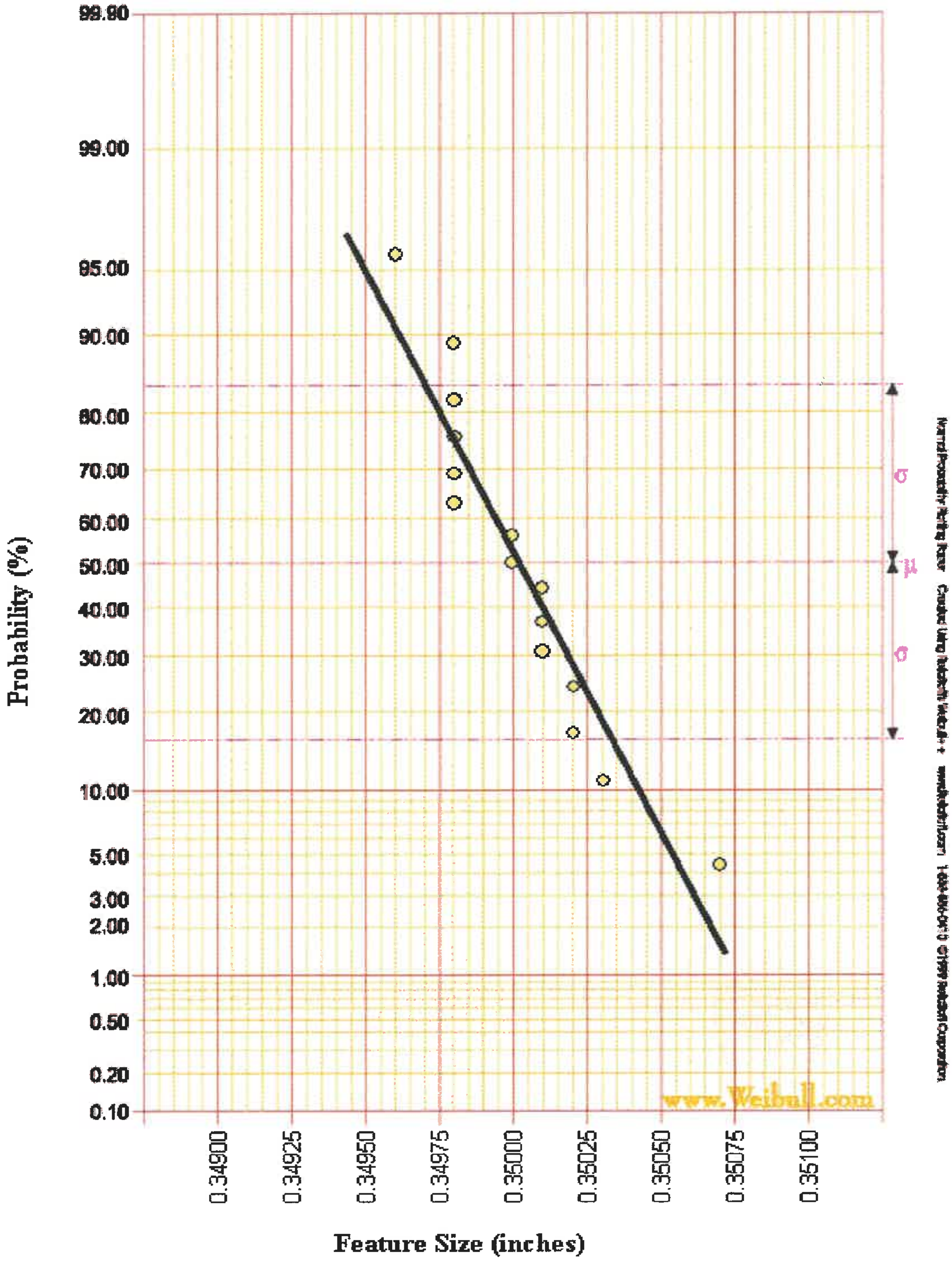


Figure B1. Uniform Probability Plot, Feature Size, Mold Master (0.35" Vertical).

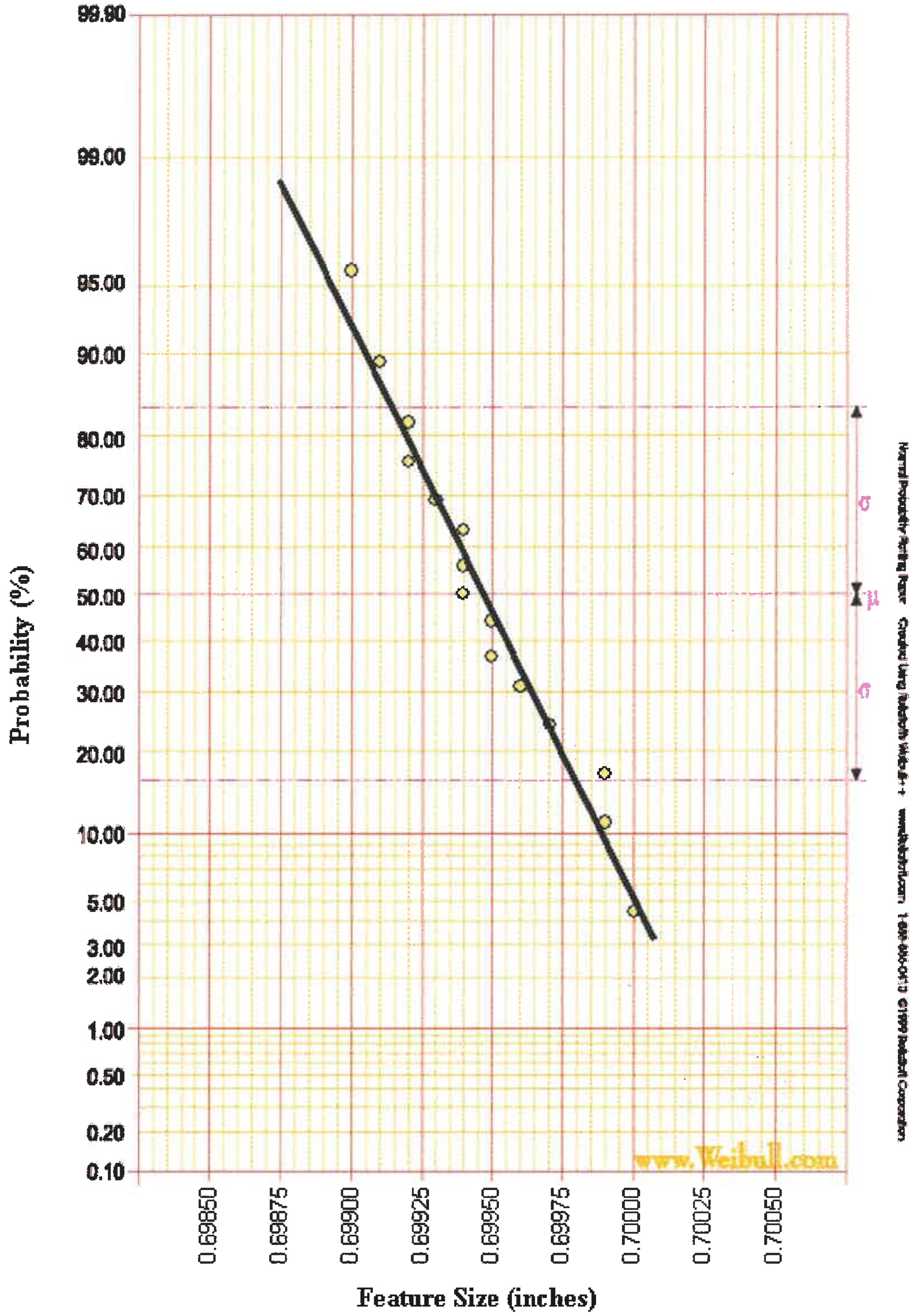


Figure B2. Uniform Probability Plot, Feature Size, Mold Master (0.7" Vertical).

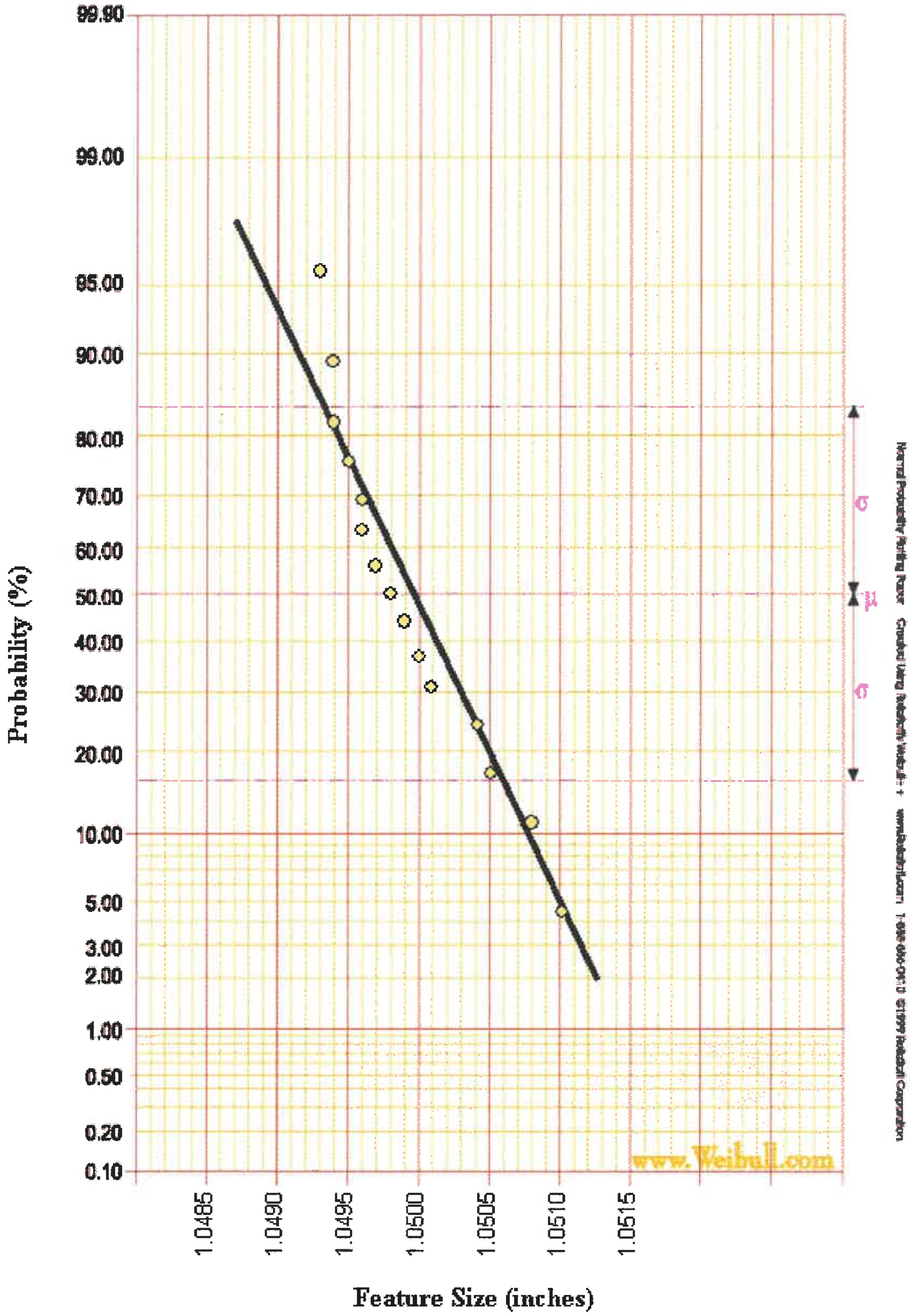


Figure B3. Uniform Probability Plot, Feature Size, Mold Master (1.05" Vertical).

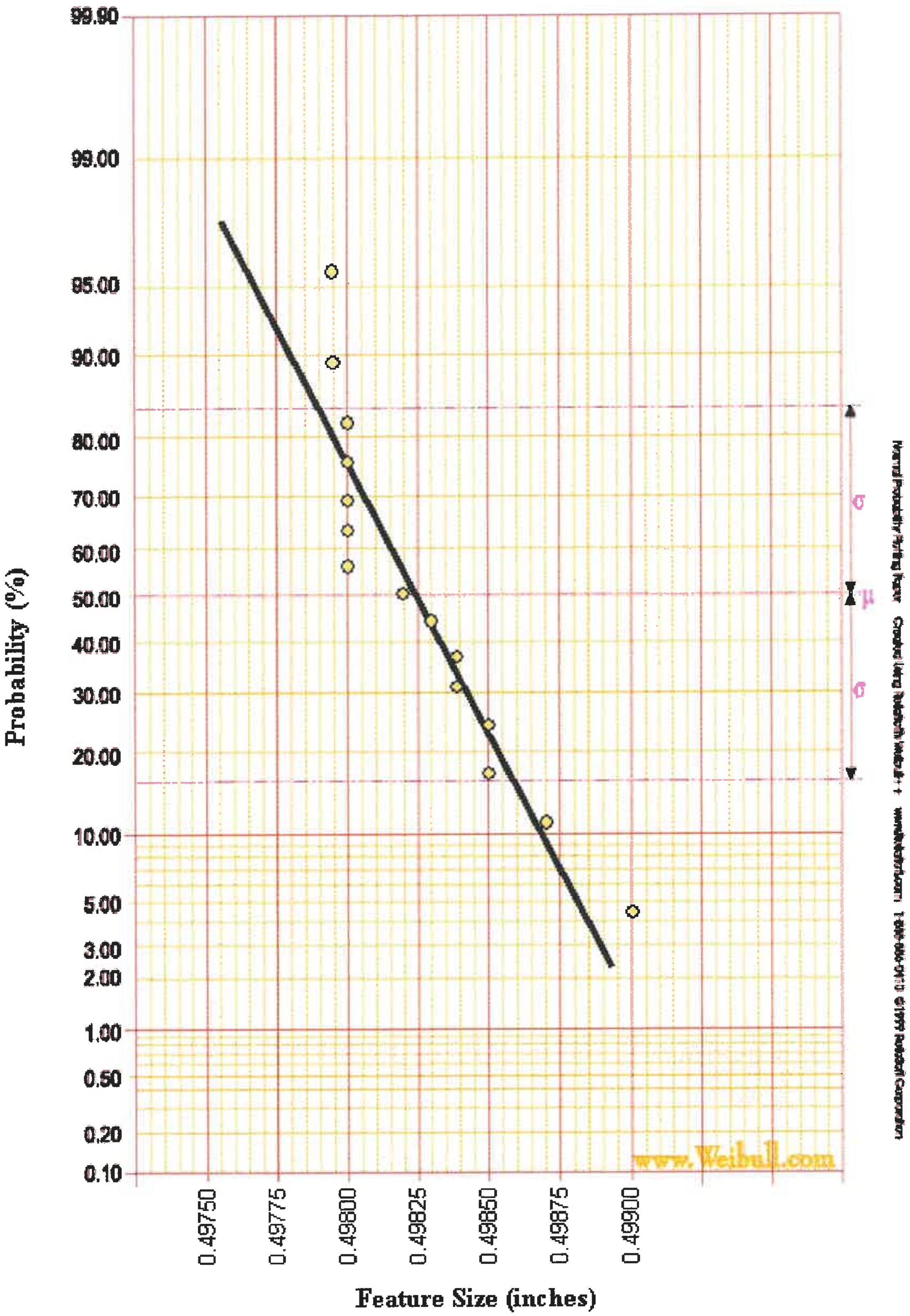


Figure B4. Uniform Probability Plot, Feature Size, Mold Master (0.5" Horizontal).

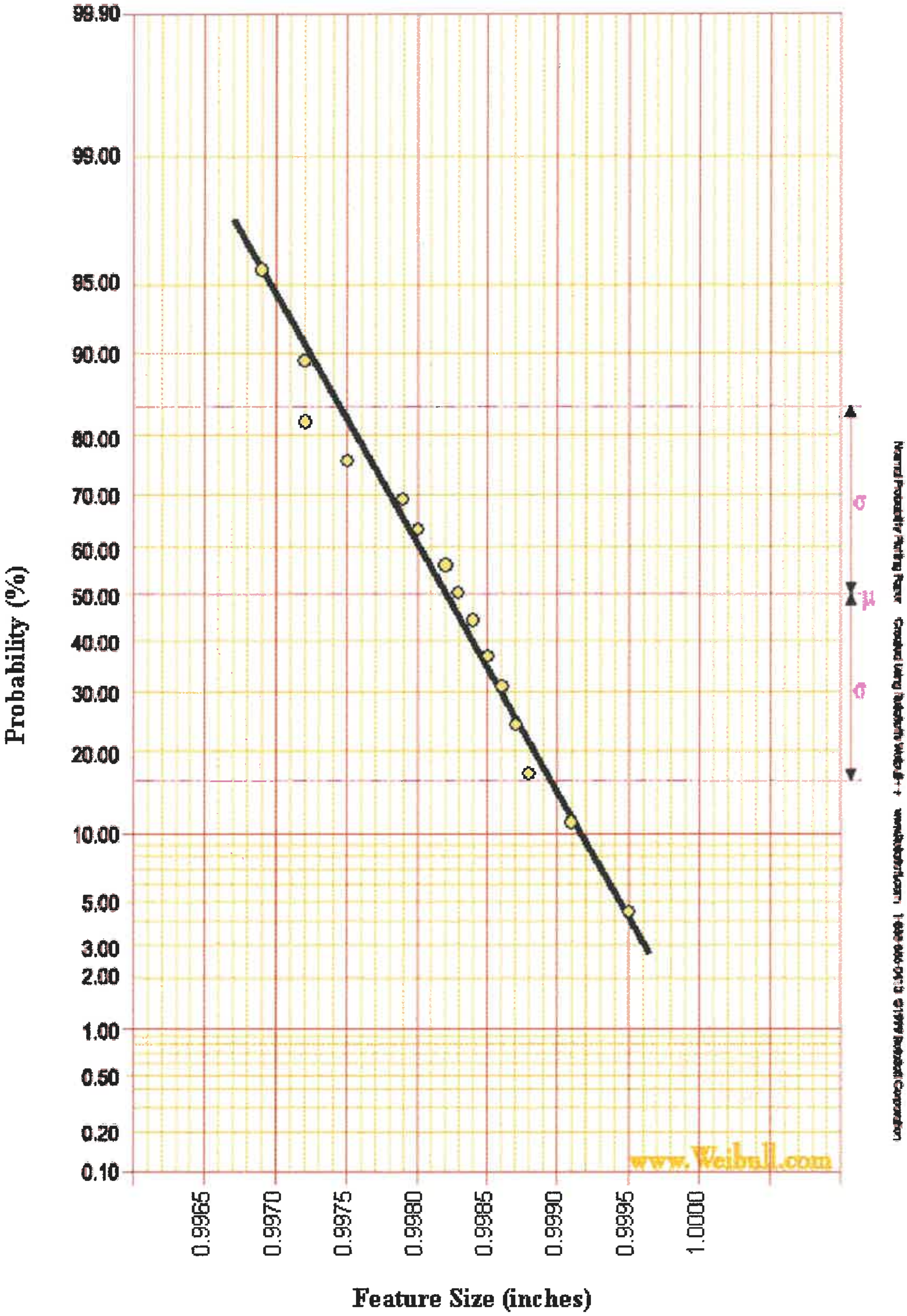


Figure B5. Uniform Probability Plot, Feature Size, Mold Master (1.0" Horizontal).

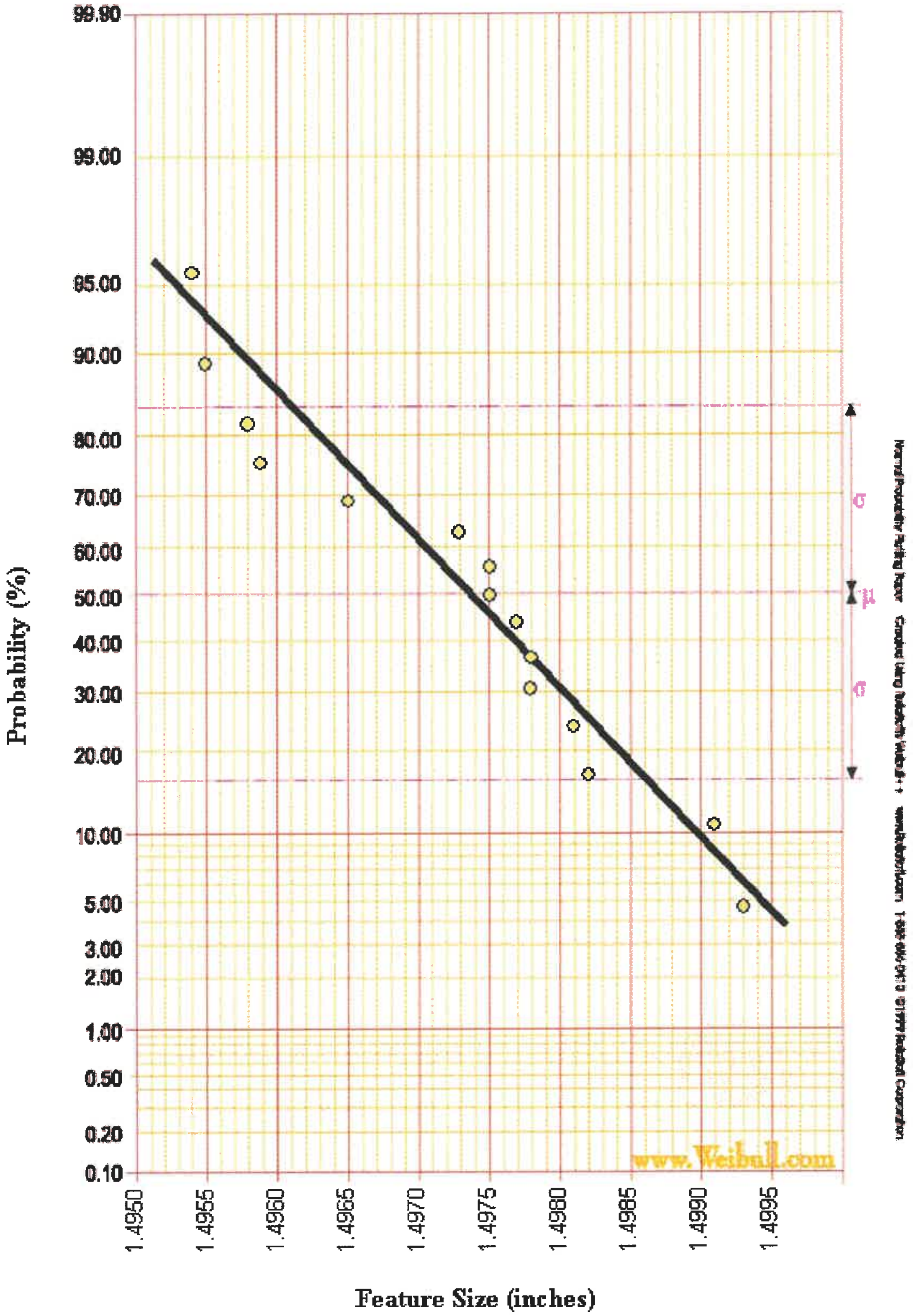


Figure B6. Uniform Probability Plot, Feature Size, Mold Master (1.5" Horizontal).

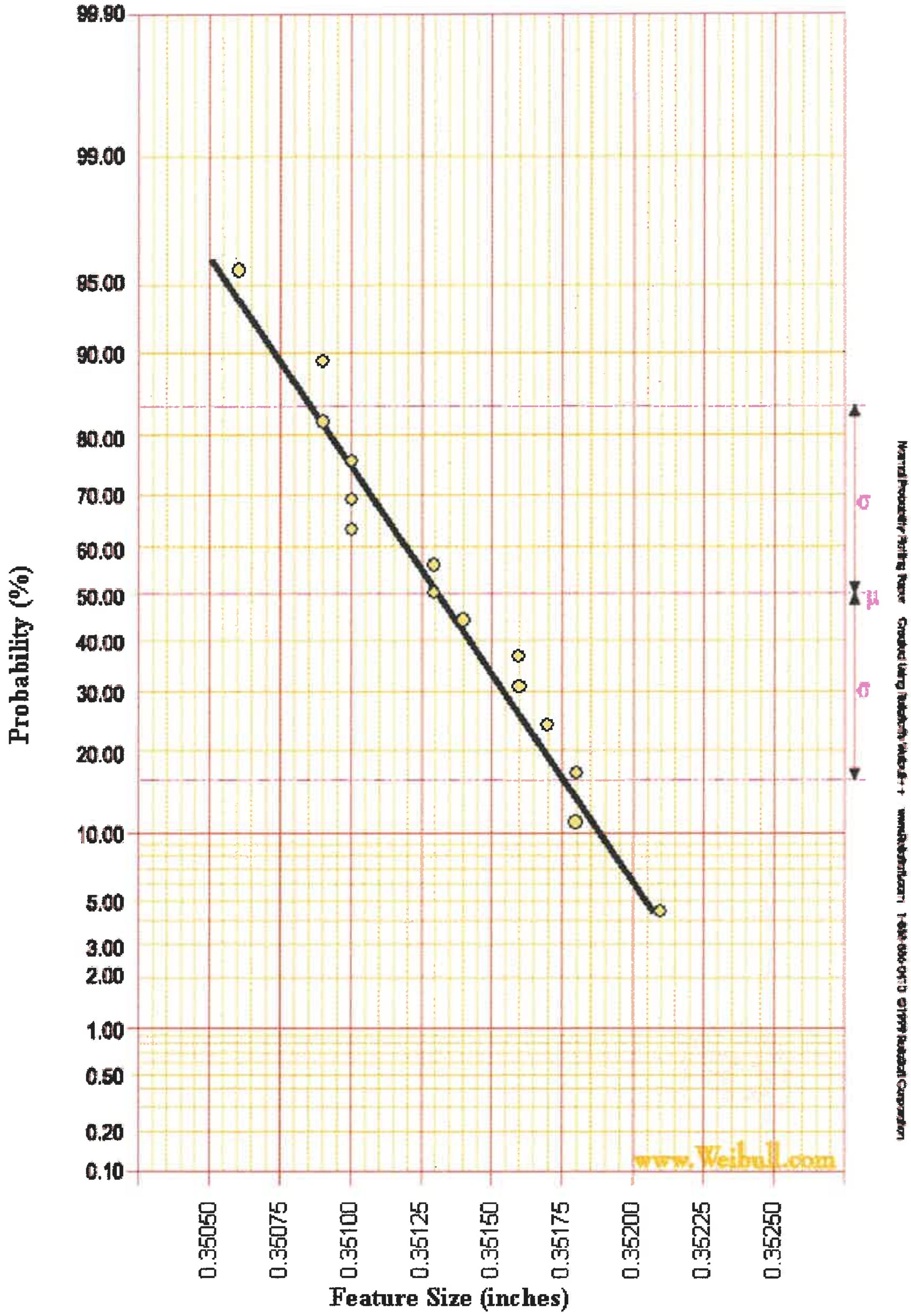


Figure B7. Uniform Probability Plot, Feature Size, Ceramic (0.35" Vertical).

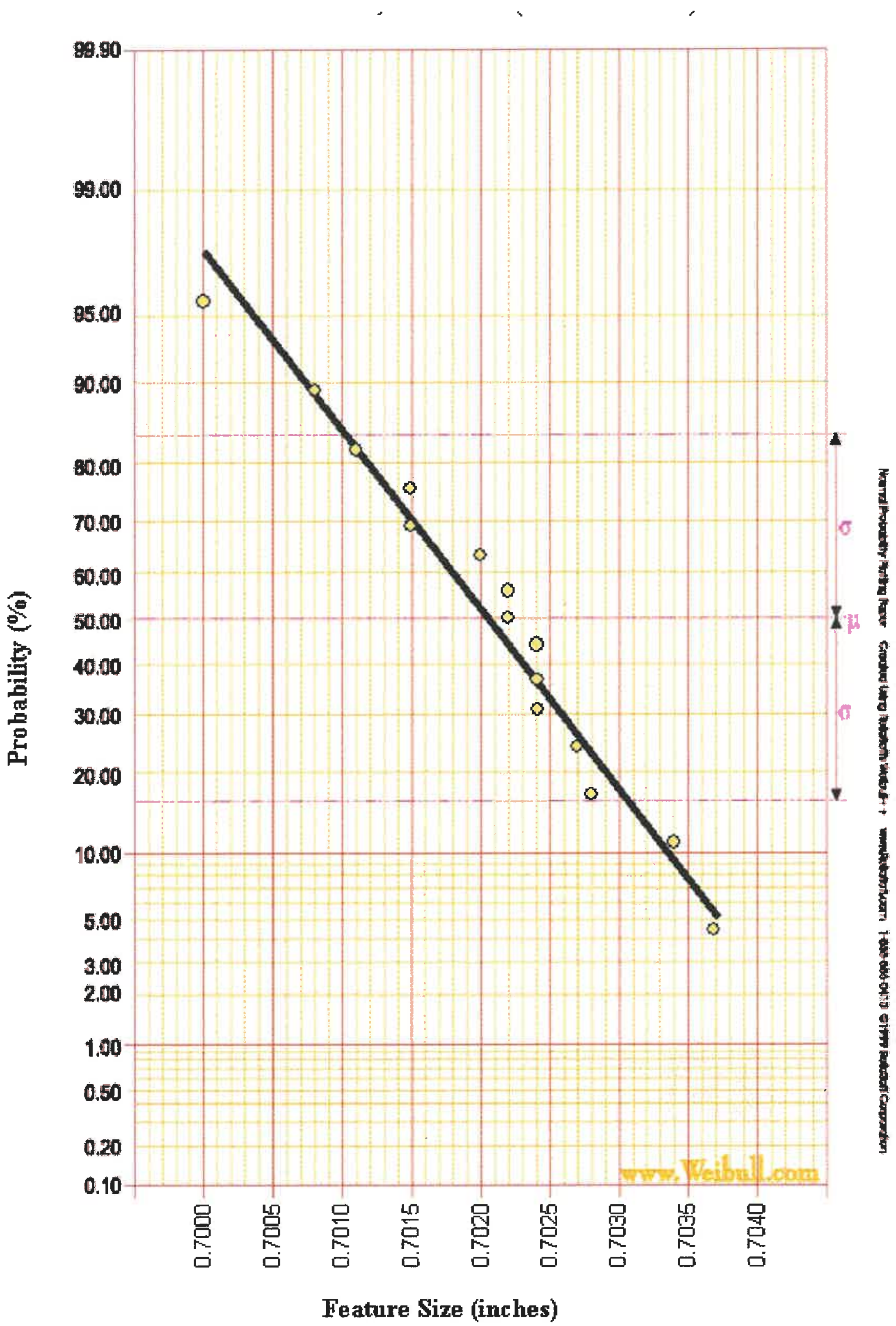


Figure B8. Uniform Probability Plot, Feature Size, Ceramic (0.7" Vertical).

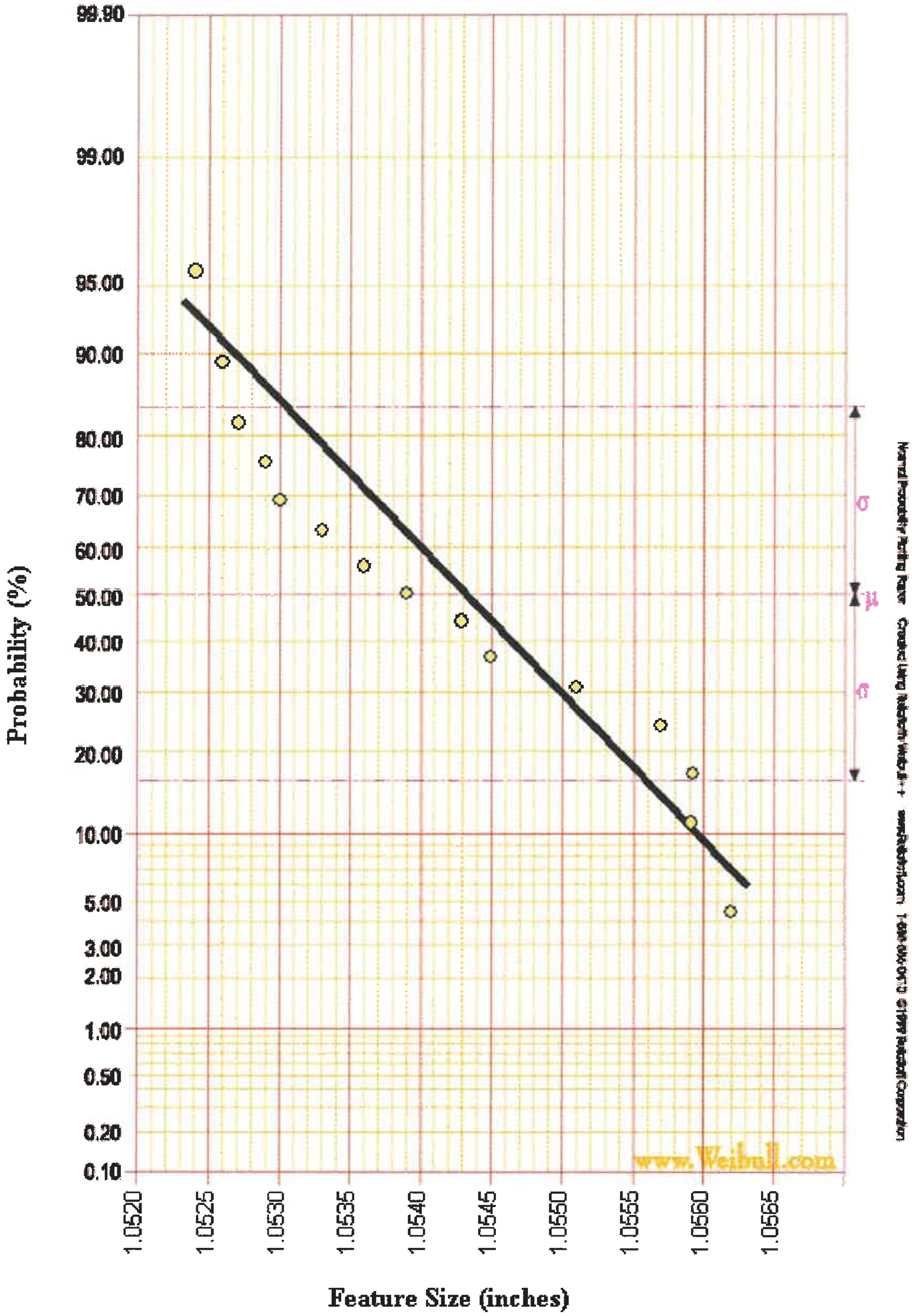


Figure B9. Uniform Probability Plot, Feature Size, Ceramic (1.05" Vertical).

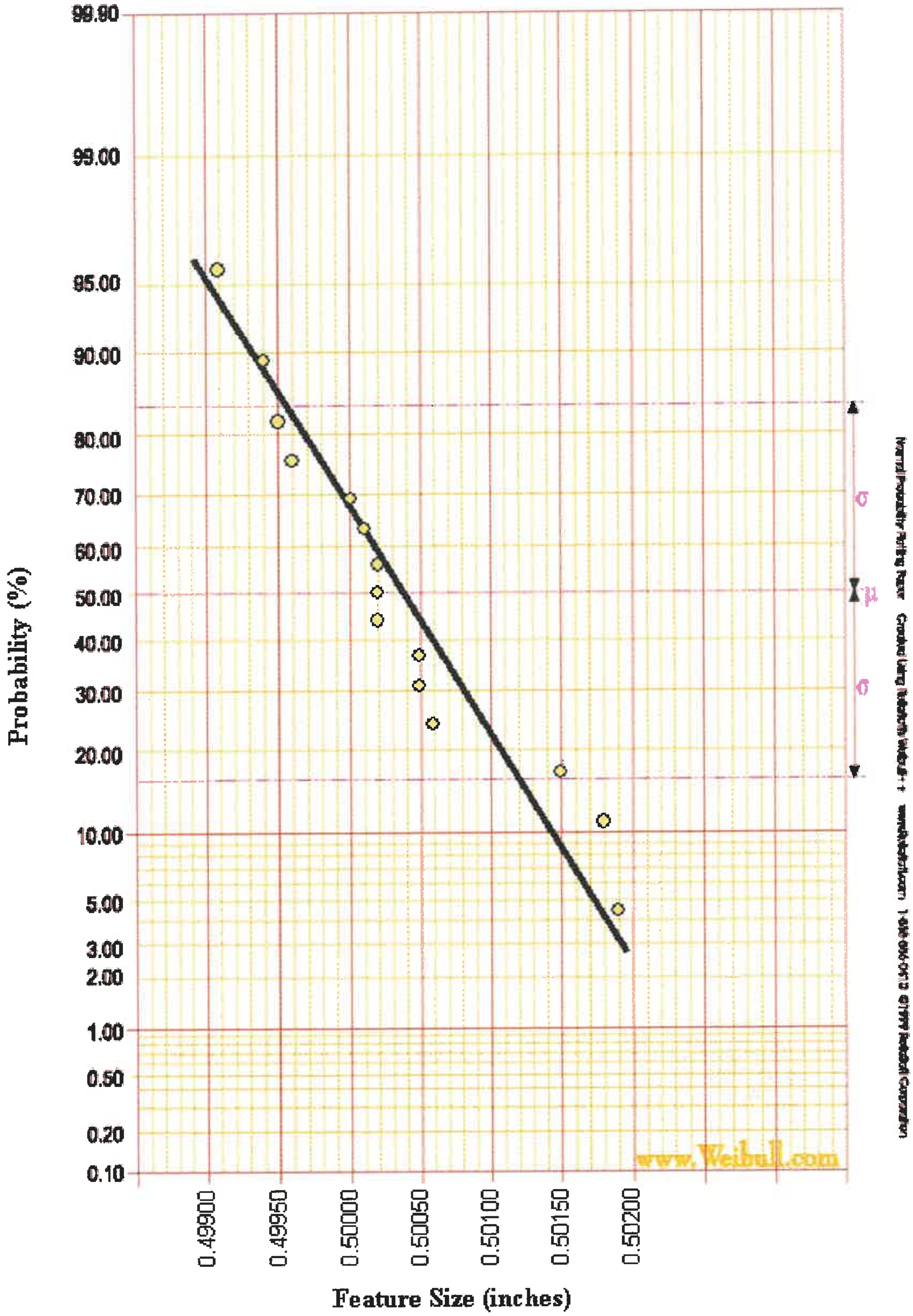


Figure B10. Uniform Probability Plot, Feature Size, Ceramic (0.5" Horizontal).

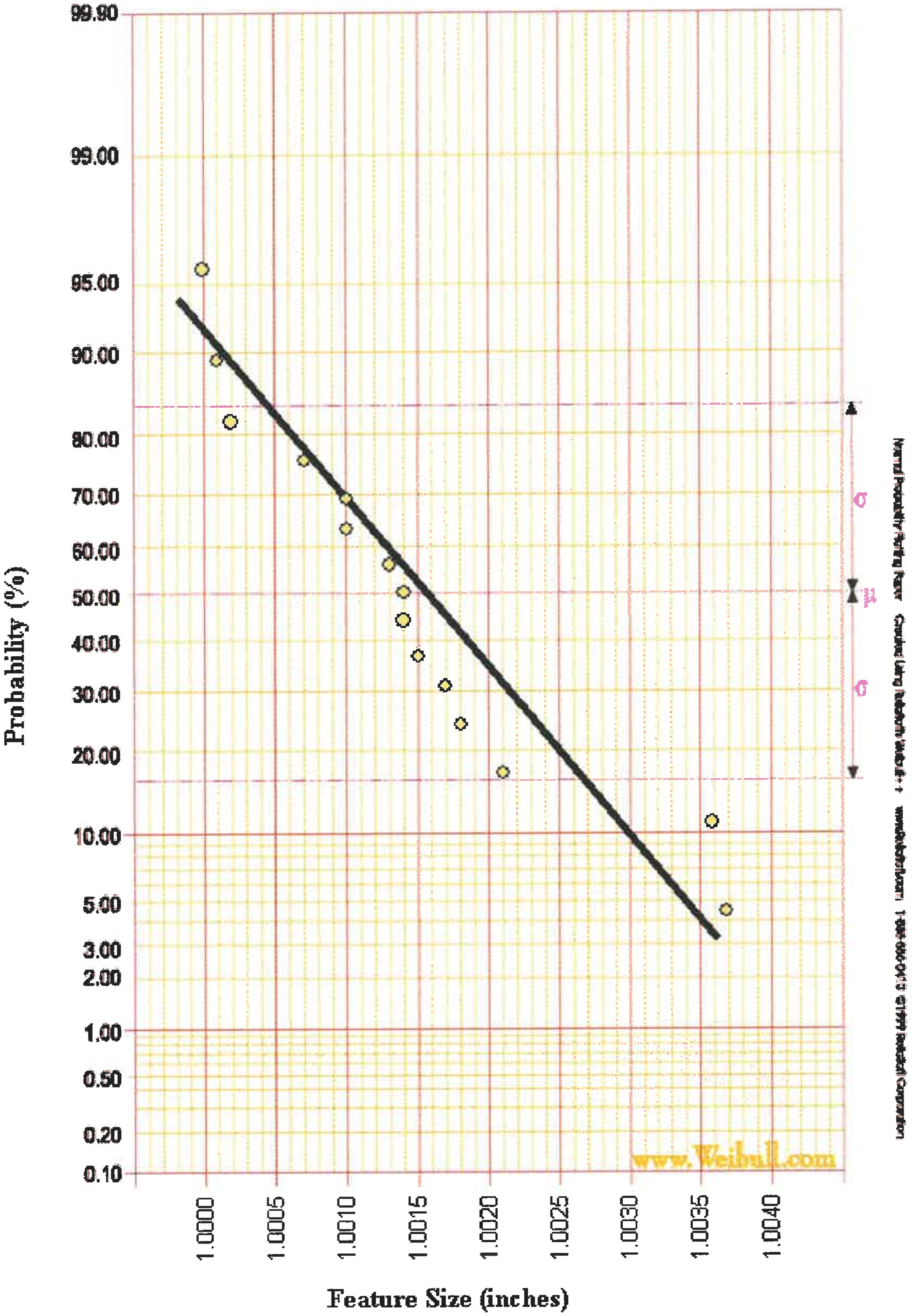


Figure B11. Uniform Probability Plot, Feature Size, Ceramic (1.0" Horizontal).

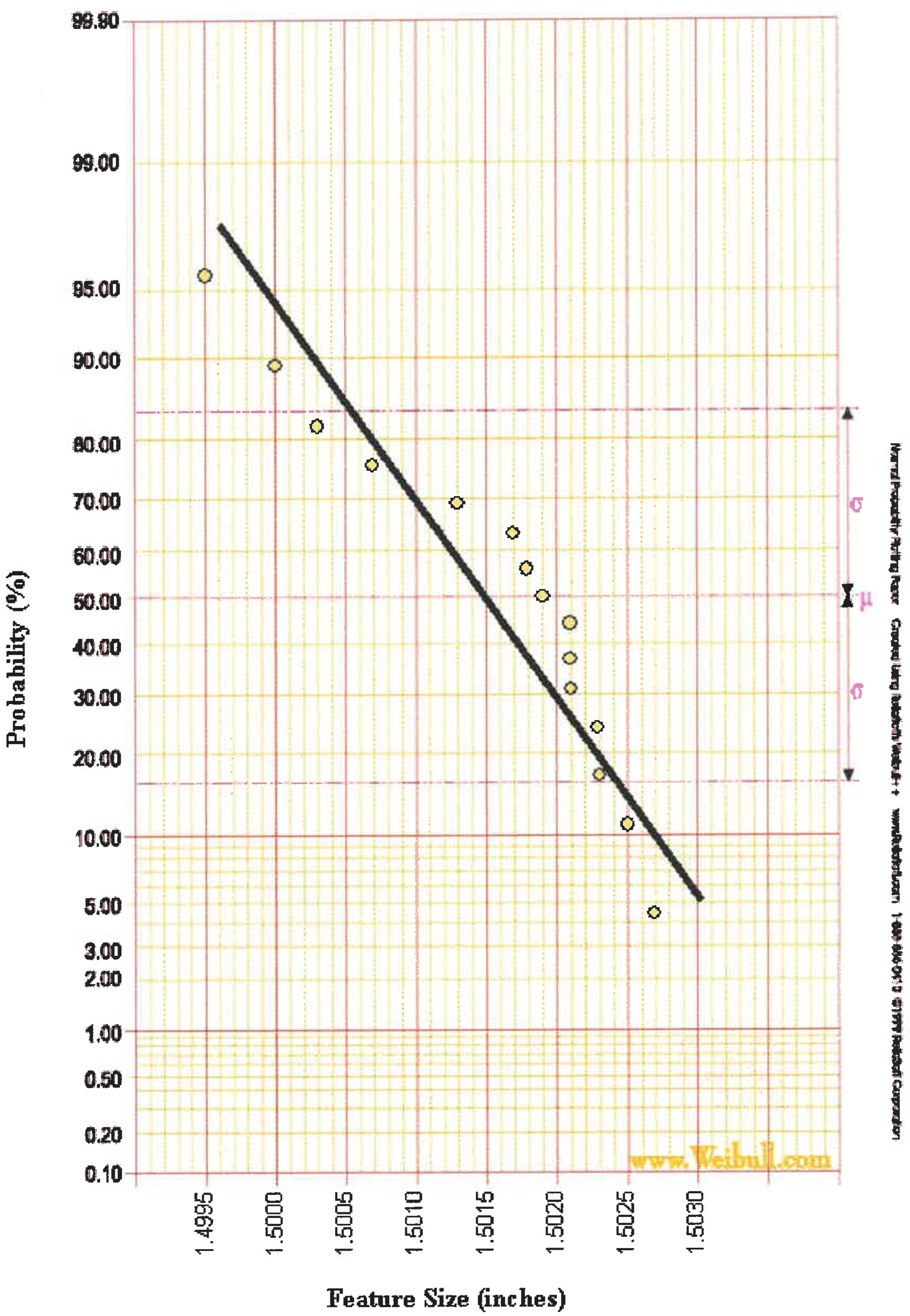


Figure B12. Uniform Probability Plot, Feature Size, Ceramic (1.5" Horizontal).

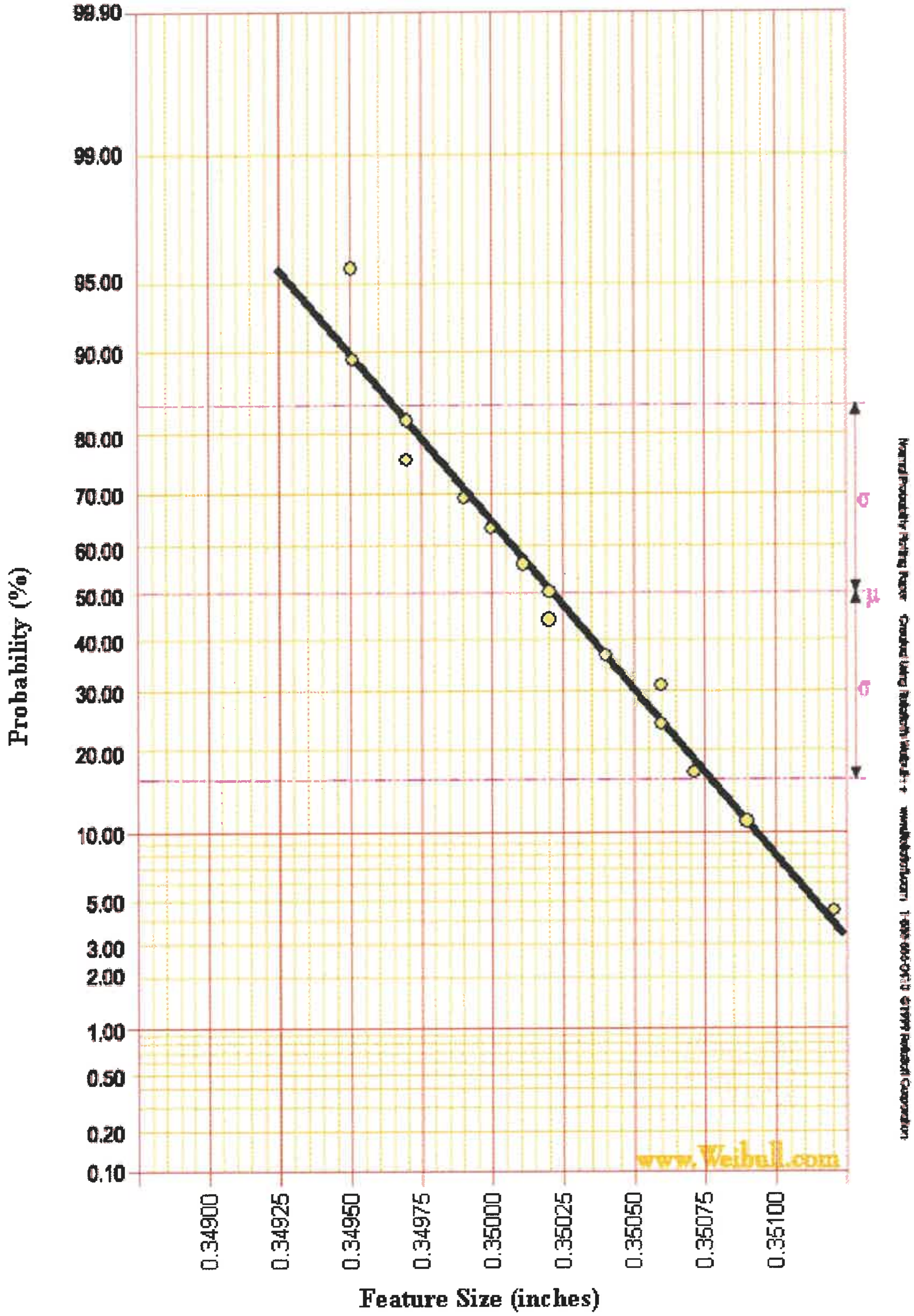


Figure B13. Uniform Probability Plot, Feature Size, Fired Ceramic (0.35" Vertical).

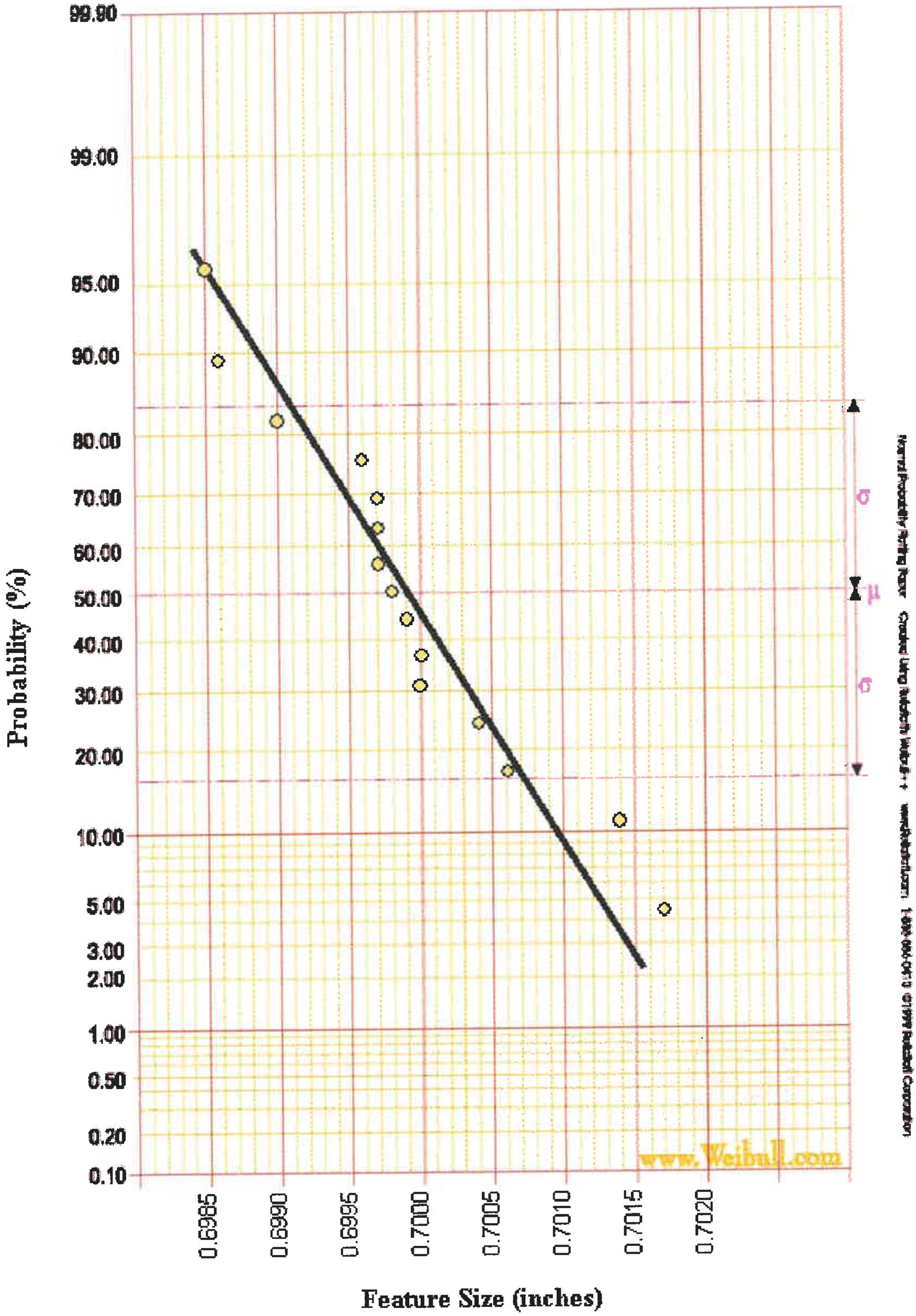


Figure B14. Uniform Probability Plot, Feature Size, Fired Ceramic (0.70" Vertical).

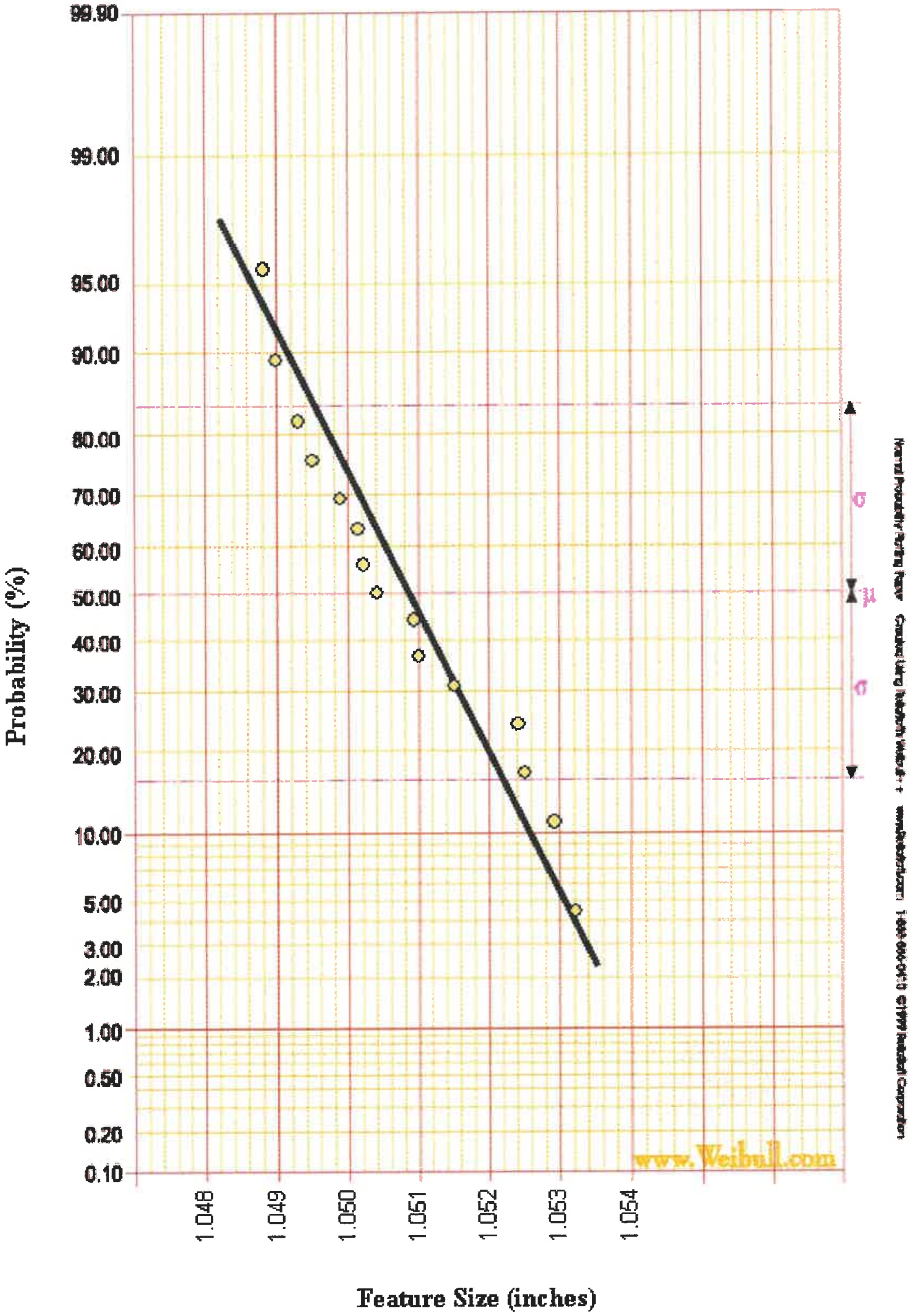


Figure B15. Uniform Probability Plot, Feature Size, Fired Ceramic (1.05" Vertical).

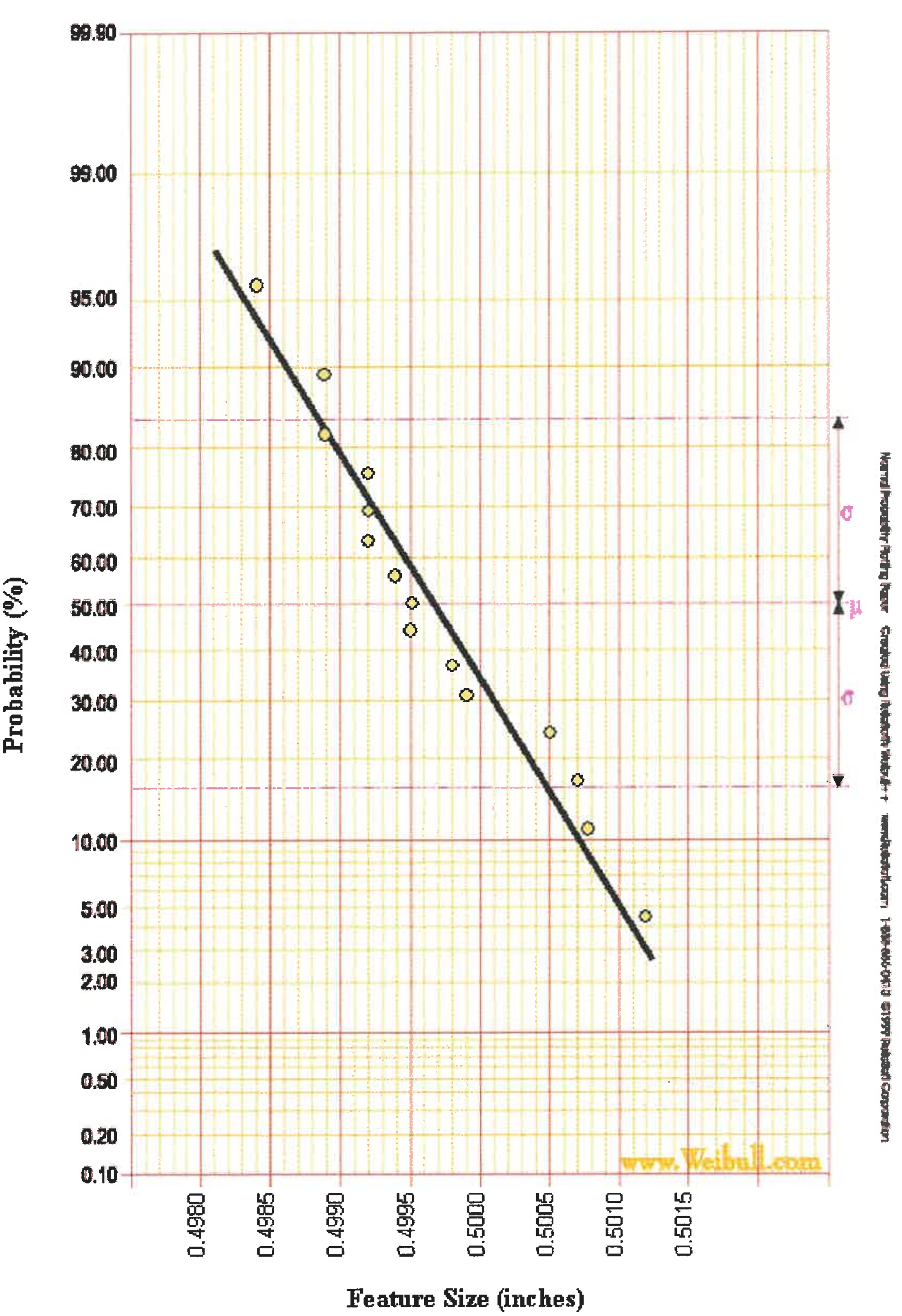


Figure B16. Uniform Probability Plot, Feature Size, Fired Ceramic (0.5" Horizontal).

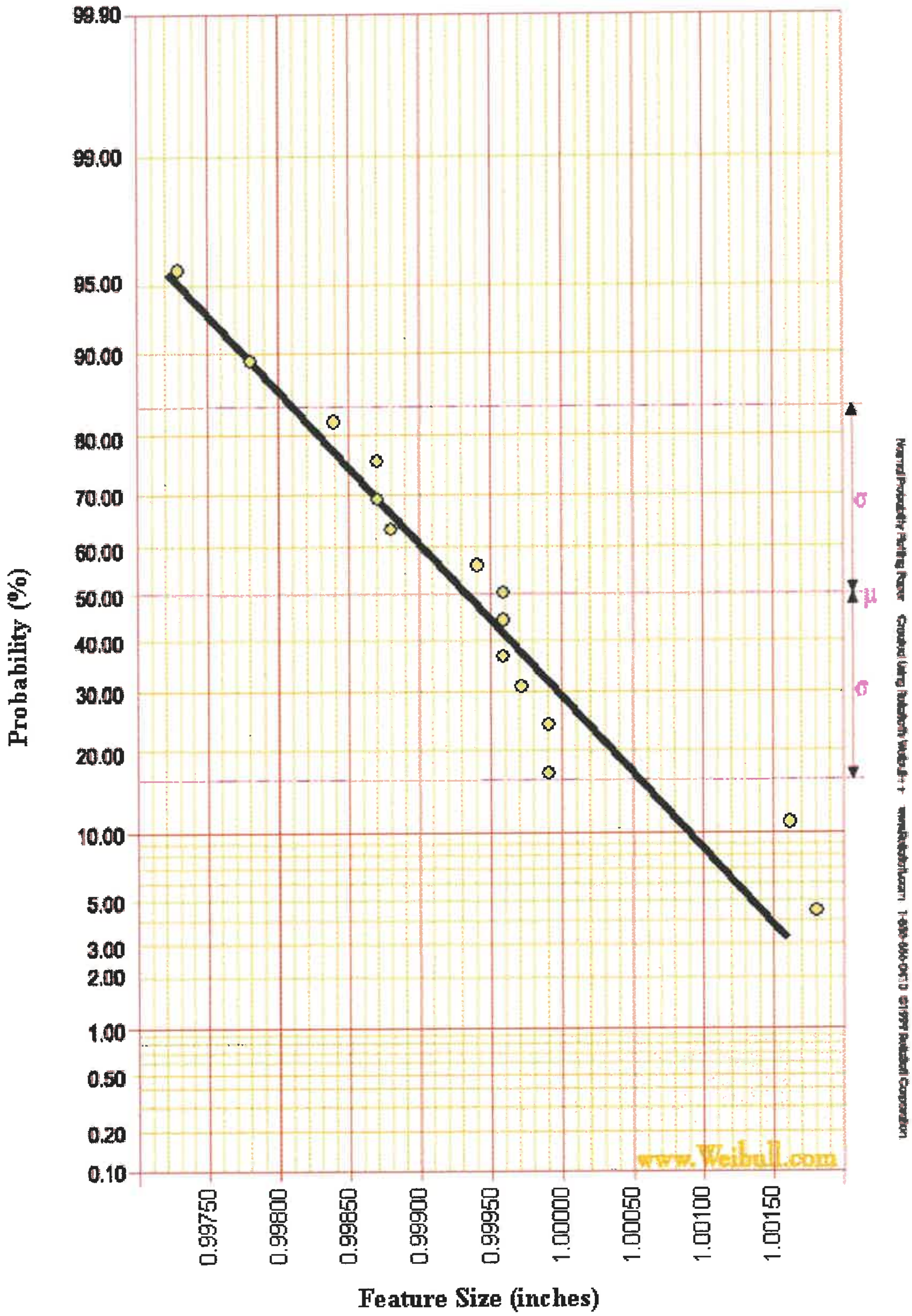


Figure B17. Uniform Probability Plot, Feature Size, Fired Ceramic (1.0" Horizontal).

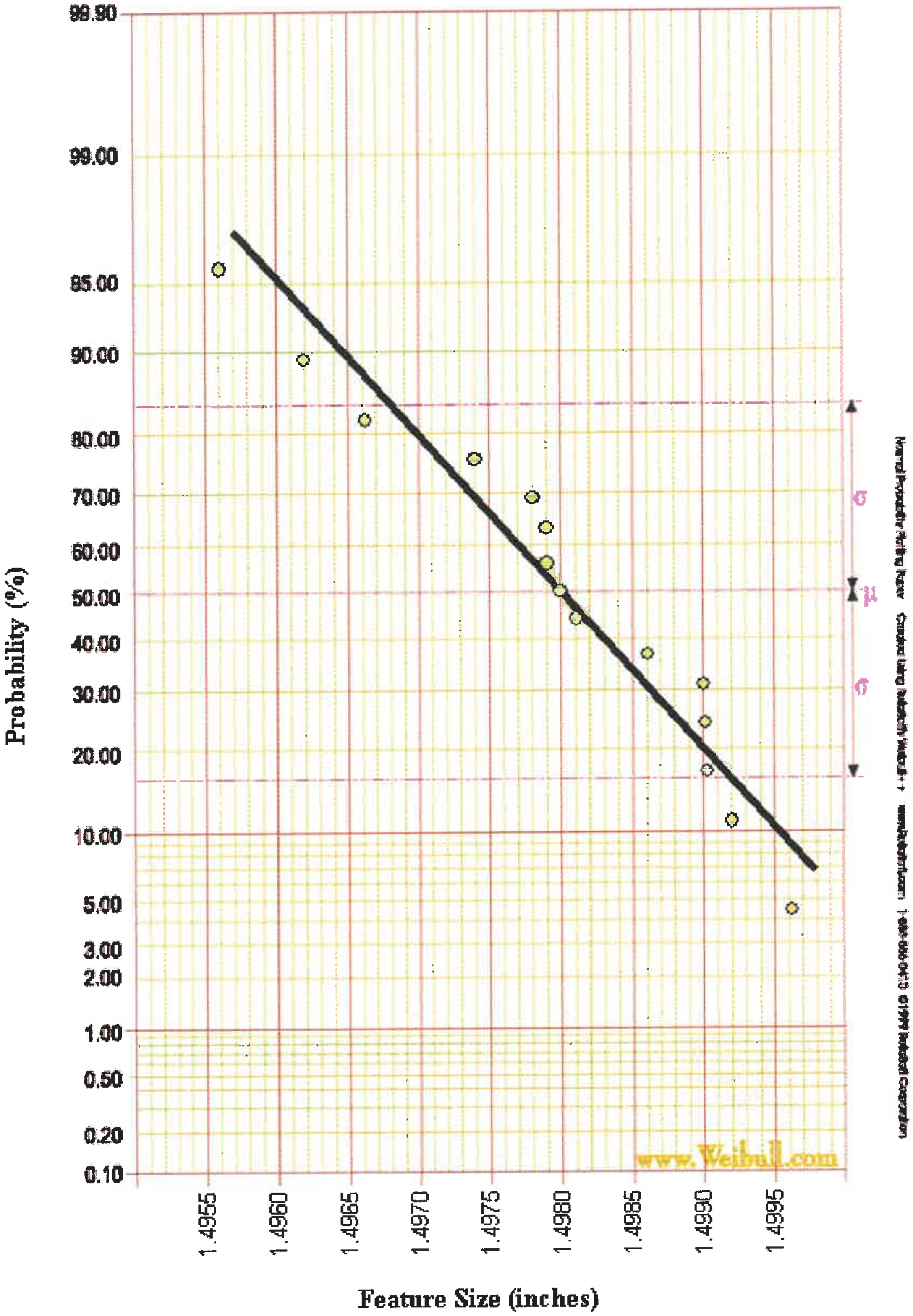


Figure B18. Uniform Probability Plot, Feature Size, Fired Ceramic (1.5" Horizontal).

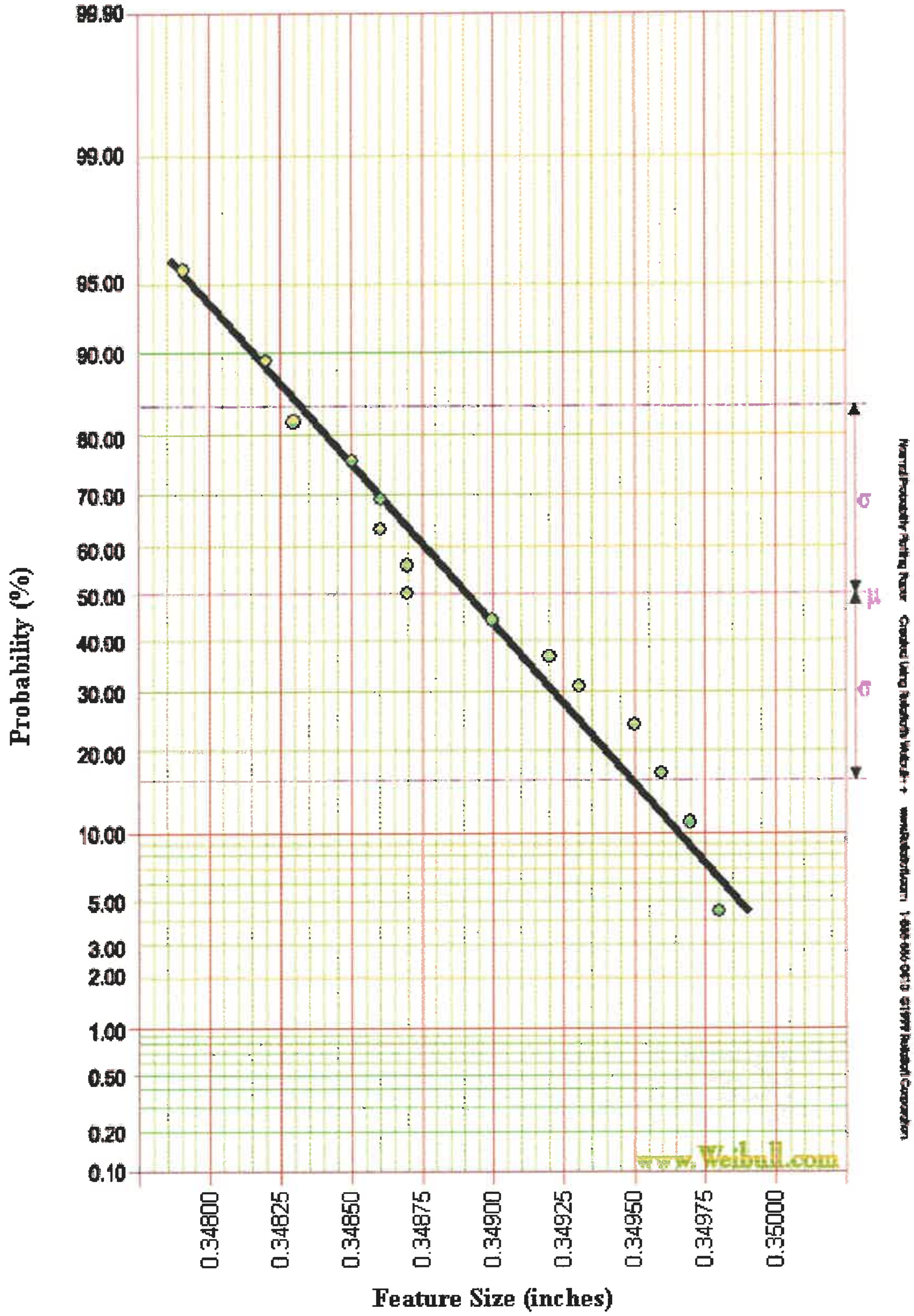


Figure B19. Uniform Probability Plot, Feature Size, MMC (0.35" Vertical).

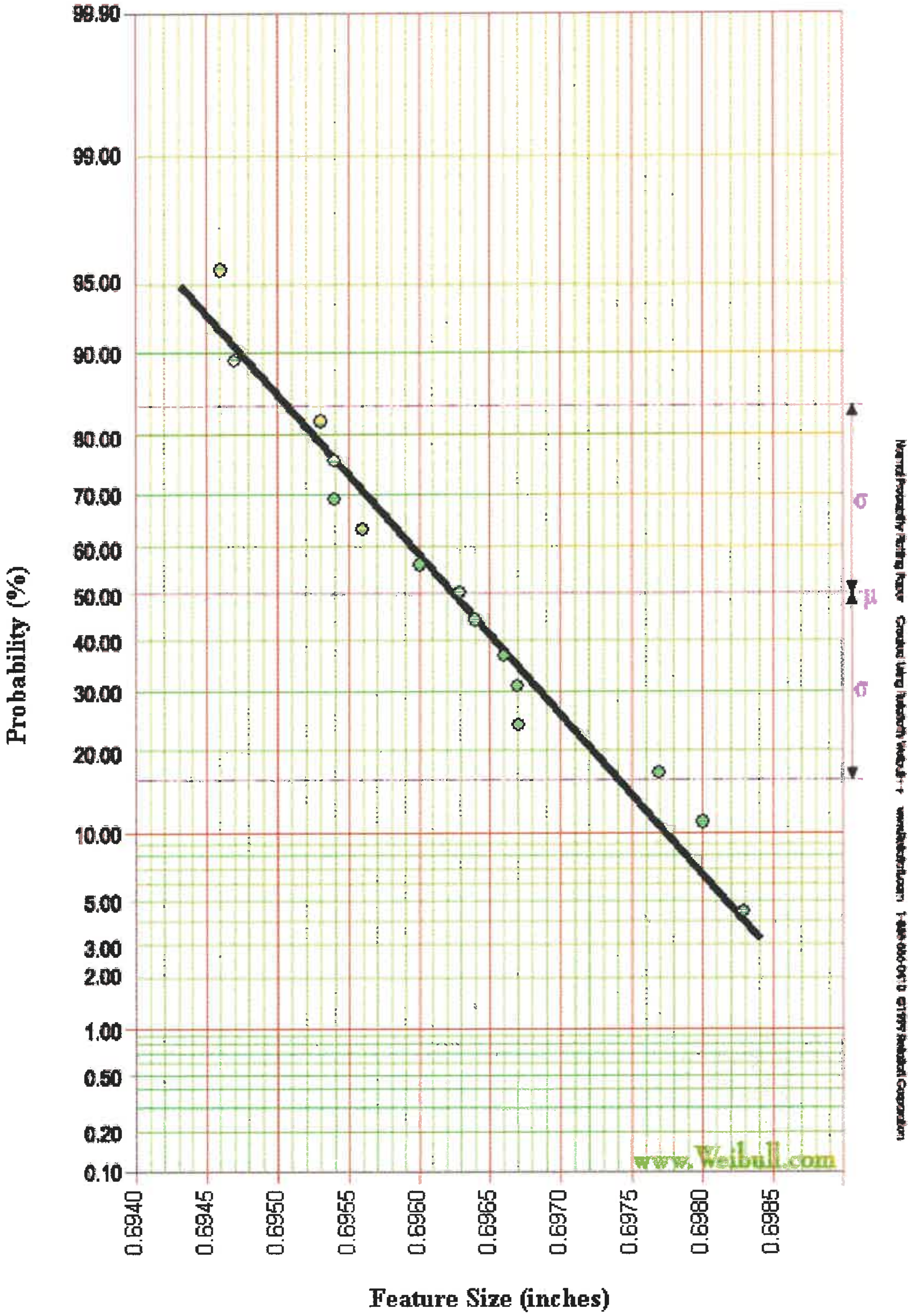


Figure B20. Uniform Probability Plot, Feature Size, MMC (0.7" Vertical).

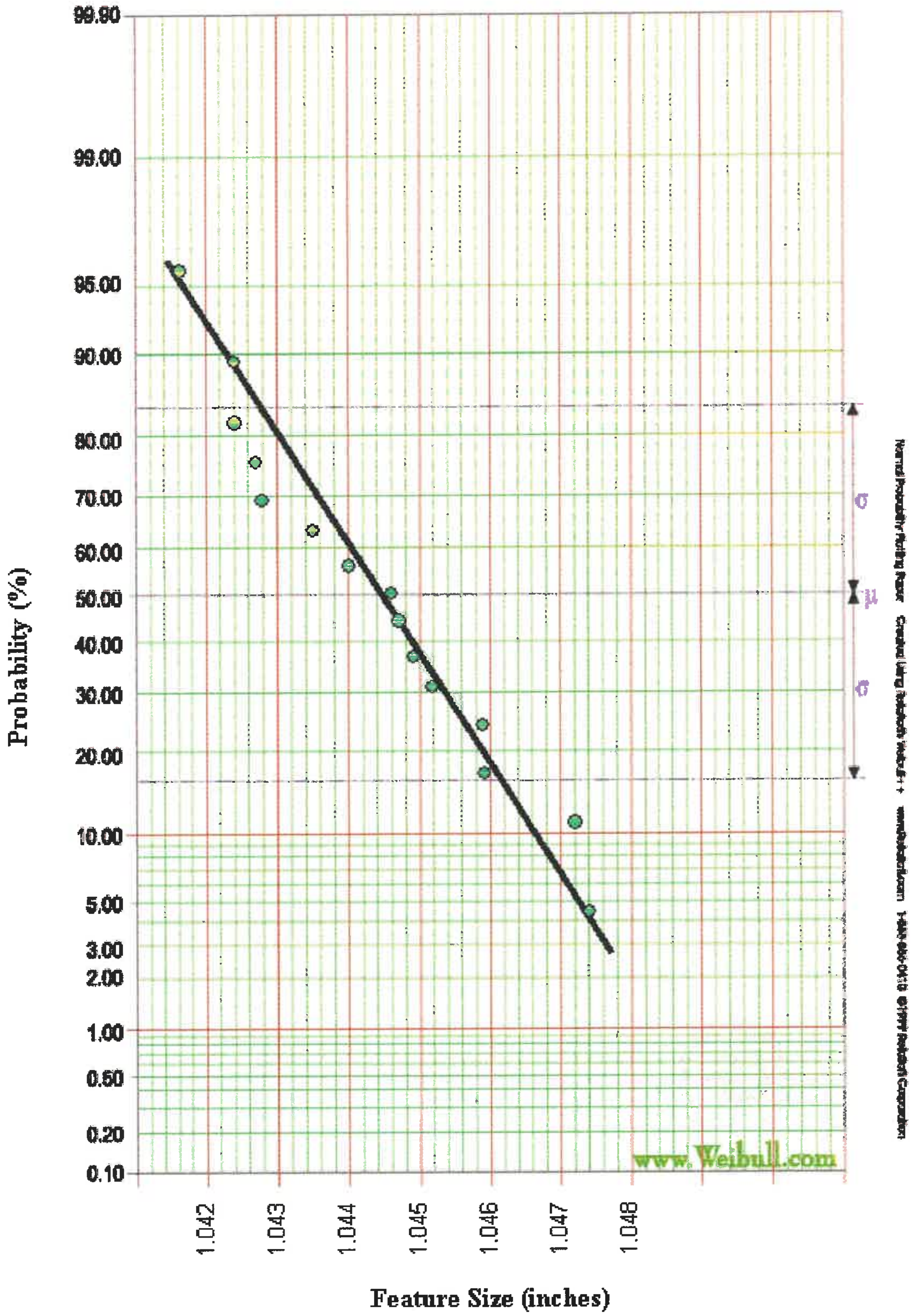


Figure B21. Uniform Probability Plot, Feature Size, MMC (1.05" Vertical).

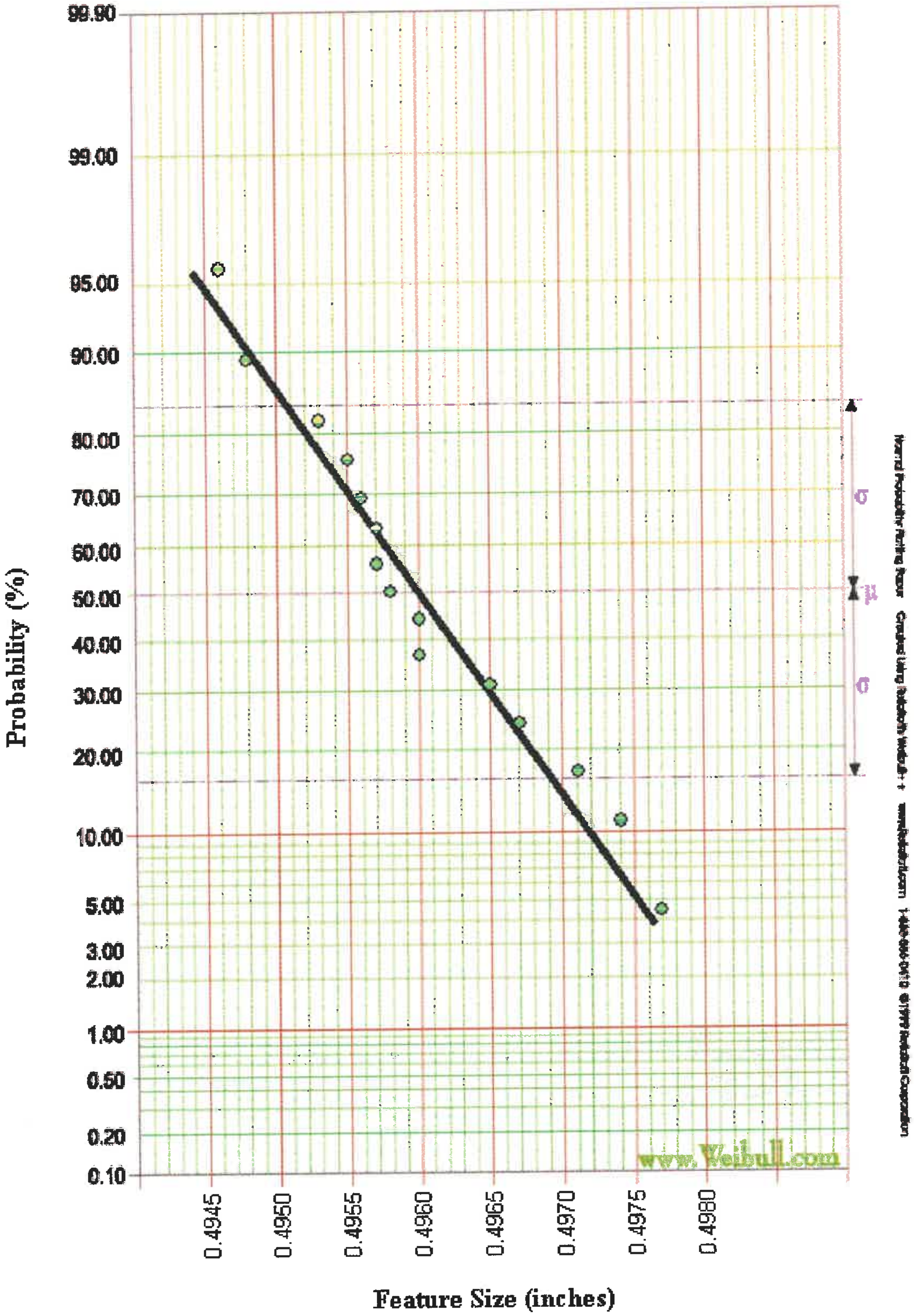


Figure B22. Uniform Probability Plot, Feature Size, MMC (0.5" Horizontal).

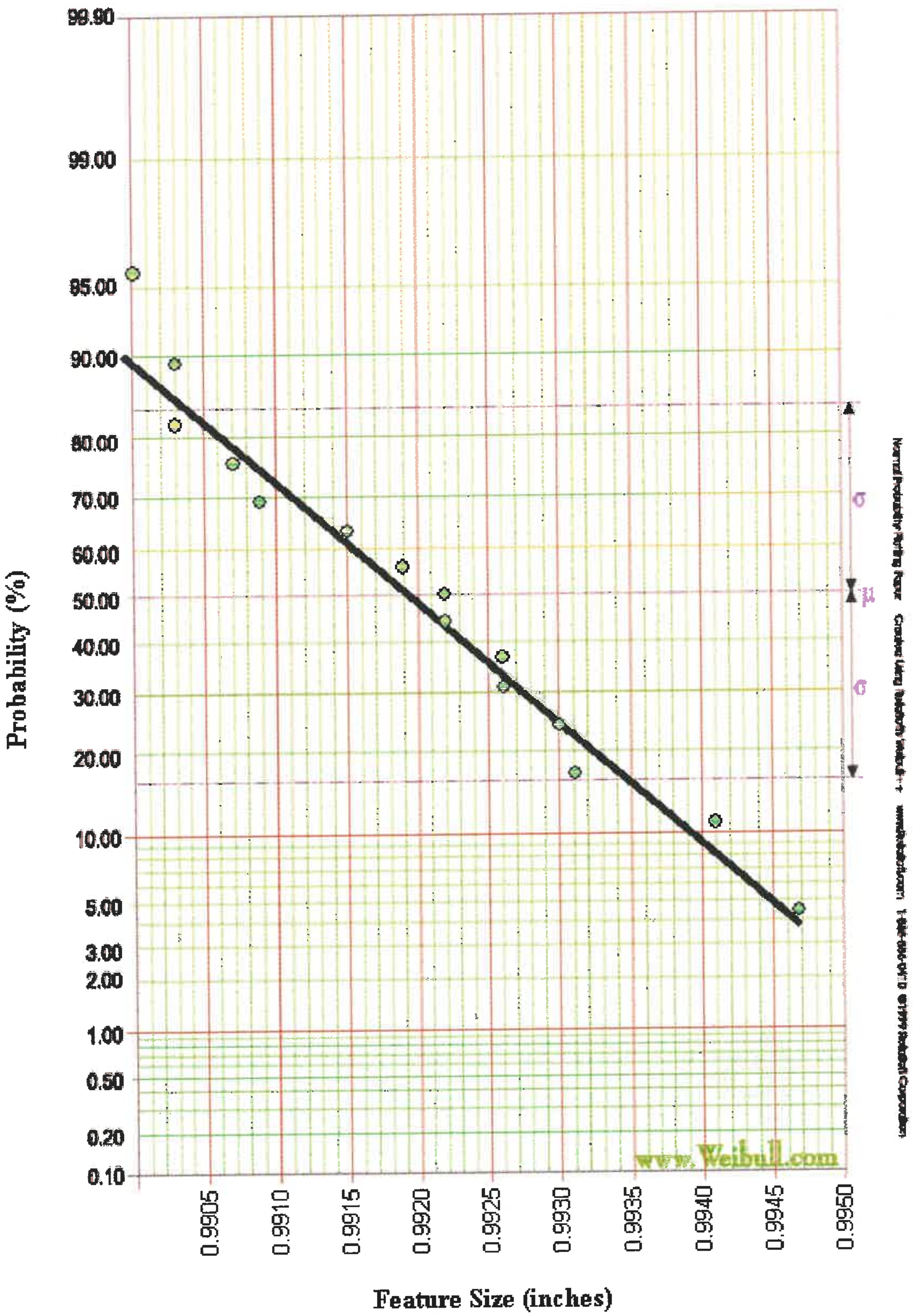


Figure B23. Uniform Probability Plot, Feature Size, MMC (1.0" Horizontal).

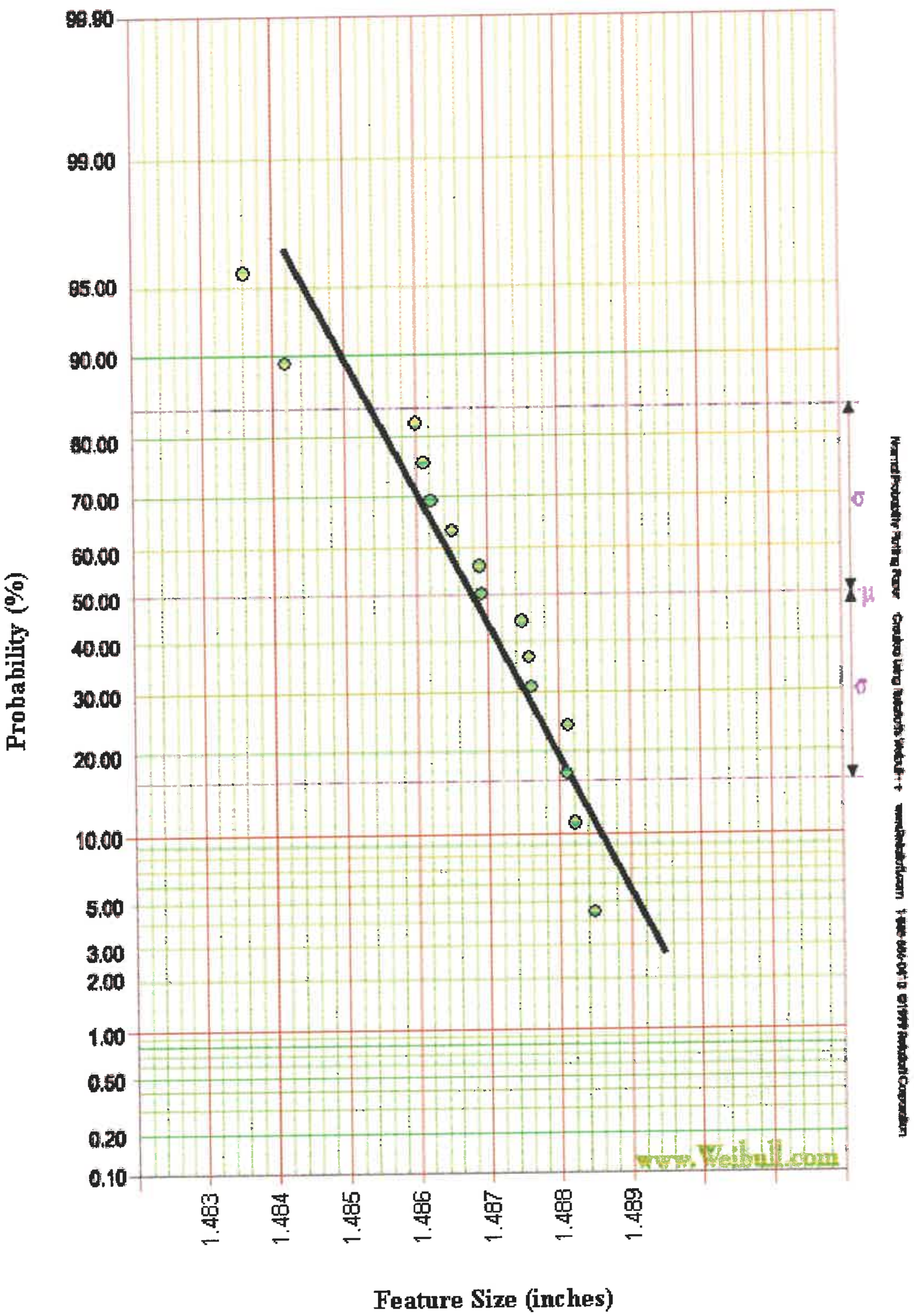
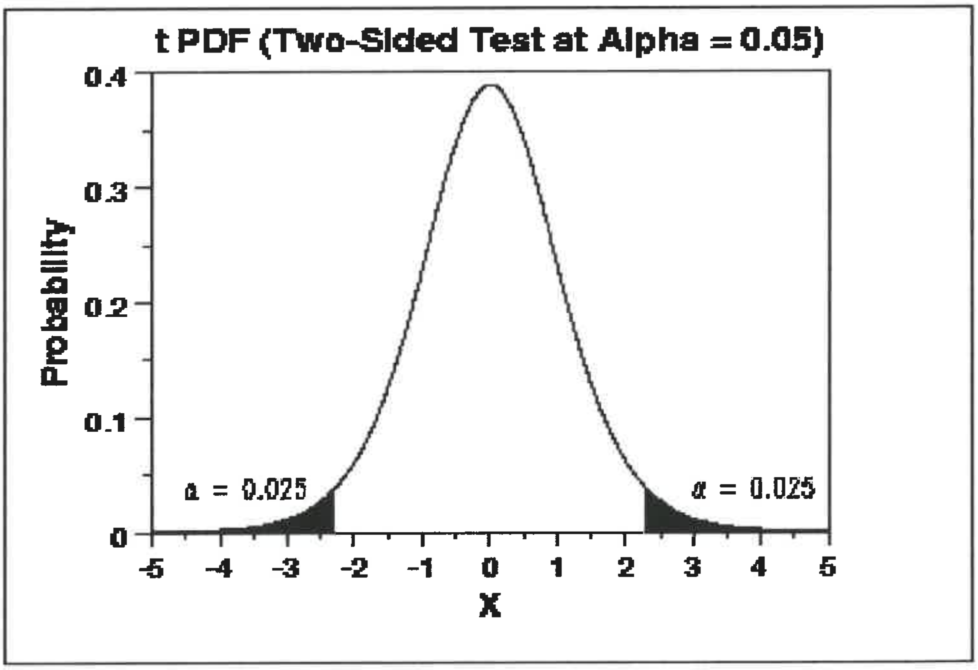


Figure B24. Uniform Probability Plot, Feature Size, MMC (1.5" Horizontal).

Appendix C. Student's t table and F table



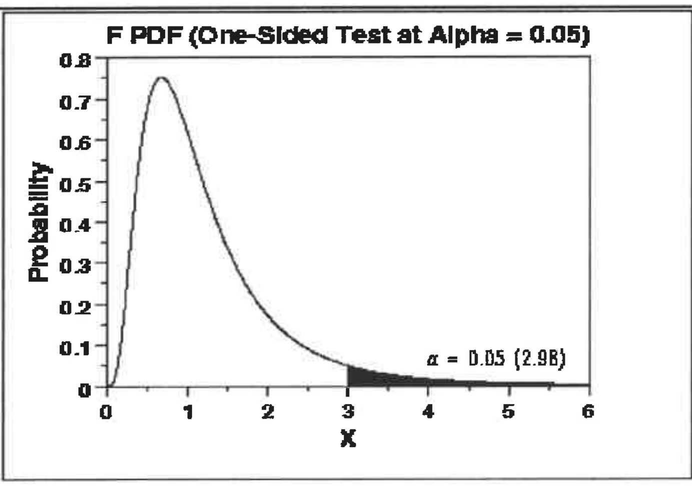
Upper Critical Values of Stuent's t Distribution, v degrees of freedom

Source:National Institute of Standards and Testing

Probability of exceeding the critical value:

v	0.1	0.05	0.025	0.01	0.005	0.001
1	3.078	6.314	12.706	31.821	63.657	318.313
2	1.886	2.92	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
⋮	⋮	⋮	⋮	⋮	⋮	⋮
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.06	2.485	2.787	3.45
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.31	1.697	2.042	2.457	2.75	3.385
40	1.303	1.684	2.021	2.423	2.704	3.307
45	1.301	1.679	2.014	2.412	2.69	3.281
50	1.299	1.676	2.009	2.403	2.678	3.261
60	1.296	1.671	2	2.39	2.66	3.232
75	1.293	1.665	1.992	2.377	2.643	3.202
90	1.291	1.662	1.987	2.368	2.632	3.183
100	1.29	1.66	1.984	2.364	2.626	3.174
Inf.	1.282	1.645	1.96	2.326	2.576	3.09

Appendix C. Student's t table and F table



Upper Critical Values of F Distribution (0.05 level of significance)

Source:National Institute of Standards and Testing

Probability of exceeding the critical value:

n2 (degrees of freedom)

n1 (degrees of freedom) \	1	2	3	4	......	13	14	15	16
1	161.448	199.5	215.707	224.583	......	244.69	245.364	245.95	246.464
2	18.513	19	19.164	19.247	......	19.419	19.424	19.429	19.433
3	10.128	9.552	9.277	9.117	......	8.729	8.715	8.703	8.692
4	7.709	6.944	6.591	6.388	......	5.891	5.873	5.858	5.844
5	6.608	5.786	5.409	5.192	......	4.655	4.636	4.619	4.604
6	5.987	5.143	4.757	4.534	......	3.976	3.956	3.938	3.922
7	5.591	4.737	4.347	4.12	......	3.55	3.529	3.511	3.494
8	5.318	4.459	4.066	3.838	......	3.259	3.237	3.218	3.202
9	5.117	4.256	3.863	3.633	......	3.048	3.025	3.006	2.989
10	4.965	4.103	3.708	3.478	......	2.887	2.865	2.845	2.828
11	4.844	3.982	3.587	3.357	......	2.761	2.739	2.719	2.701
12	4.747	3.885	3.49	3.259	......	2.66	2.637	2.617	2.599
13	4.667	3.806	3.411	3.179	......	2.577	2.554	2.533	2.515
14	4.6	3.739	3.344	3.112	......	2.507	2.484	2.463	2.445
15	4.543	3.682	**3.287**	3.056	......	2.448	2.42	**2.403**	2.385
16	4.494	3.634	3.239	3.007	......	2.397	2.373	2.352	2.333
17	4.451	3.592	3.197	2.965	......	2.353	2.329	2.308	2.289
18	4.414	3.555	3.16	2.928	......	2.314	2.29	2.269	2.25
19	4.381	3.522	3.127	2.895	......	2.28	2.256	2.234	2.215
20	4.351	3.493	3.098	2.866	......	2.25	2.225	2.203	2.184
25	4.242	3.385	2.991	2.759	......	2.136	2.111	2.089	2.069

Appendix D. Shrinkages

Unfired Ceramic --> Fired Ceramic (relative shrinkage)

		a	b	c	d	e		
Vertical	0.3500	-0.26%	-0.33%	-0.36%	-0.32%	-0.29%		
		-0.33%	-0.35%	-0.33%	-0.31%	-0.30%	Average Relative Shrinkage=	-0.0032
		-0.34%	-0.26%	-0.38%	-0.33%	-0.27%	Standard Deviation=	0.0003
	0.7000	-0.27%	-0.36%	-0.35%	-0.33%	-0.29%		
		-0.33%	-0.34%	-0.34%	-0.27%	-0.30%	Average Relative Shrinkage=	-0.0032
		-0.34%	-0.27%	-0.35%	-0.31%	-0.33%	Standard Deviation=	0.0003
	1.0500	-0.28%	-0.34%	-0.34%	-0.32%	-0.31%		
		-0.34%	-0.32%	-0.34%	-0.30%	-0.28%	Average Relative Shrinkage=	-0.0032
		-0.33%	-0.31%	-0.35%	-0.28%	-0.32%	Standard Deviation=	0.0002

Unfired Ceramic --> Fired Ceramic (relative shrinkage)

		a	b	c	d	e		
Horizontal	0.5000	-0.09%	-0.21%	-0.12%	-0.15%	-0.14%		
		-0.12%	-0.07%	-0.13%	-0.14%	-0.09%	Average Relative Shrinkage=	-0.0013
		-0.17%	-0.01%	-0.14%	-0.19%	-0.16%	Standard Deviation=	0.0005
	1.0000	-0.19%	-0.23%	-0.21%	-0.18%	-0.17%		
		-0.24%	-0.17%	-0.22%	-0.21%	-0.18%	Average Relative Shrinkage=	-0.0021
		-0.28%	-0.18%	-0.18%	-0.22%	-0.21%	Standard Deviation=	0.0003
	1.5000	-0.25%	-0.27%	-0.26%	-0.21%	-0.22%		
		-0.26%	-0.21%	-0.28%	-0.23%	-0.21%	Average Relative Shrinkage=	-0.0024
		-0.29%	-0.21%	-0.23%	-0.20%	-0.25%	Standard Deviation=	0.0003

Mold-Master-->Unfired Ceramic (relative shrinkage)

Fired Ceramic-->Metal Matrix Composite (relative shrinkage)

		a	b	c	d	e		
Vertical	0.3500	-0.13%	-0.62%	-0.44%	-0.45%	-0.41%		
		-0.52%	-0.42%	-0.34%	-0.38%	-0.28%	Average Relative Shrinkage=	-0.0038
		-0.48%	-0.13%	-0.60%	-0.17%	-0.55%	Standard Deviation=	0.0008
	0.7000	-0.26%	-0.77%	-0.52%	-0.67%	-0.54%		
		-0.61%	-0.55%	-0.47%	-0.64%	-0.27%	Average Relative Shrinkage=	-0.0053
		-0.49%	-0.36%	-0.70%	-0.49%	-0.80%	Standard Deviation=	0.0010
	1.0500	-0.30%	-0.76%	-0.54%	-0.71%	-0.52%		
		-0.68%	-0.56%	-0.49%	-0.80%	-0.35%	Average Relative Shrinkage=	-0.0062
		-0.56%	-0.45%	-0.78%	-0.68%	-0.86%	Standard Deviation=	0.0008

Fired Ceramic-->Metal Matrix Composite (relative shrinkage)

		a	b	c	d	e		
Horizontal	0.5000	-0.66%	-0.79%	-0.86%	-0.73%	-0.70%		
		-0.77%	-0.78%	-0.73%	-0.71%	-0.67%	Average Relative Shrinkage=	-0.0073
		-0.69%	-0.75%	-0.79%	-0.68%	-0.70%	Standard Deviation=	0.0005
	1.0000	-0.71%	-0.78%	-0.80%	-0.77%	-0.75%		
		-0.75%	-0.70%	-0.73%	-0.69%	-0.70%	Average Relative Shrinkage=	-0.0074
		-0.71%	-0.74%	-0.81%	-0.68%	-0.79%	Standard Deviation=	0.0004
	1.5000	-0.70%	-0.79%	-0.80%	-0.73%	-0.74%		
		-0.74%	-0.74%	-0.71%	-0.72%	-0.74%	Average Relative Shrinkage=	-0.0074
		-0.73%	-0.77%	-0.73%	-0.73%	-0.77%	Standard Deviation=	0.0003

Appendix E. Uniform Probability Plots of Shrinkage

Probability Plots For Shrinkage For Each Feature Size Were Prepared And Plots Follow

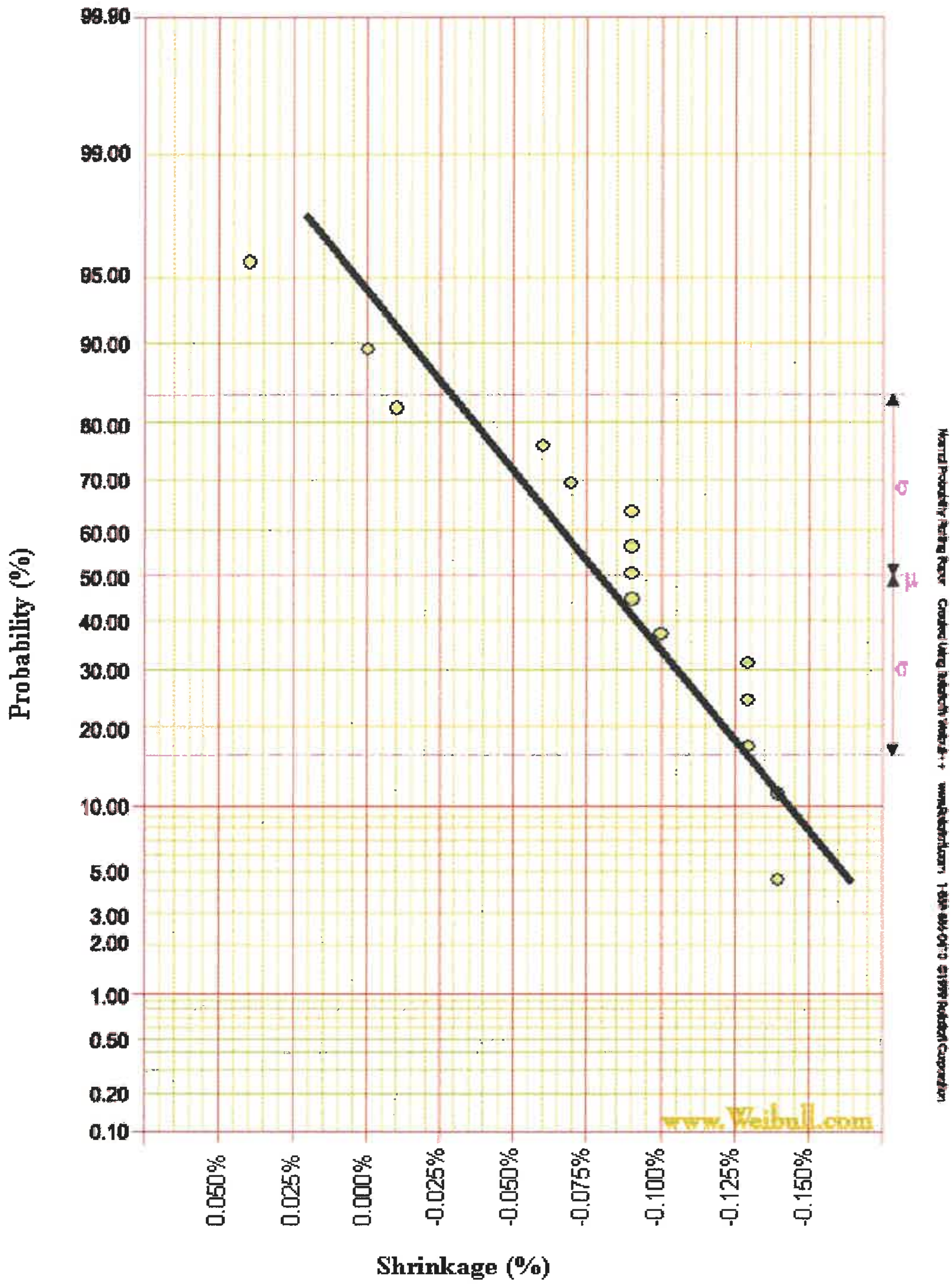


Figure E1. Uniform Probability Plot, Shrinkage, Master Pattern To Mold Master (0.35" Vertical).

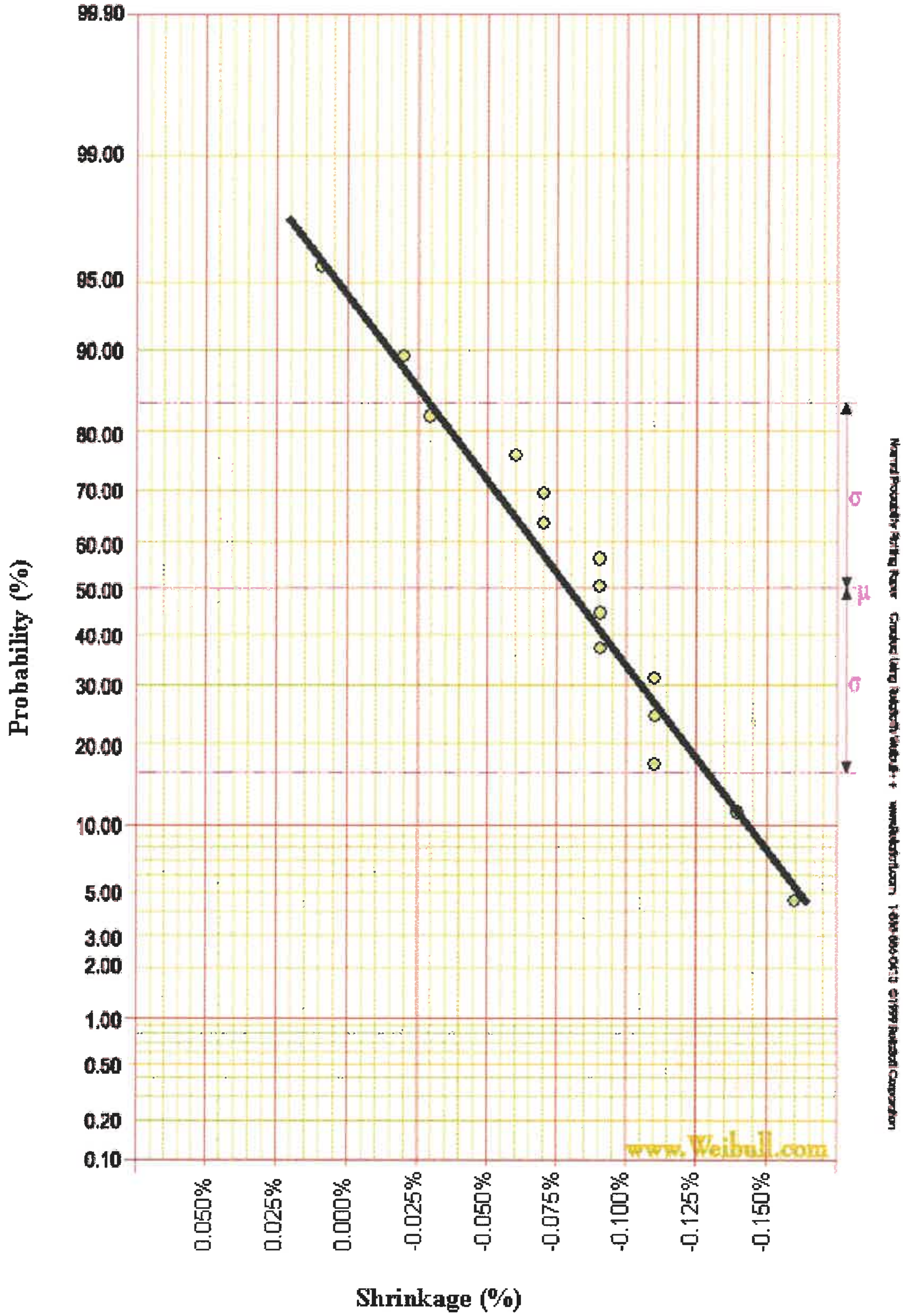


Figure E2. Uniform Probability Plot, Shrinkage, Master Pattern To Mold Master (0.7" Vertical).

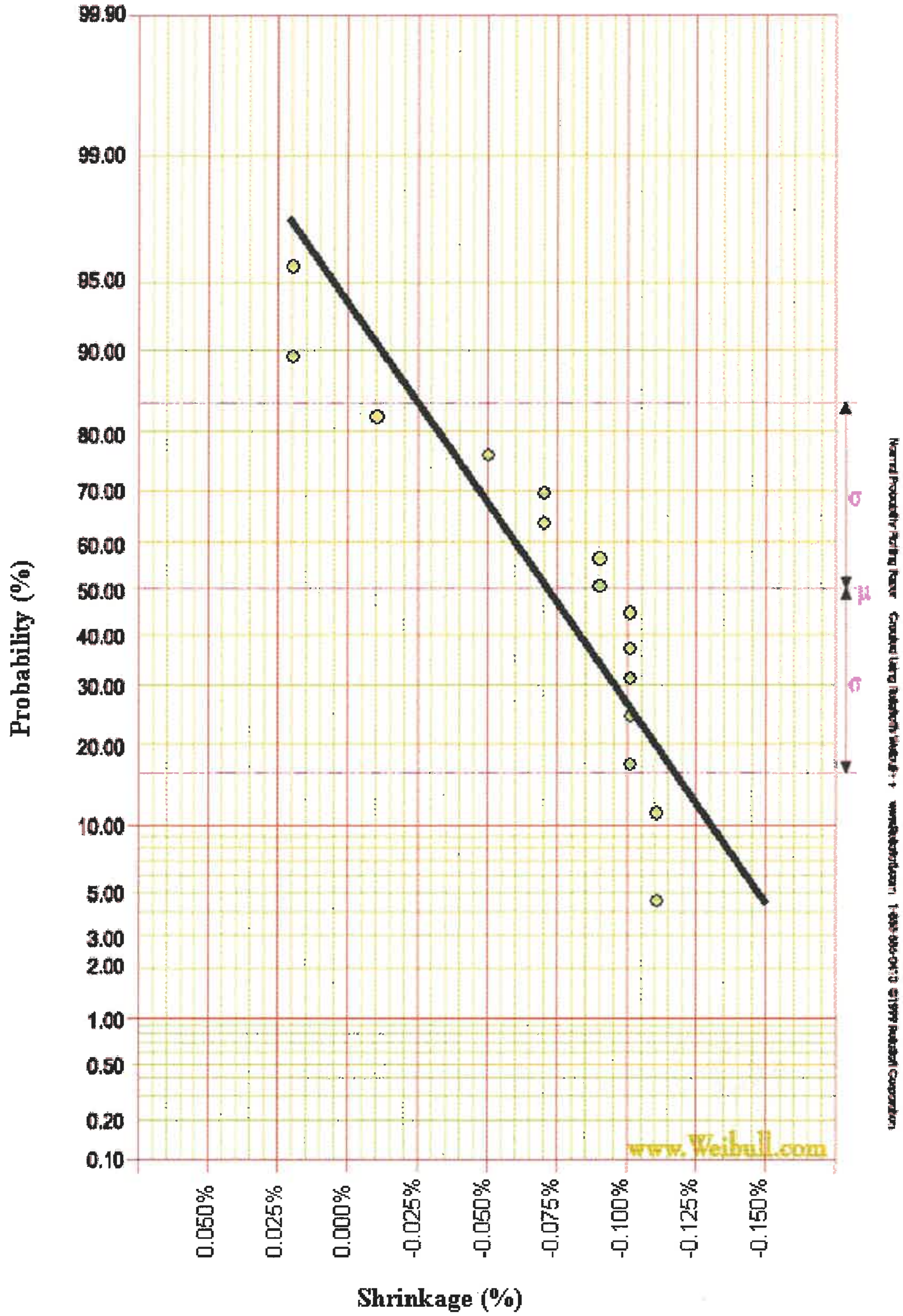


Figure E3. Uniform Probability Plot, Shrinkage, Master Pattern To Mold Master (1.05" Vertical).

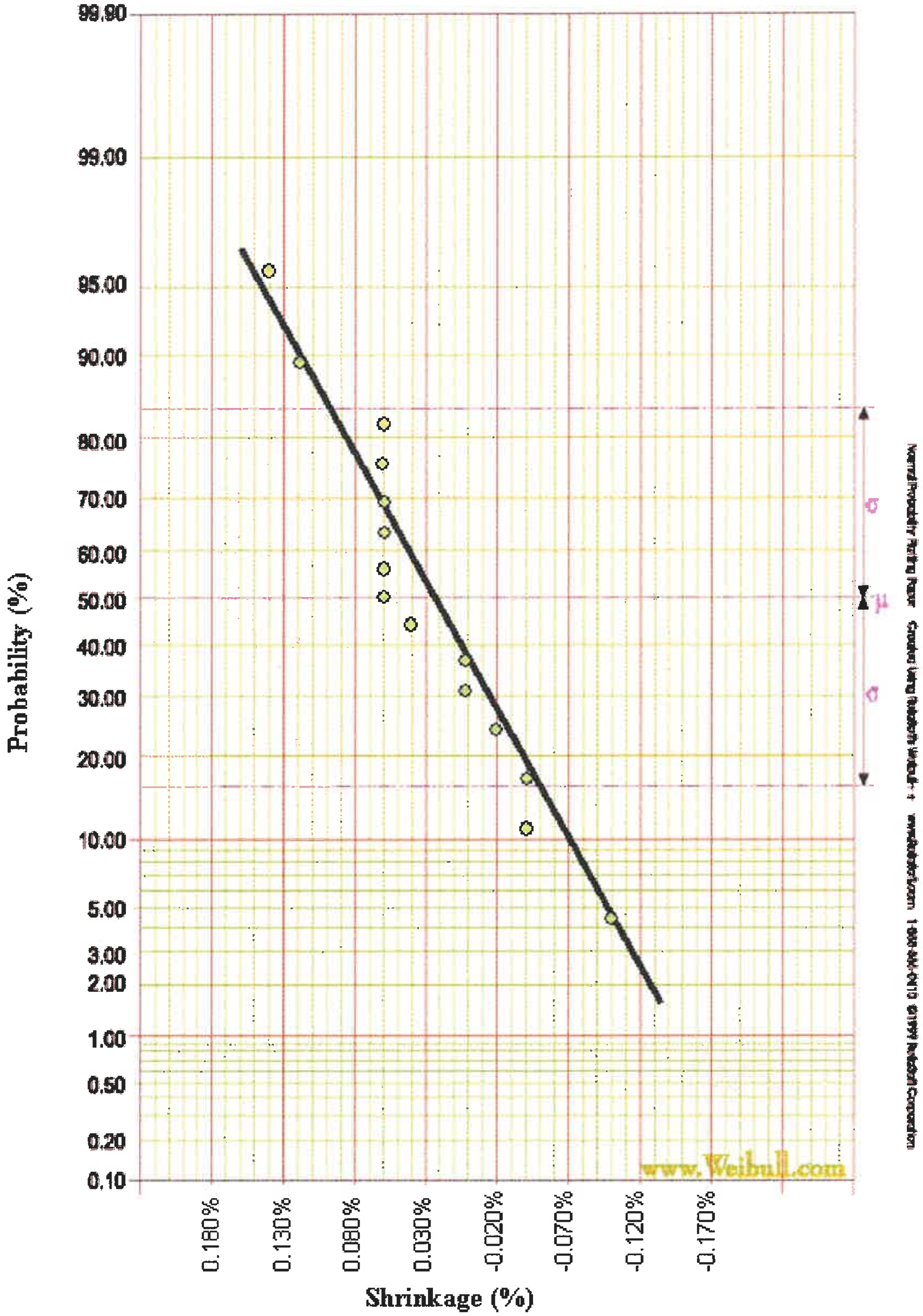


Figure E4. Uniform Probability Plot, Shrinkage, Master Pattern To Mold Master (0.5" Horizontal).

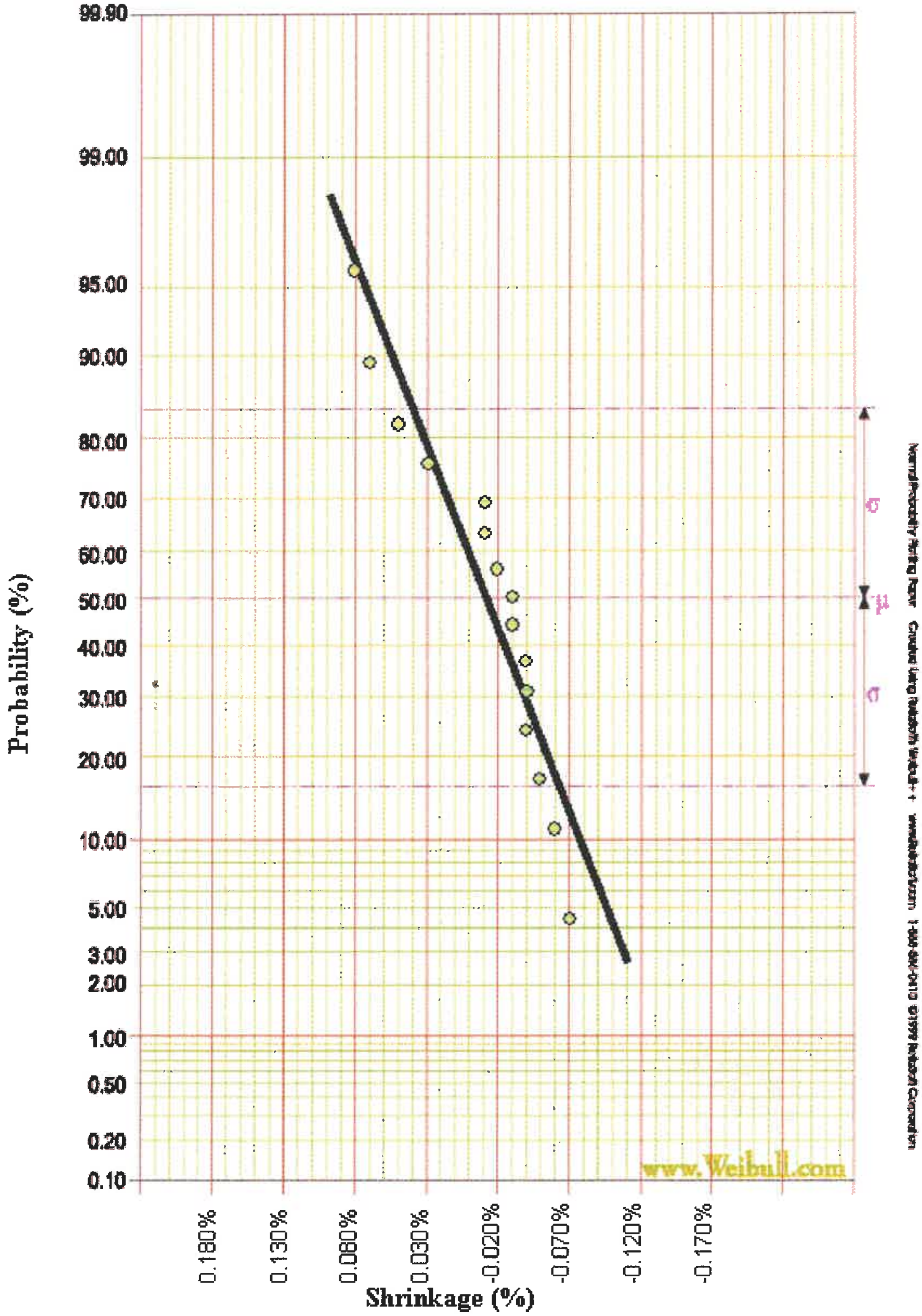


Figure E5. Uniform Probability Plot, Shrinkage, Master Pattern To Mold Master (1.0" Horizontal).

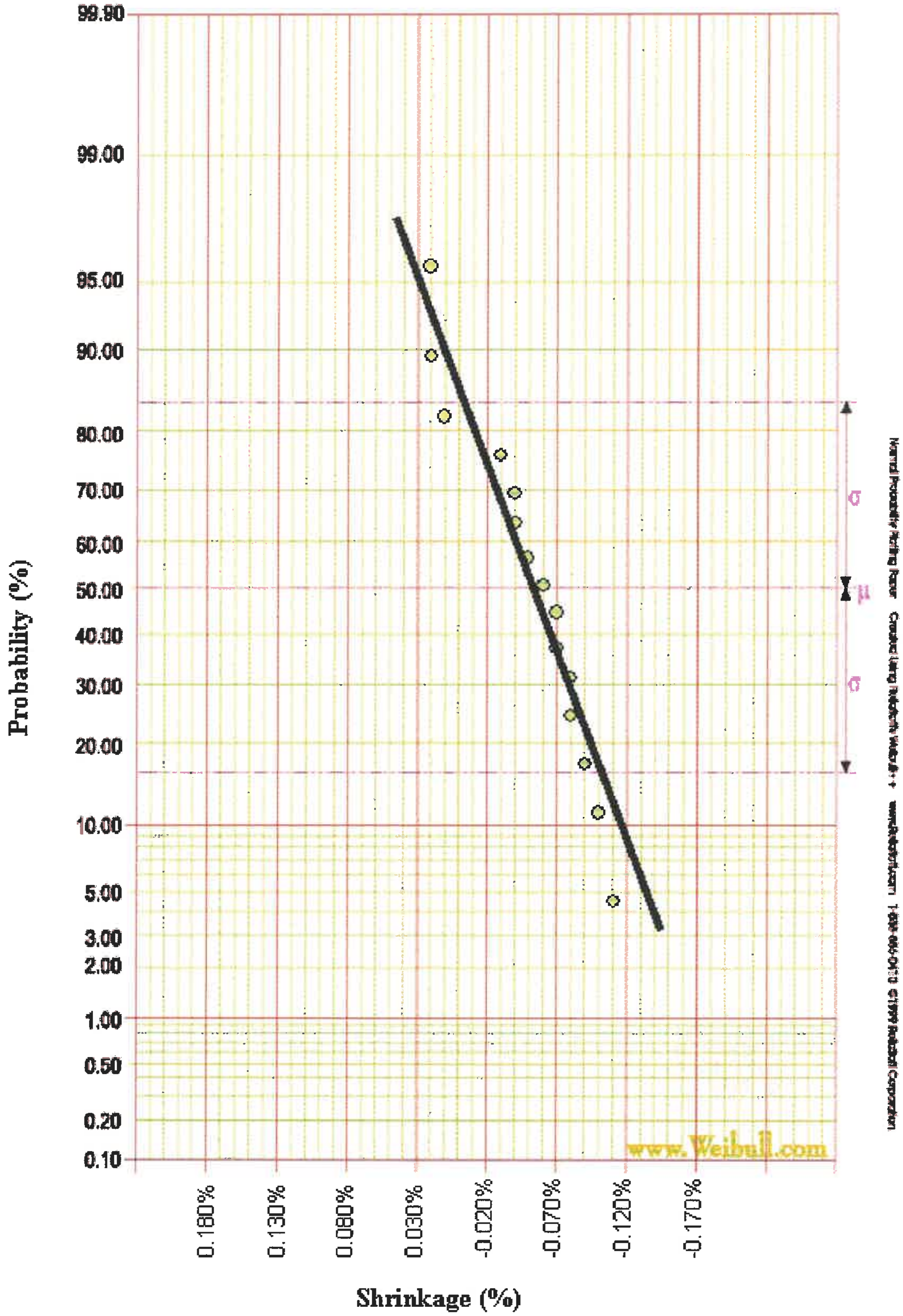


Figure E6. Uniform Probability Plot, Shrinkage, Master Pattern To Mold Master (1.5" Horizontal).

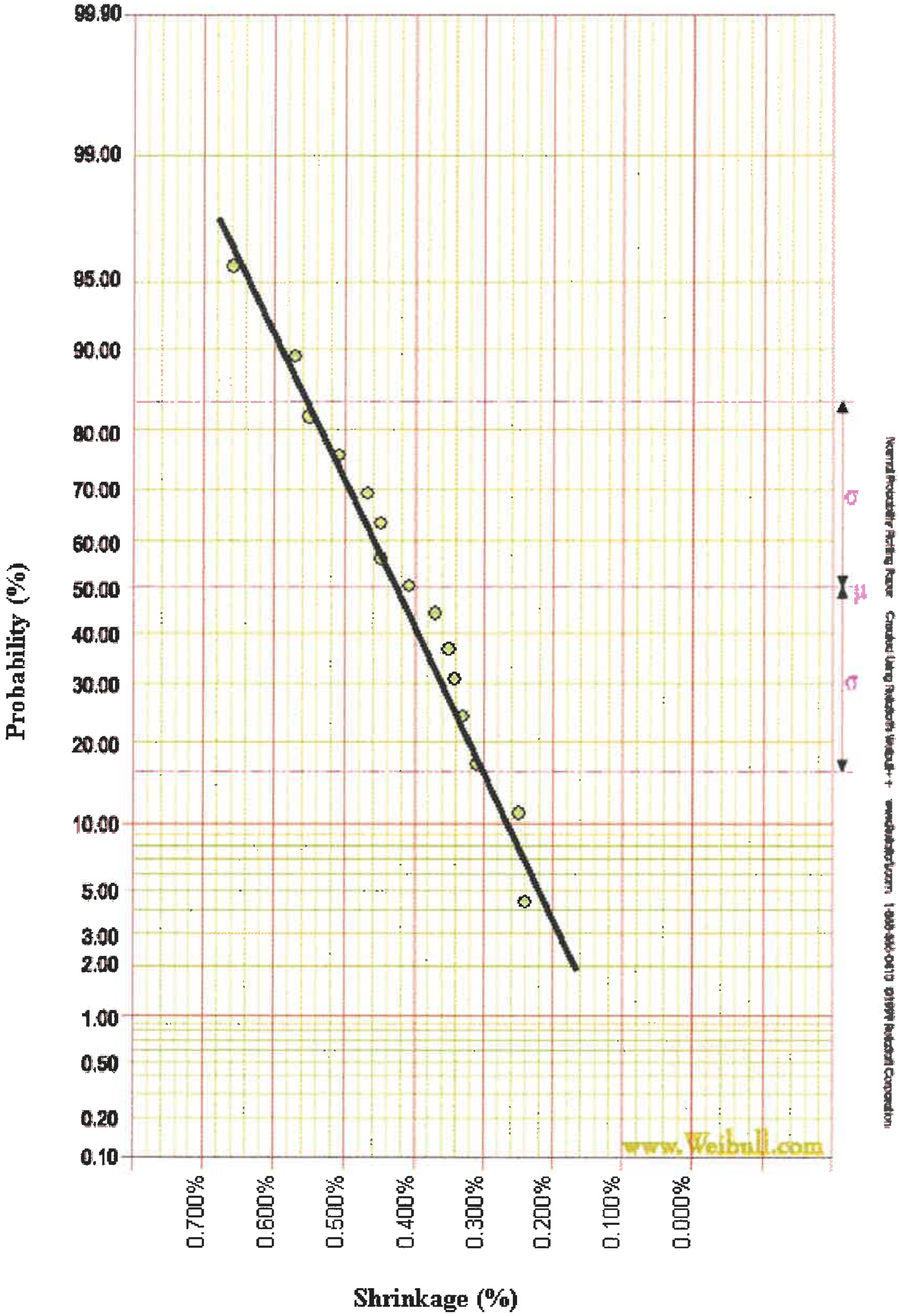


Figure E7. Uniform Probability Plot, Shrinkage, Mold Master To Ceramic (0.35" Vertical).

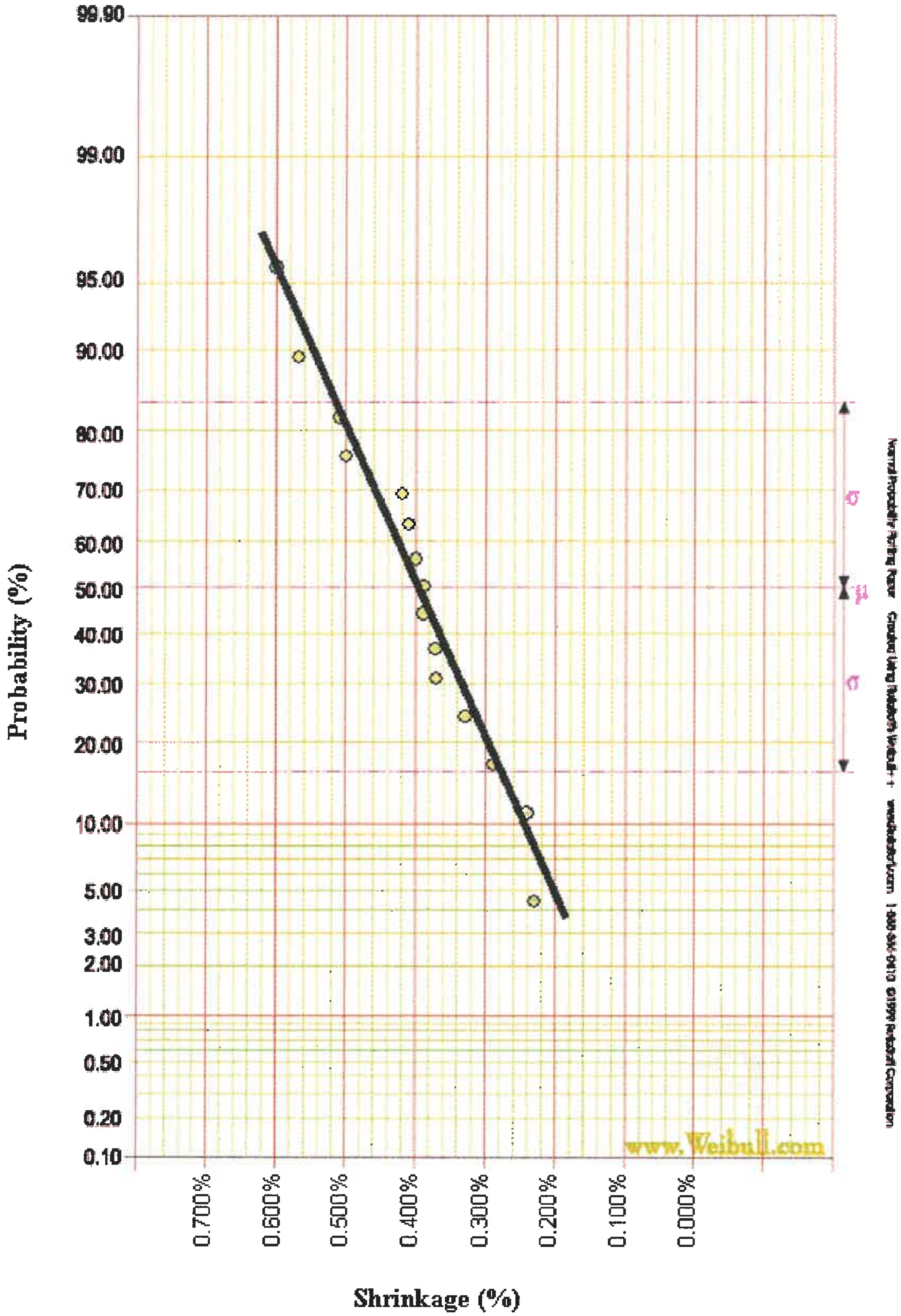


Figure E8. Uniform Probability Plot, Shrinkage, Mold Master To Ceramic (0.7" Vertical).

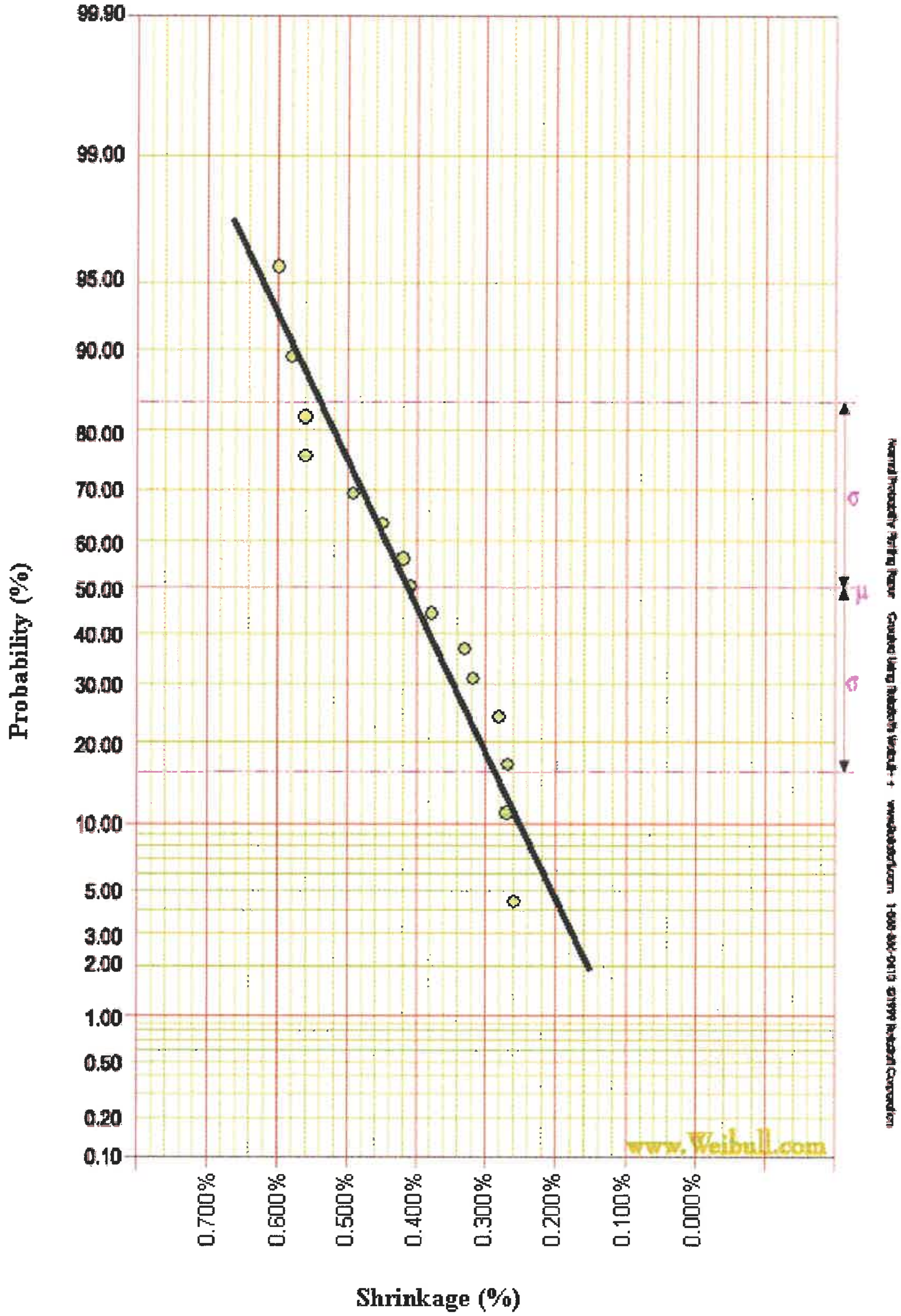


Figure E9. Uniform Probability Plot, Shrinkage, Mold Master To Ceramic (1.05" Vertical).

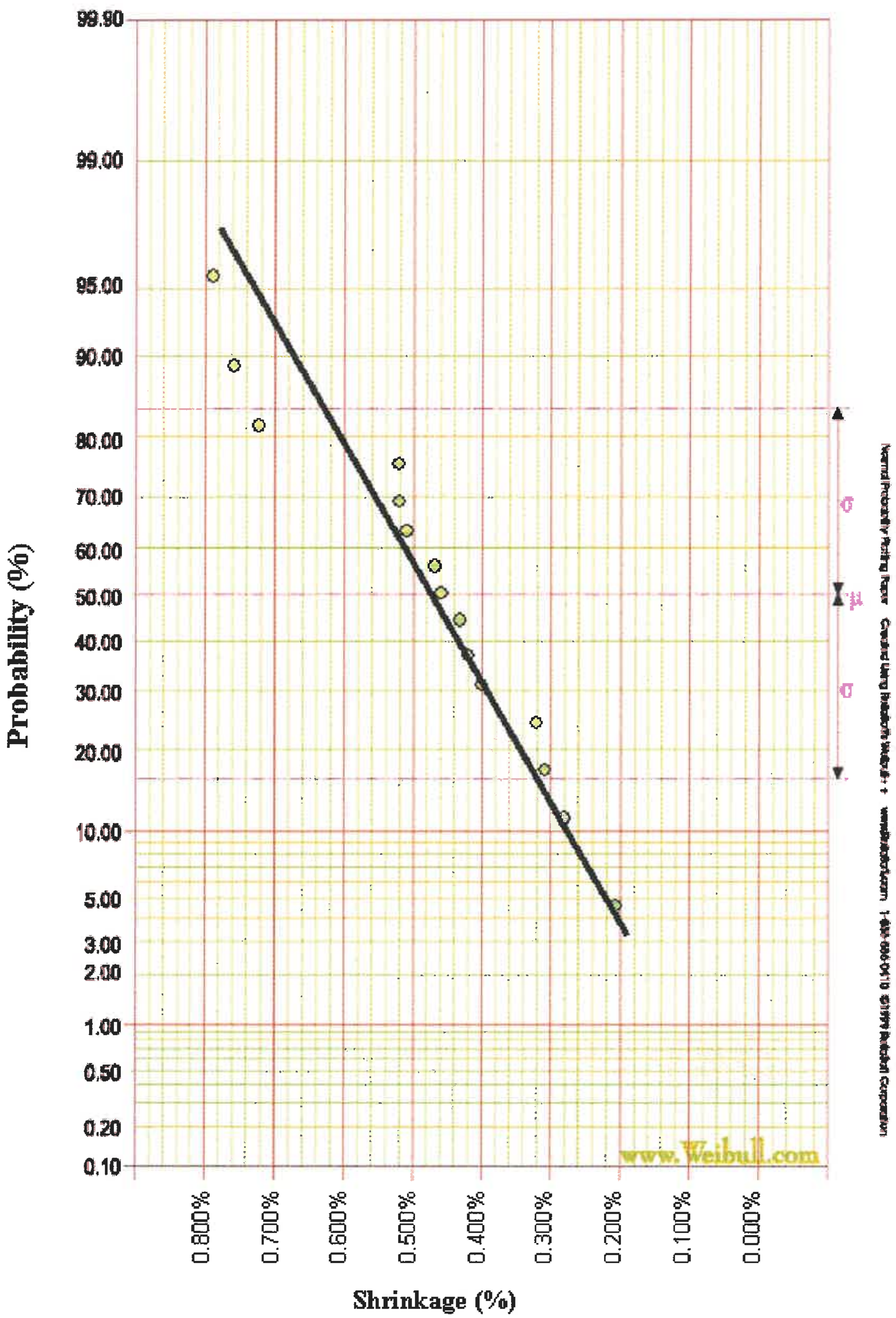


Figure E10. Uniform Probability Plot, Shrinkage, Mold Master To Ceramic (0.5" Horizontal).

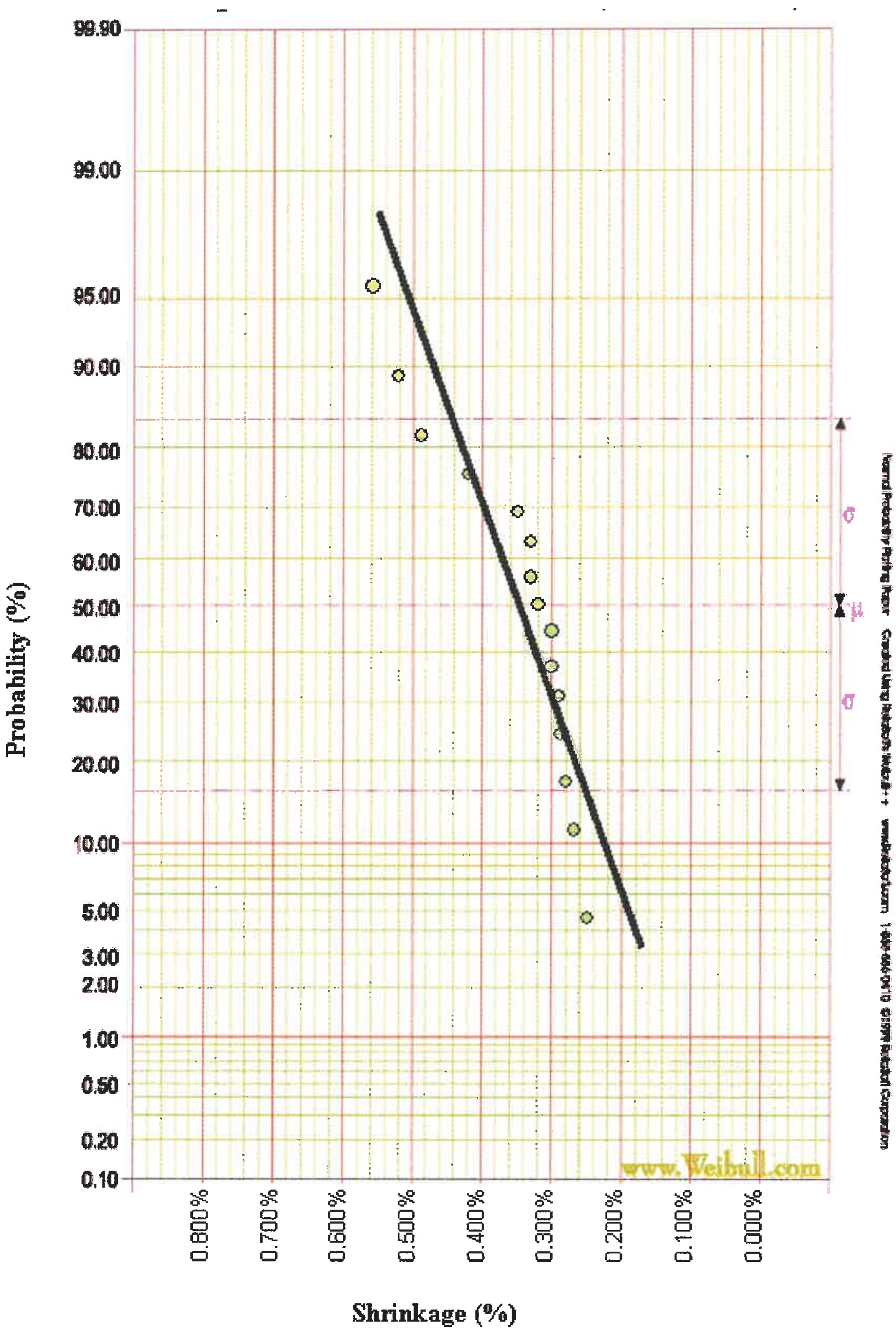


Figure E11. Uniform Probability Plot, Shrinkage, Mold Master To Ceramic (1.0" Horizontal).

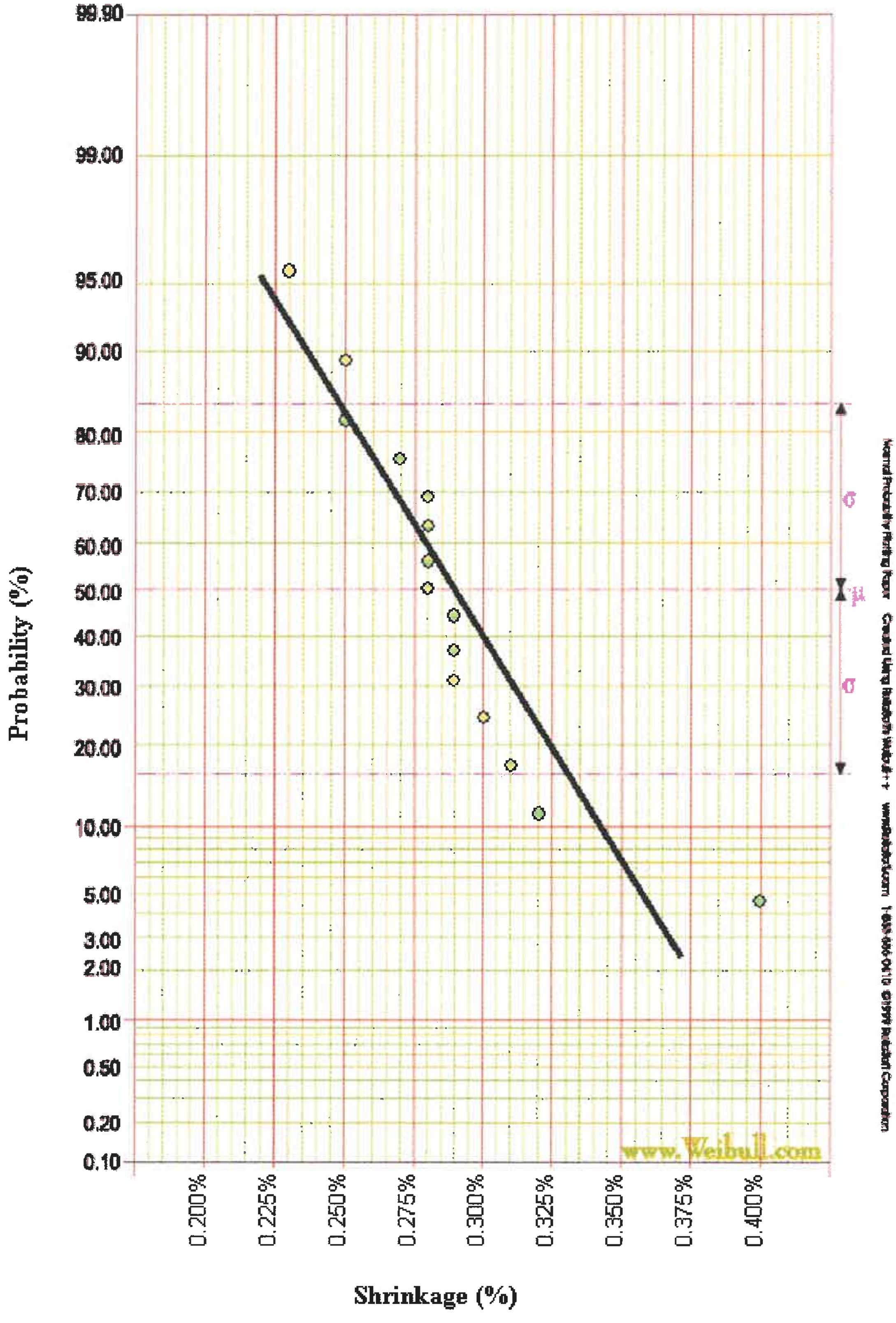


Figure E12. Uniform Probability Plot, Shrinkage, Mold Master To Ceramic (1.5" Horizontal).

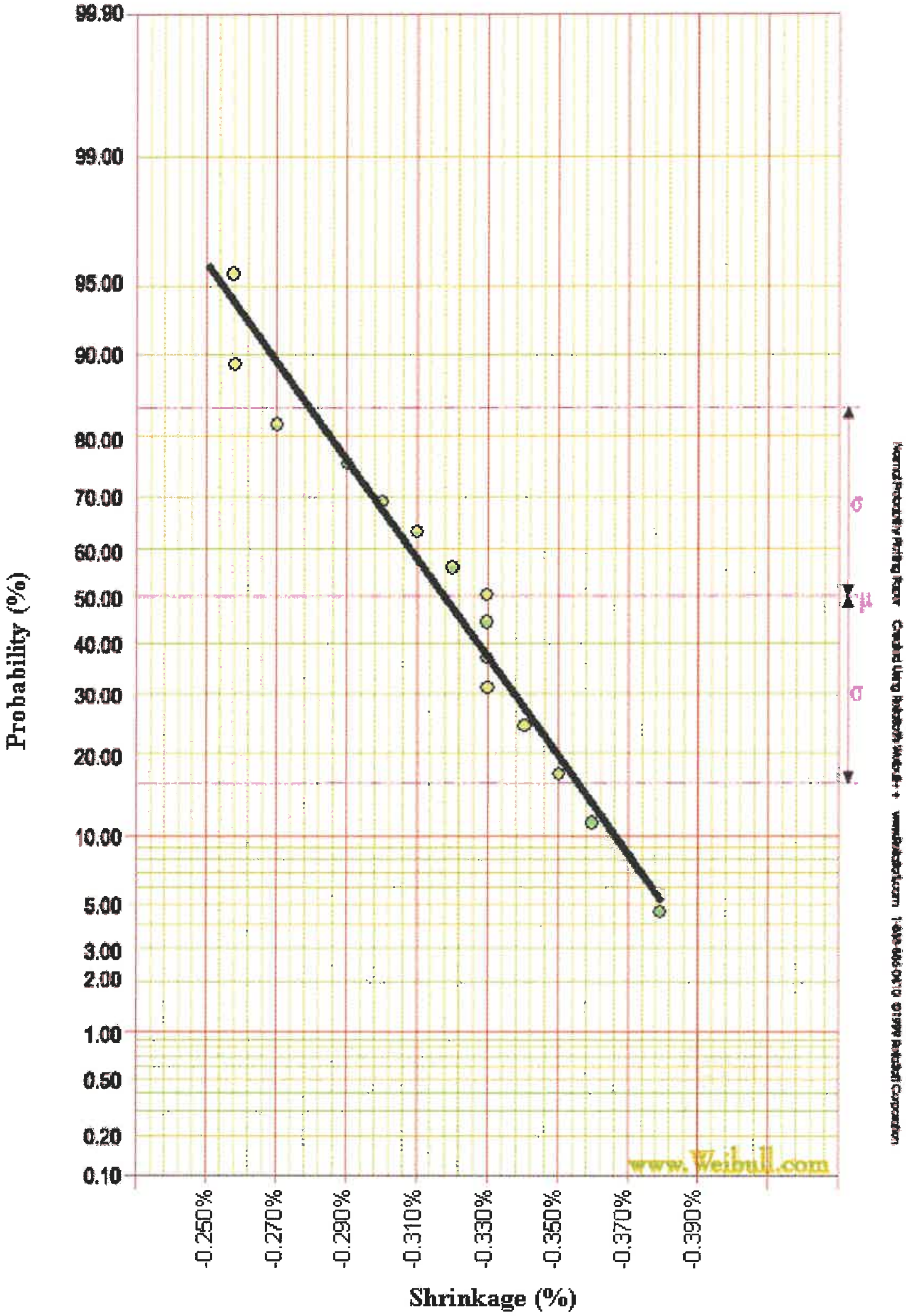


Figure E13. Uniform Probability Plot, Shrinkage, Ceramic To Fired Ceramic (0.35" Vertical).

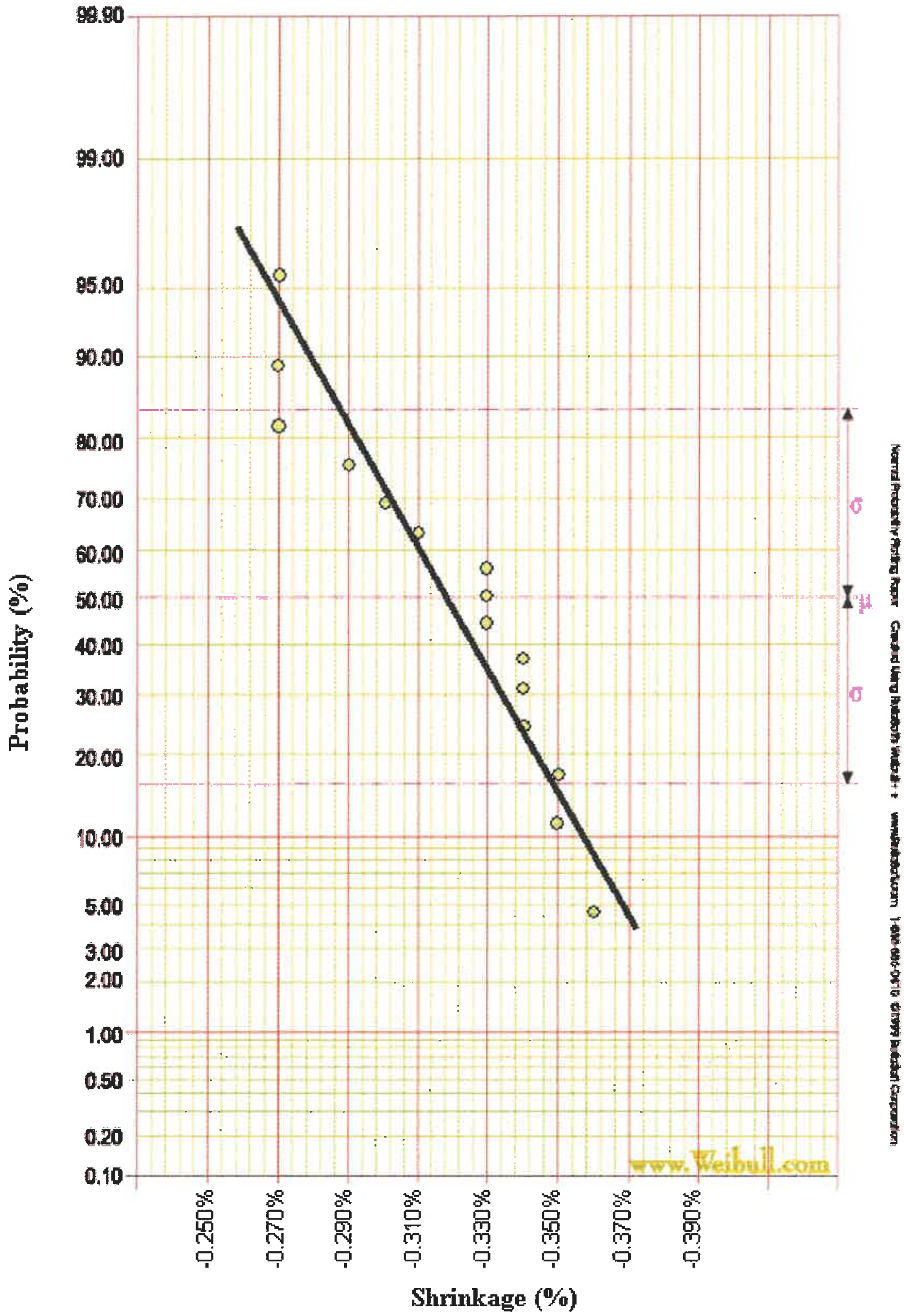


Figure E14. Uniform Probability Plot, Shrinkage, Ceramic To Fired Ceramic (0.7" Vertical).

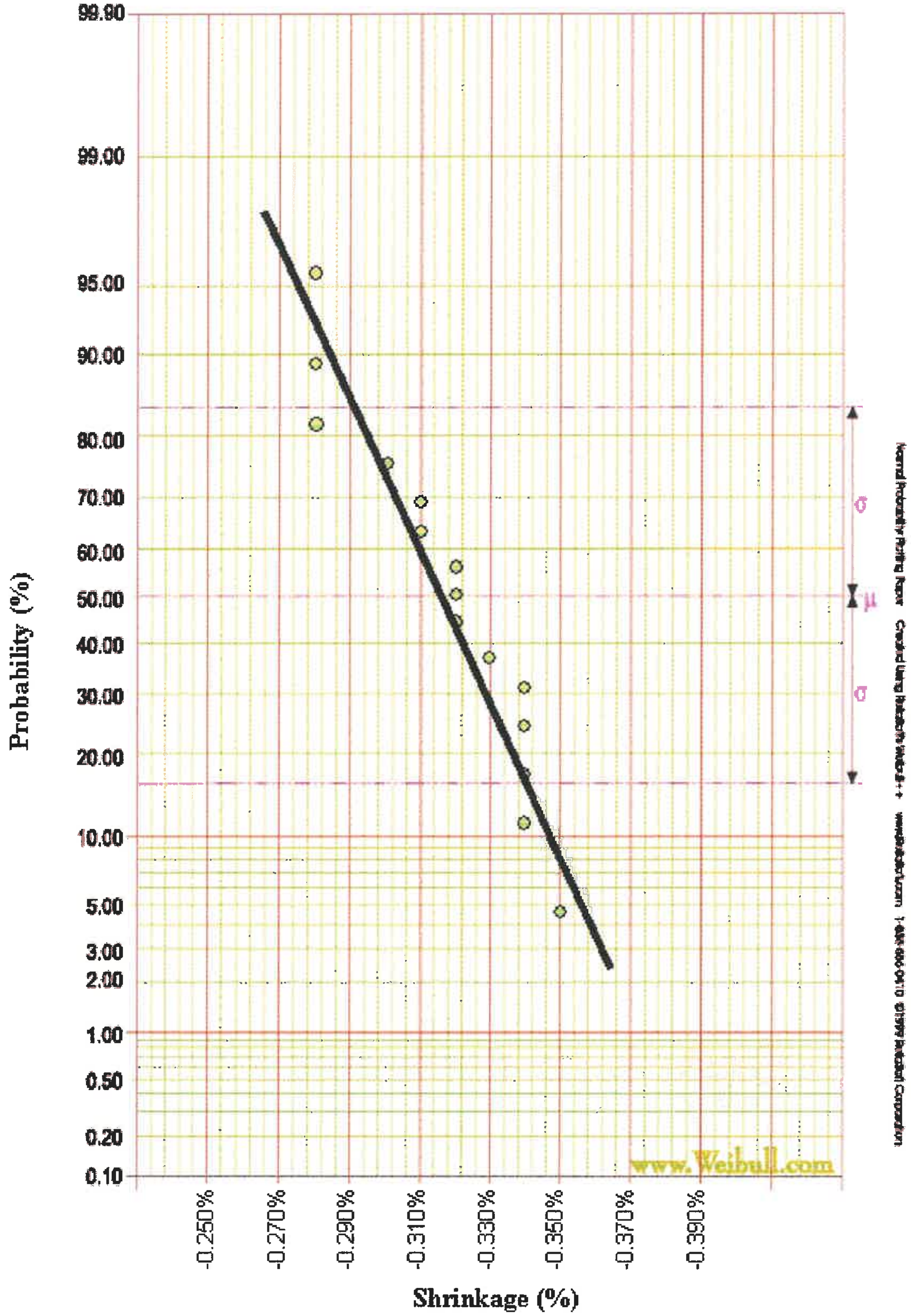


Figure E15. Uniform Probability Plot, Shrinkage, Ceramic To Fired Ceramic (1.05" Vertical).

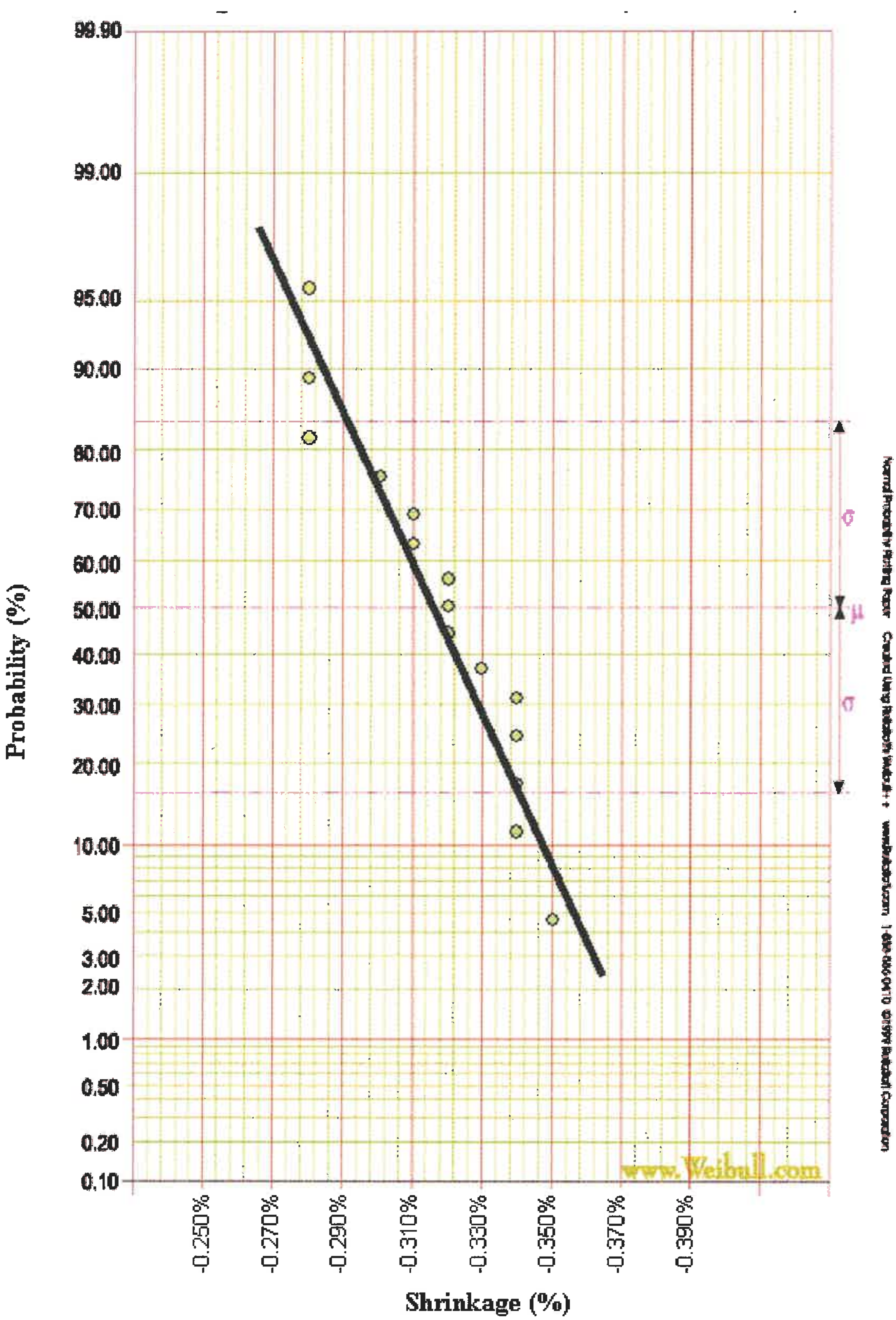


Figure E16. Uniform Probability Plot, Shrinkage, Ceramic To Fired Ceramic (0.5" Horizontal).

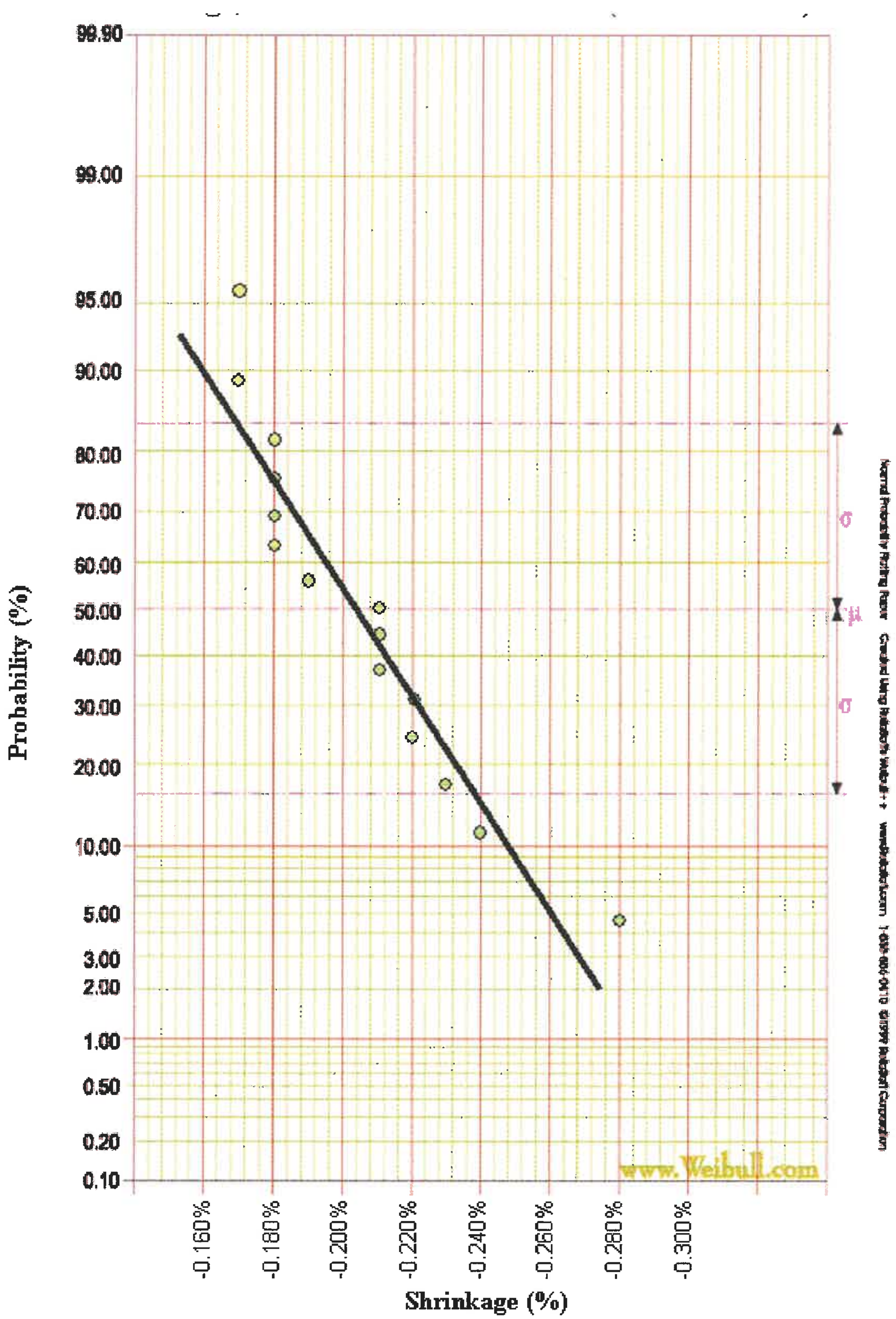


Figure E17. Uniform Probability Plot, Shrinkage, Ceramic To Fired Ceramic (1.0" Horizontal).

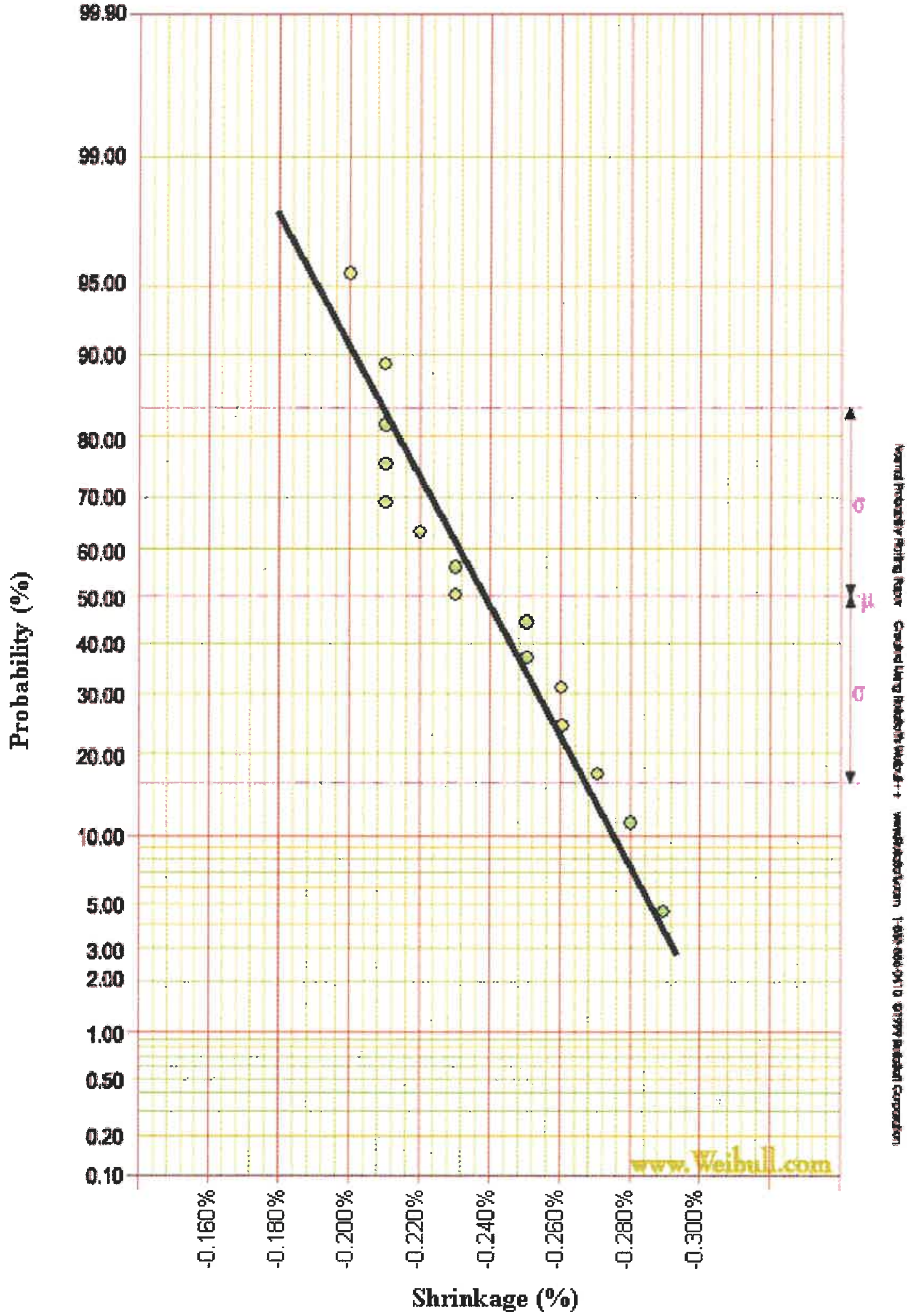


Figure E18. Uniform Probability Plot, Shrinkage, Ceramic To Fired Ceramic (1.5" Horizontal).

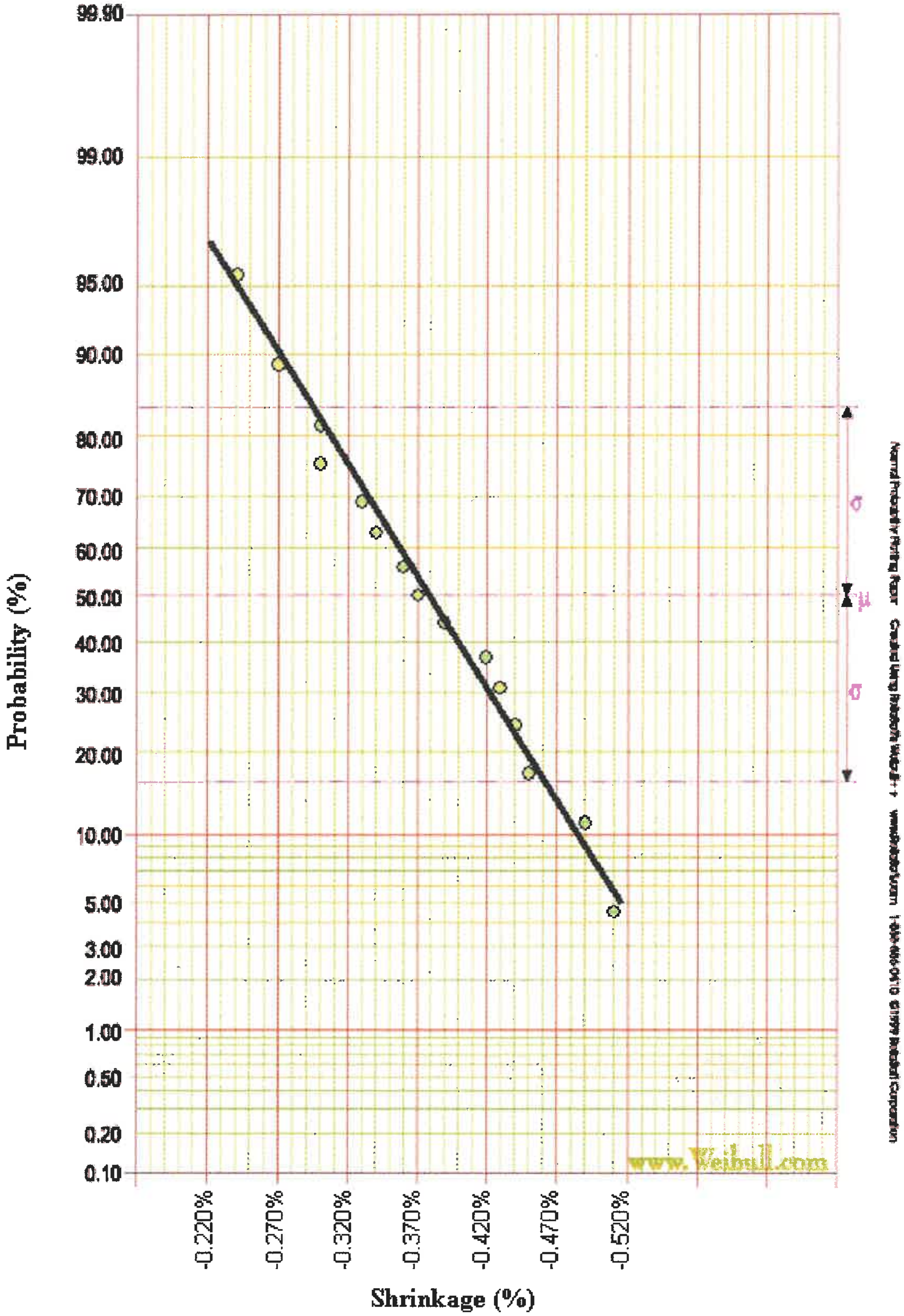


Figure E19. Uniform Probability Plot, Shrinkage, Fired Ceramic To MMC (0.35" Vertical).

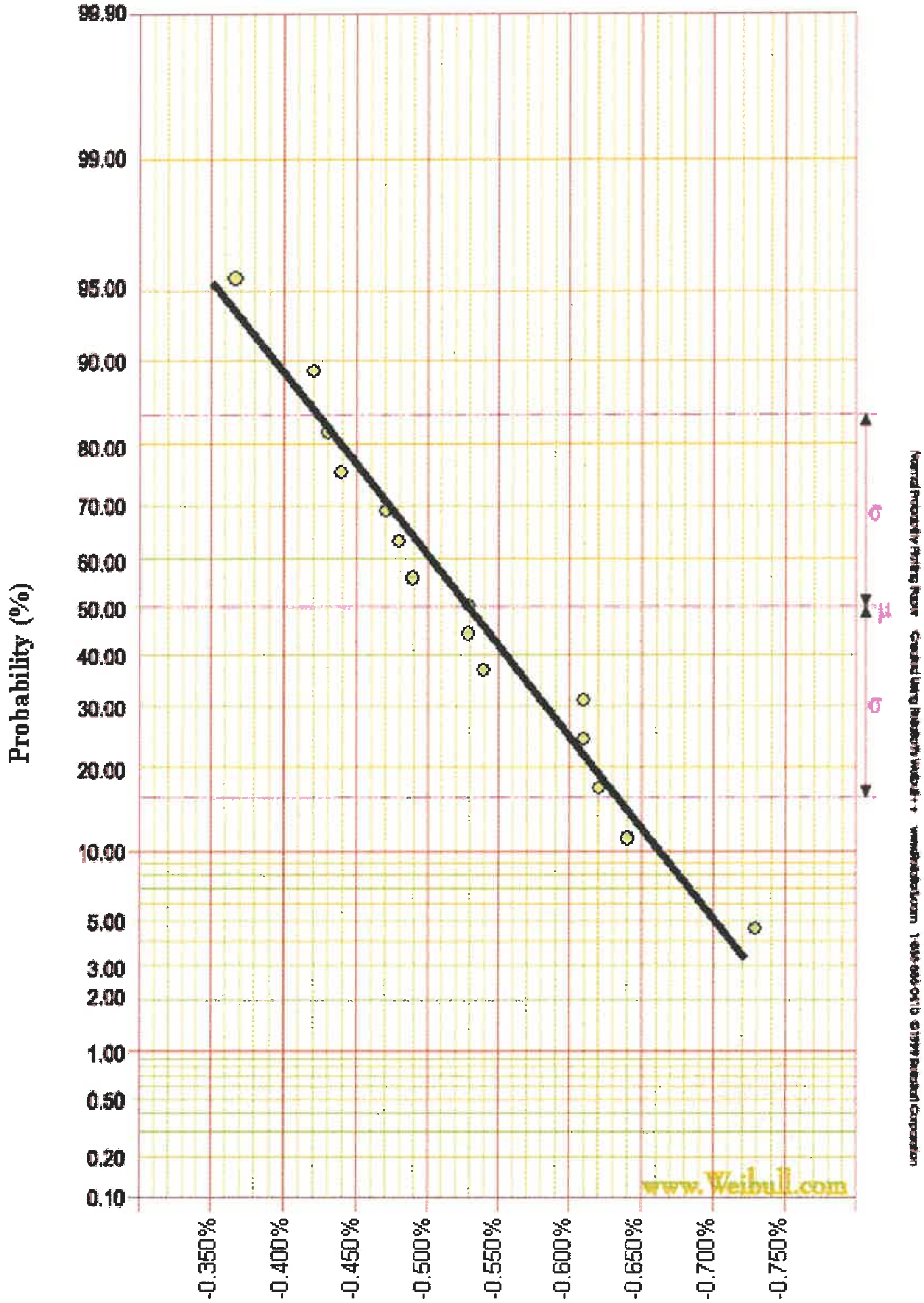


Figure E20. Uniform Probability Plot, Shrinkage, Fired Ceramic To MMC (0.7" Vertical).

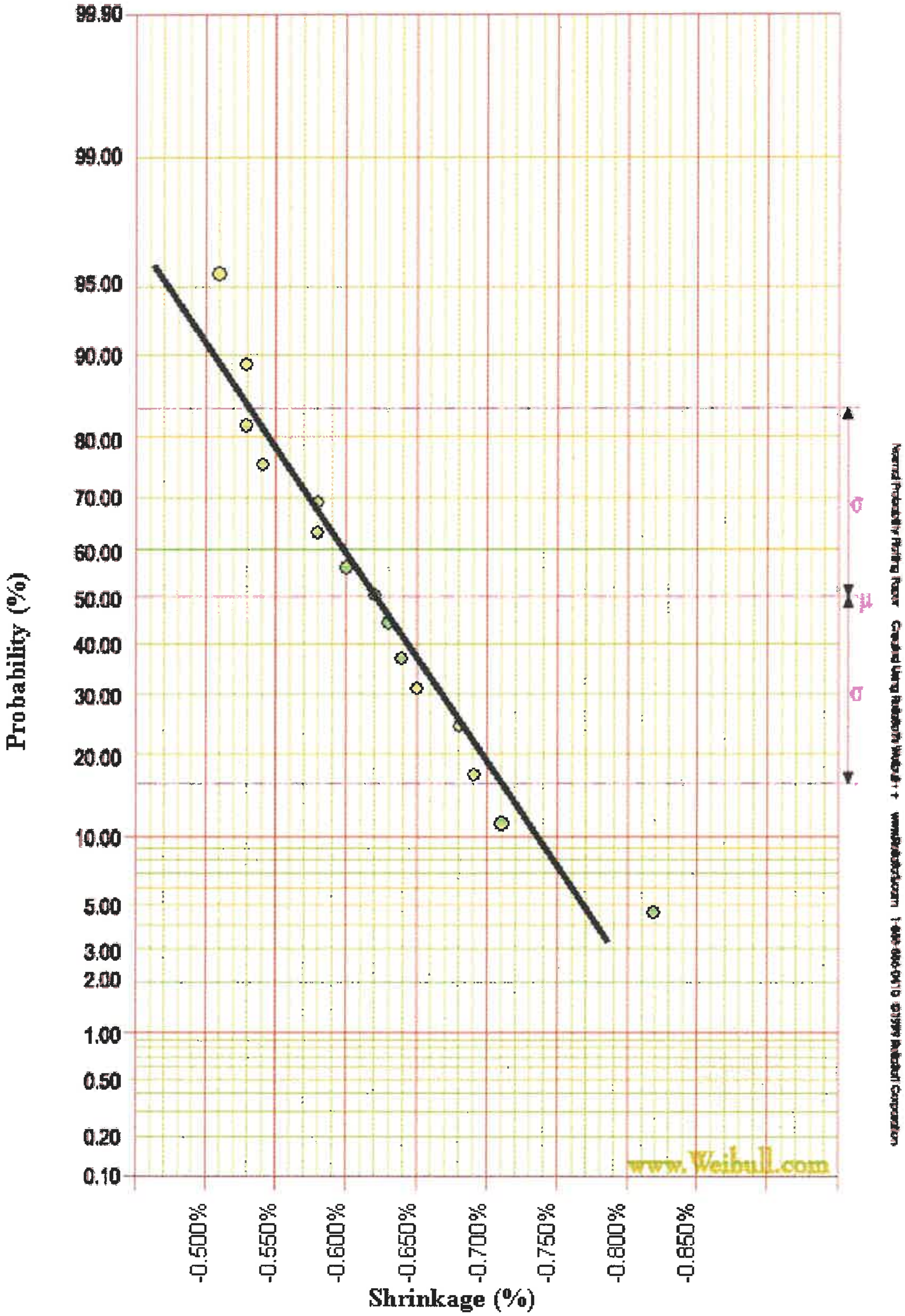


Figure E21. Uniform Probability Plot, Shrinkage, Fired Ceramic To MMC (1.05" Vertical).

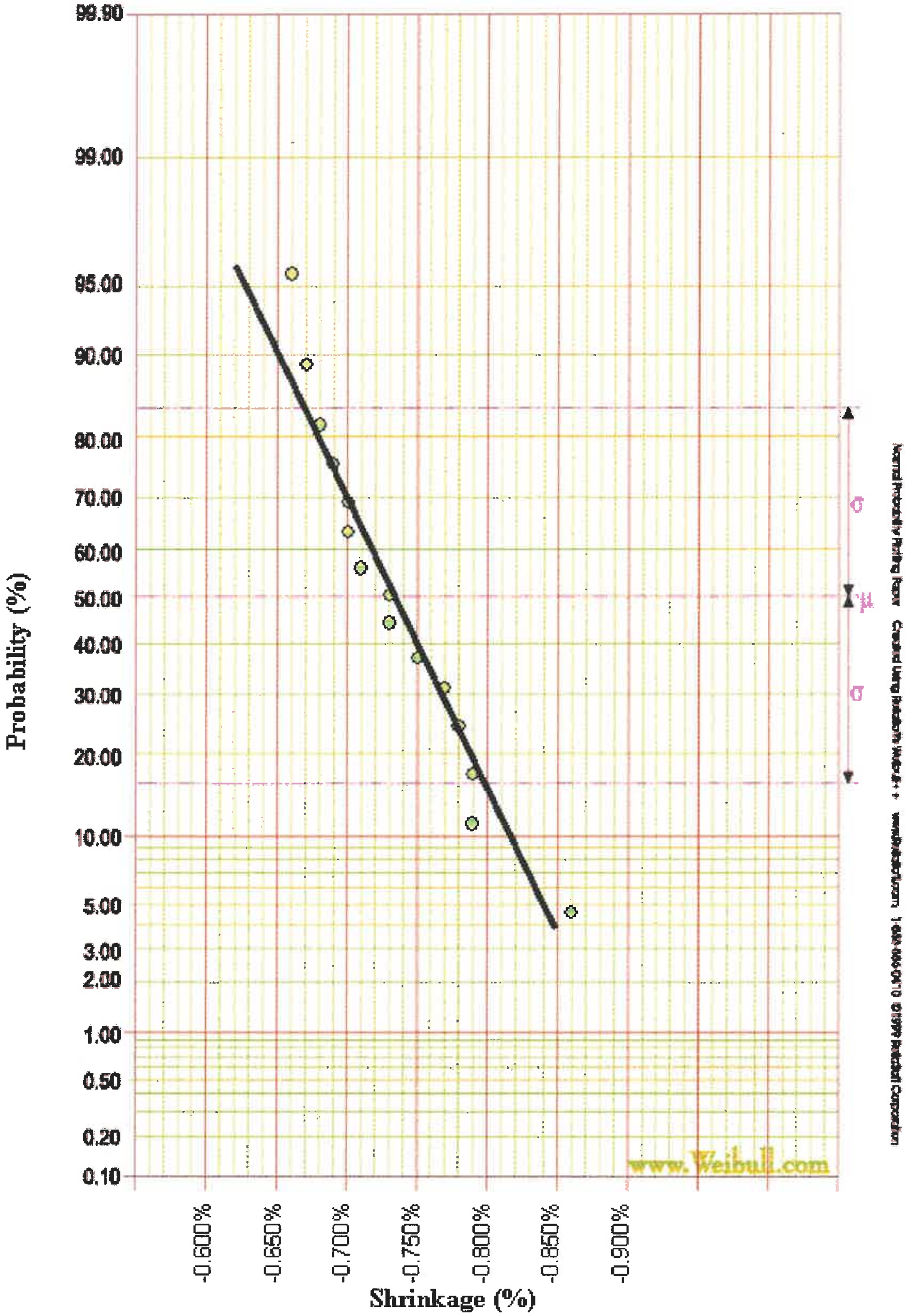


Figure E22. Uniform Probability Plot, Shrinkage, Fired Ceramic To MMC (0.5" Horizontal).

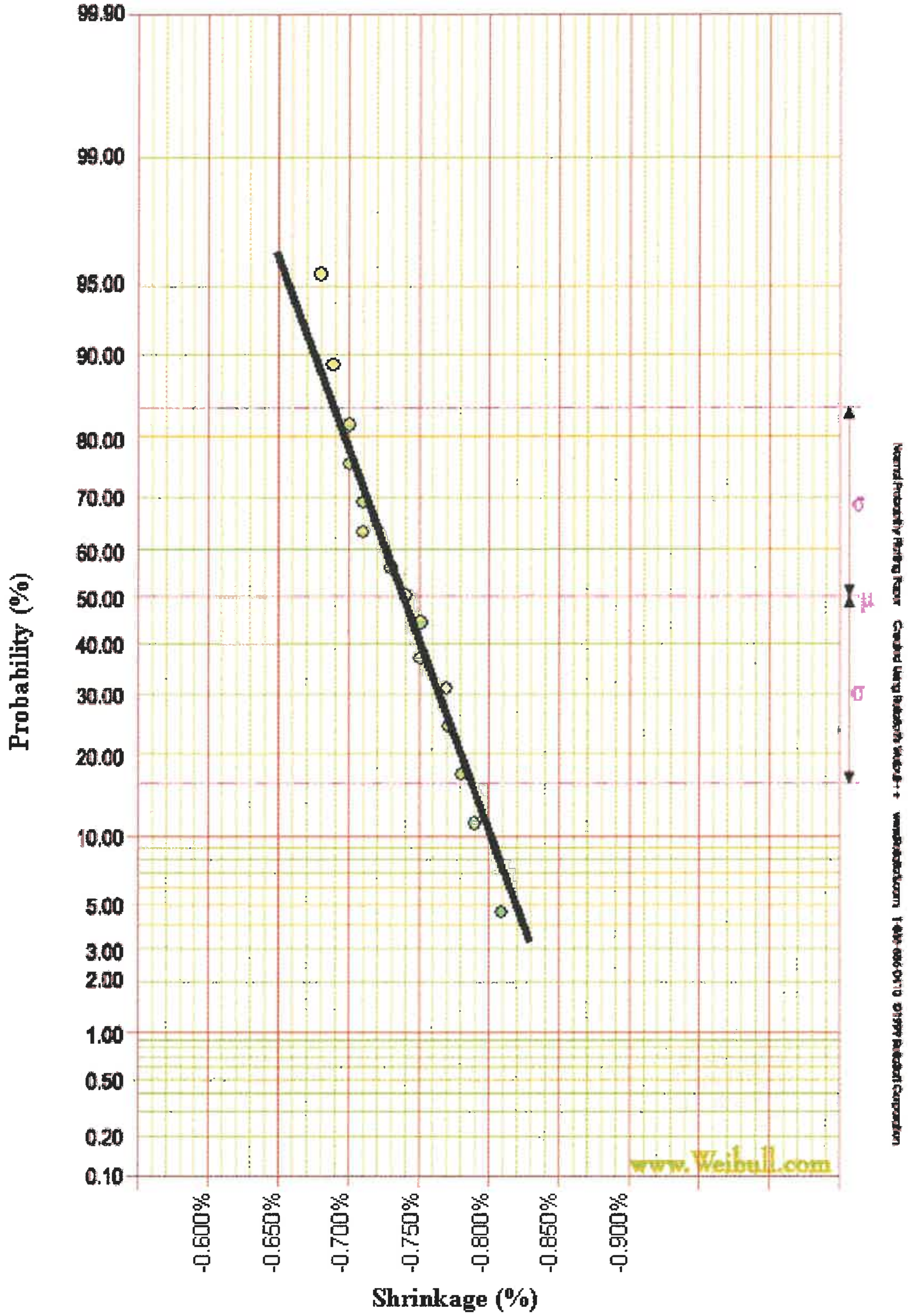


Figure E23. Uniform Probability Plot, Shrinkage, Fired Ceramic To MMC (1.0” Horizontal).

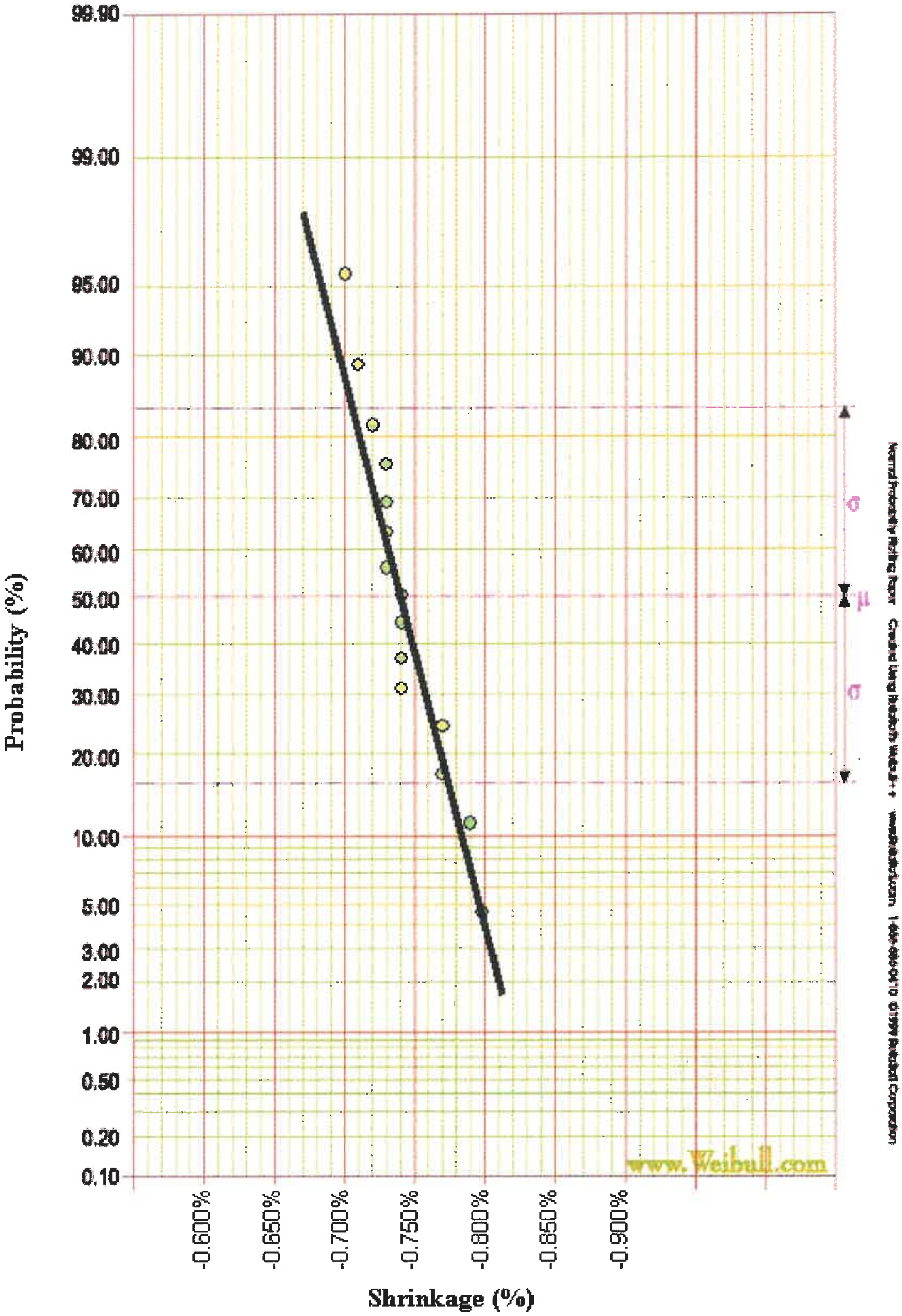


Figure E24. Uniform Probability Plot, Shrinkage, Fired Ceramic To MMC (1.5" Horizontal).

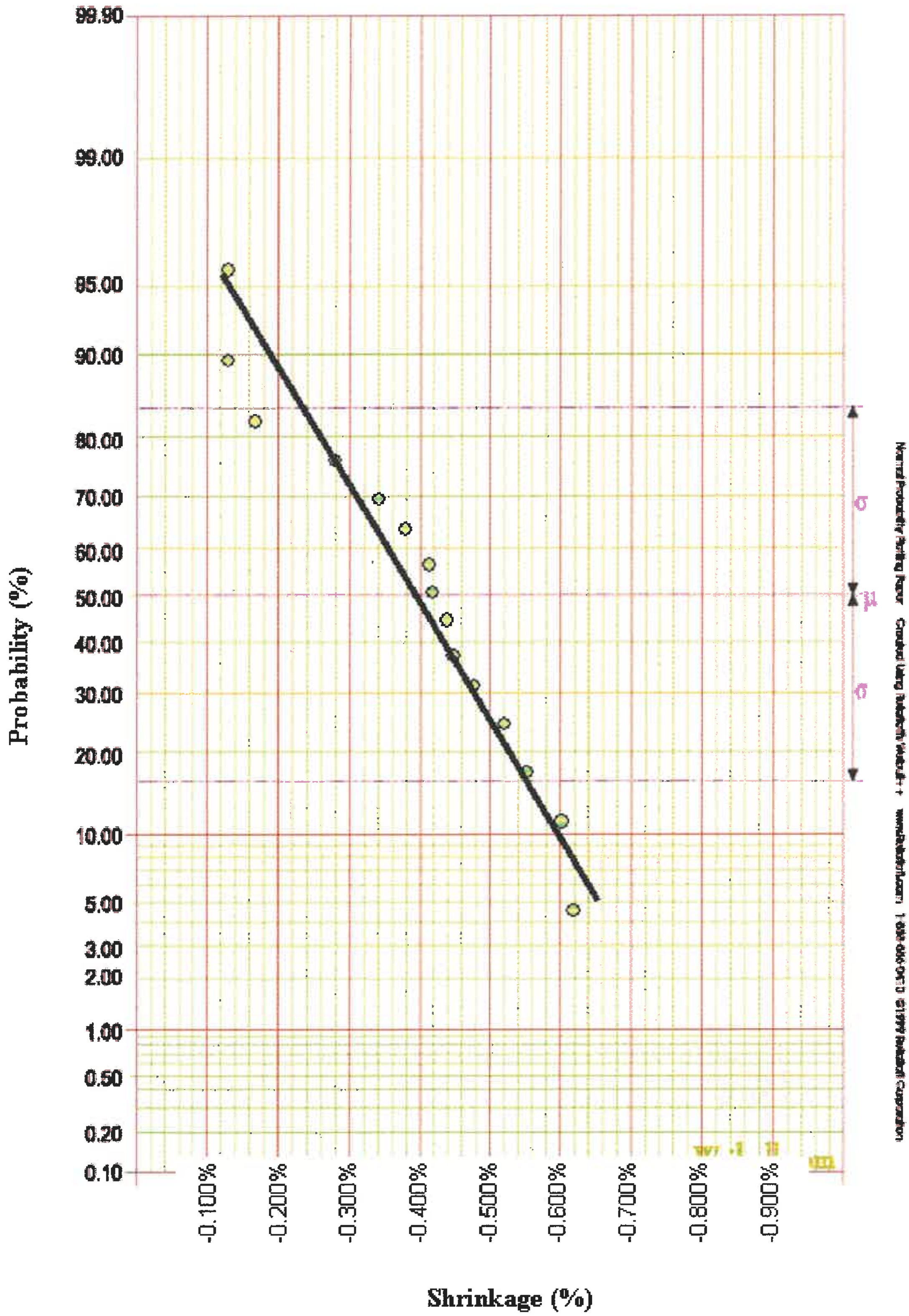


Figure E25. Uniform Probability Plot, Shrinkage, Master Pattern To MMC (0.35" Vertical).

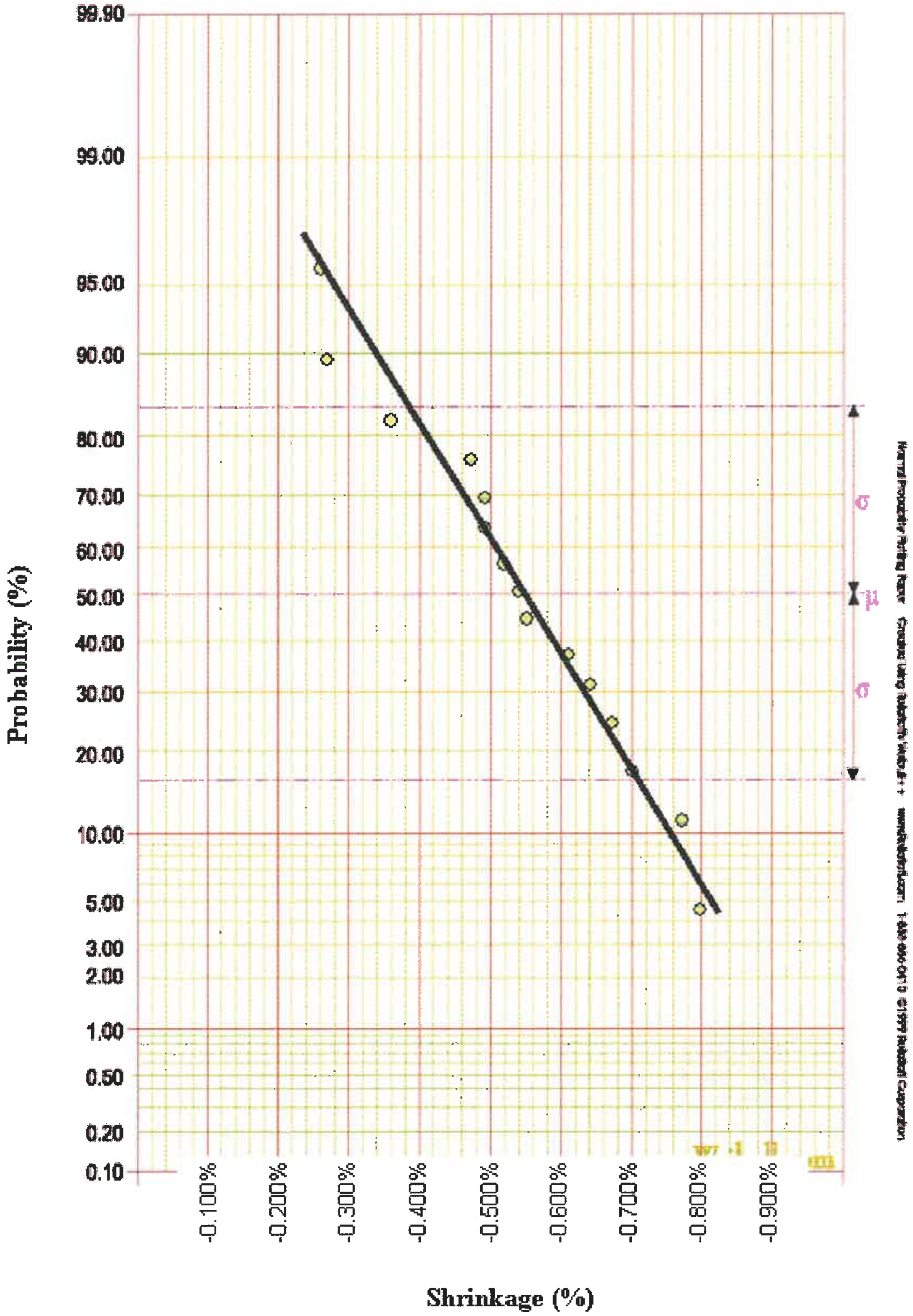


Figure E26. Uniform Probability Plot, Shrinkage, Master Pattern To MMC (0.7" Vertical).

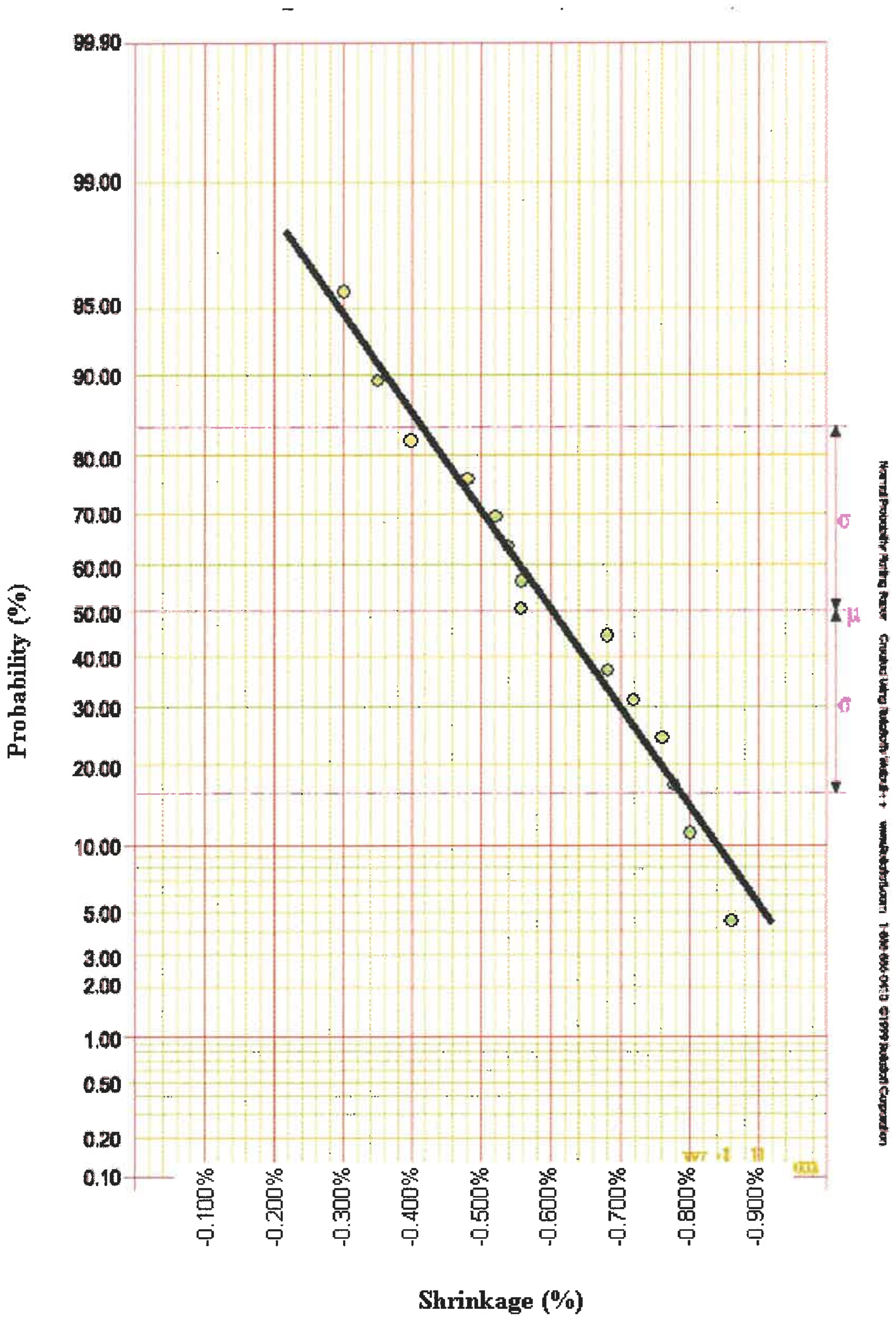


Figure E27. Uniform Probability Plot, Shrinkage, Master Pattern To MMC (1.05" Vertical).

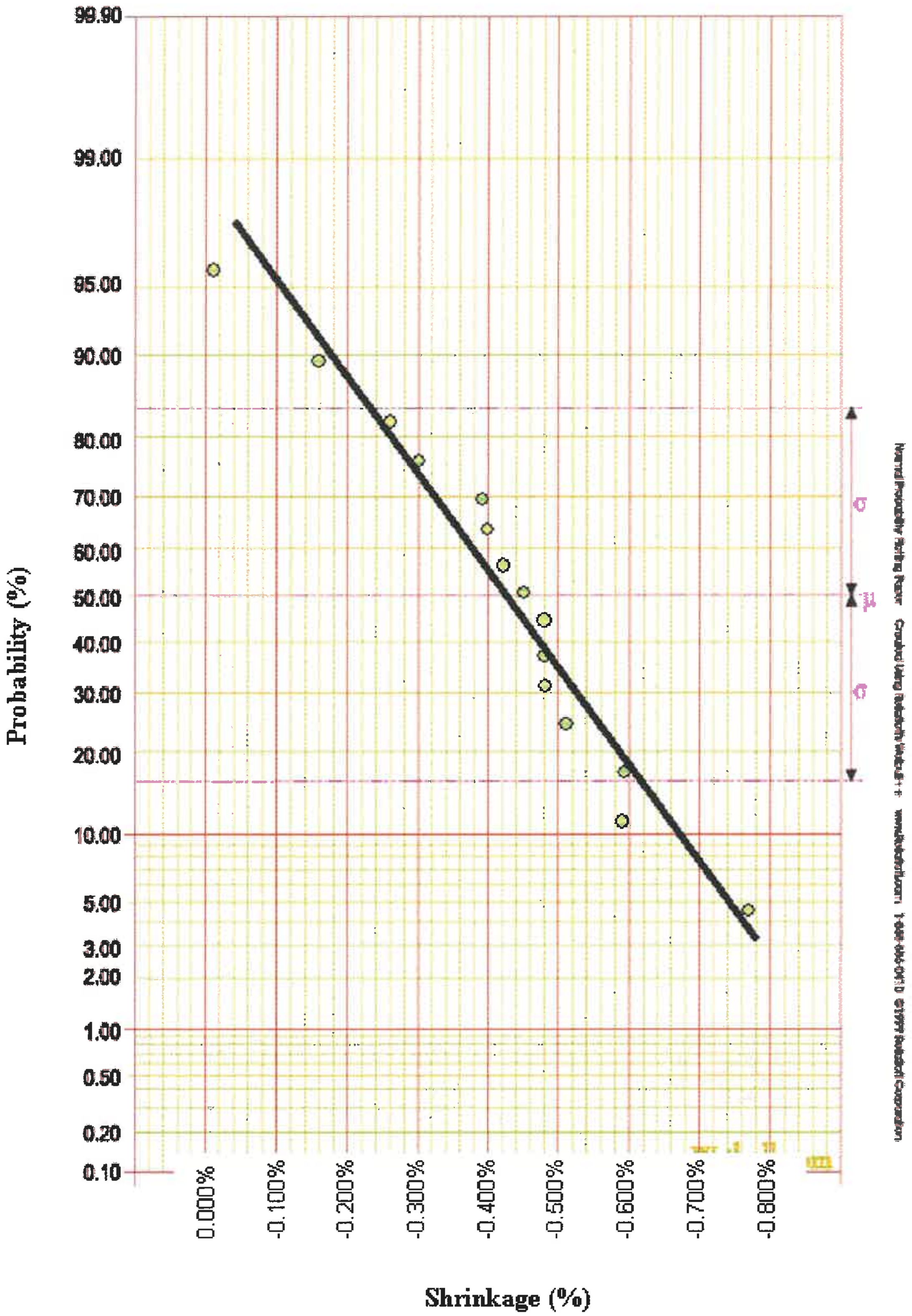


Figure E28. Uniform Probability Plot, Shrinkage, Master Pattern To MMC (0.5" Horizontal).

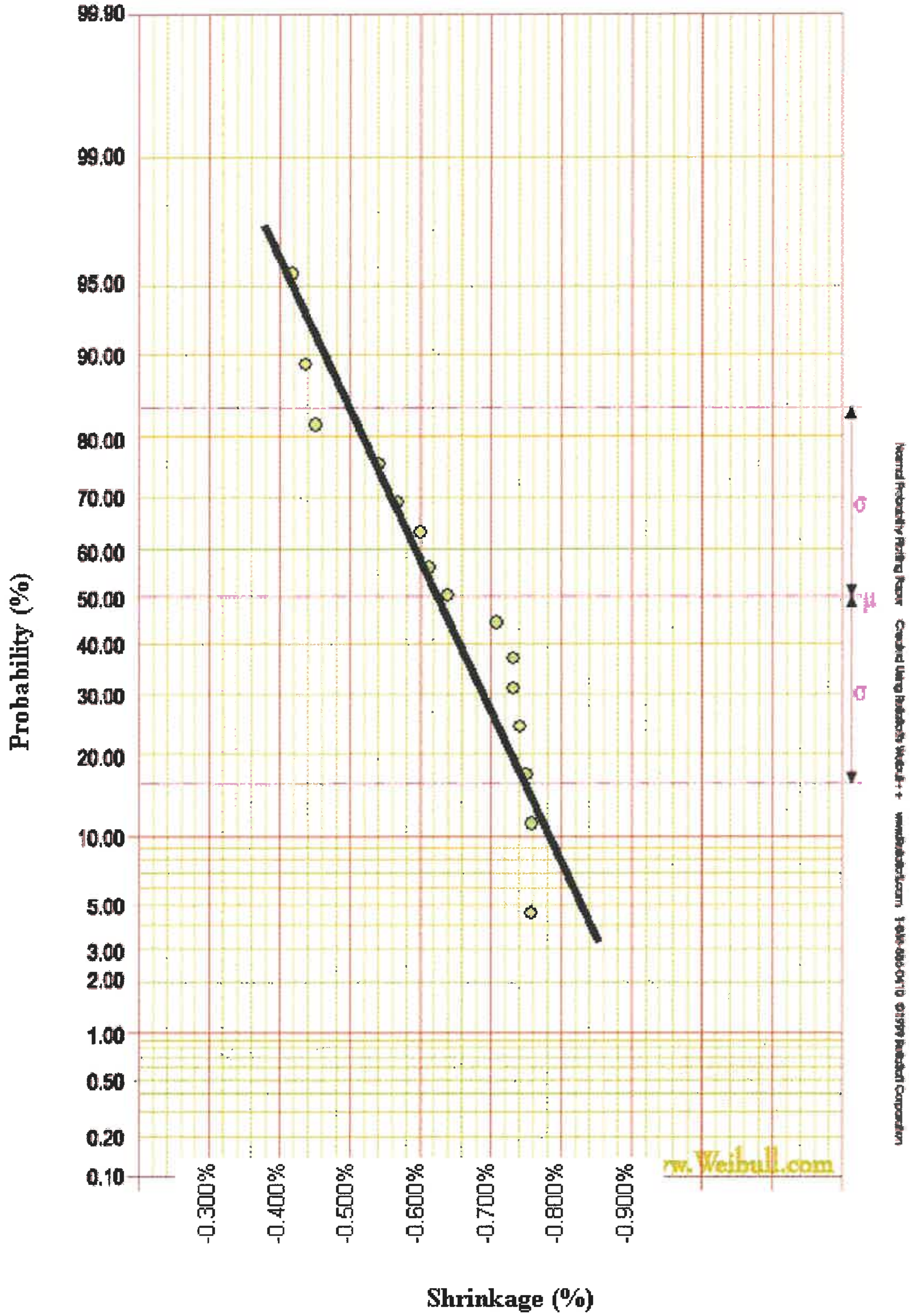


Figure E29. Uniform Probability Plot, Shrinkage, Master Pattern To MMC (1.0" Horizontal).

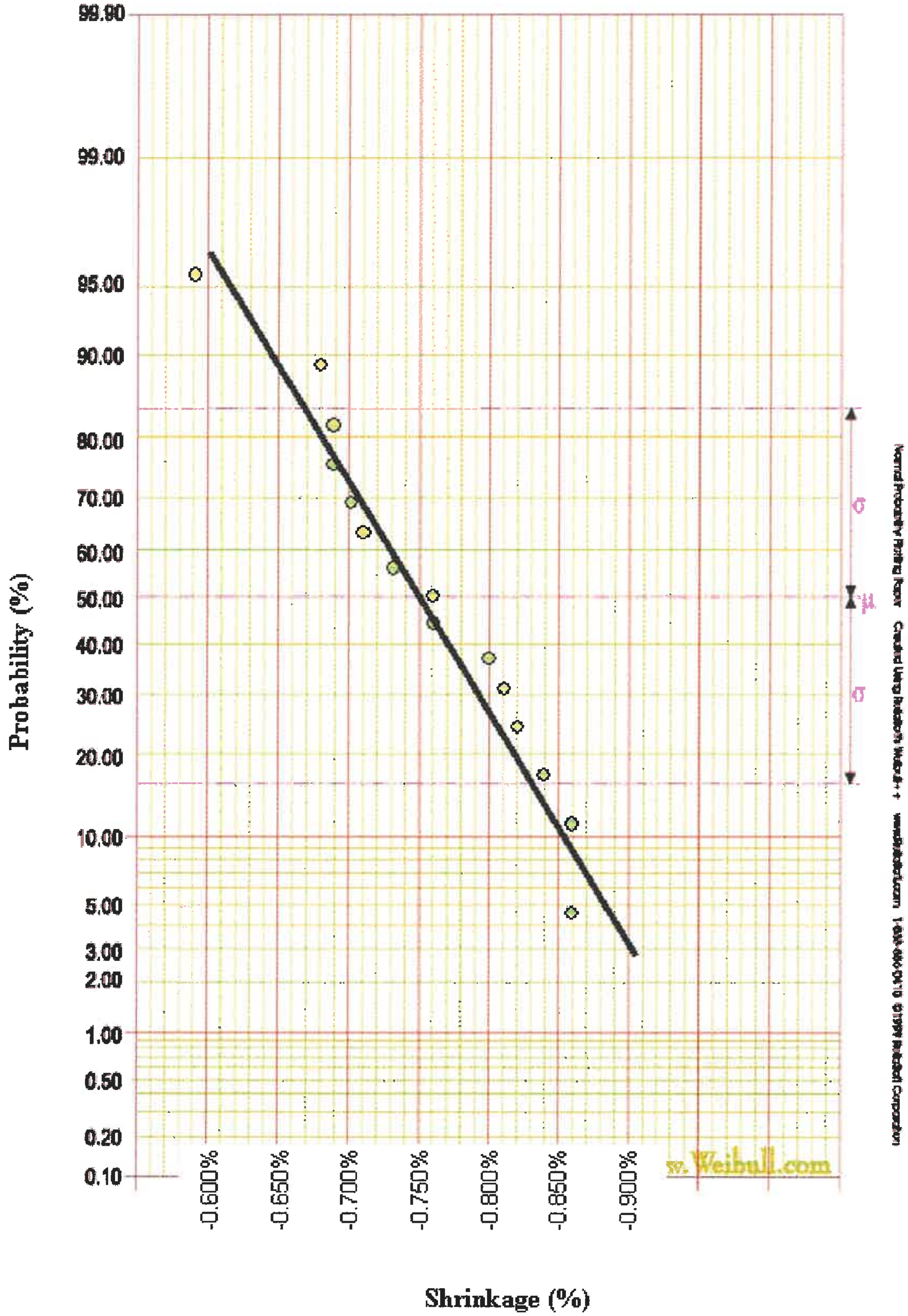
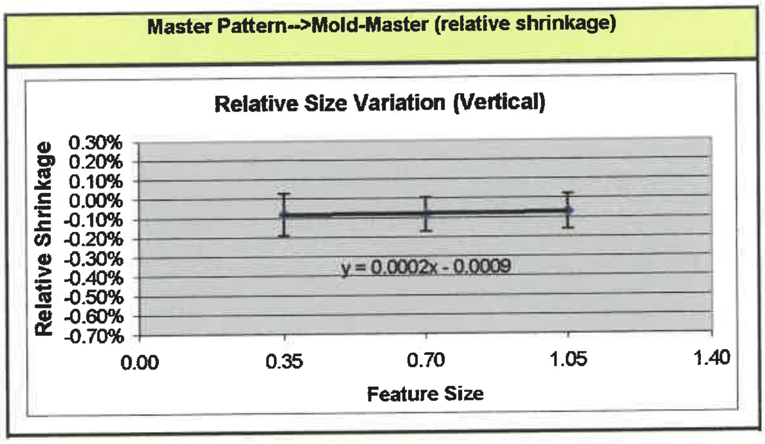
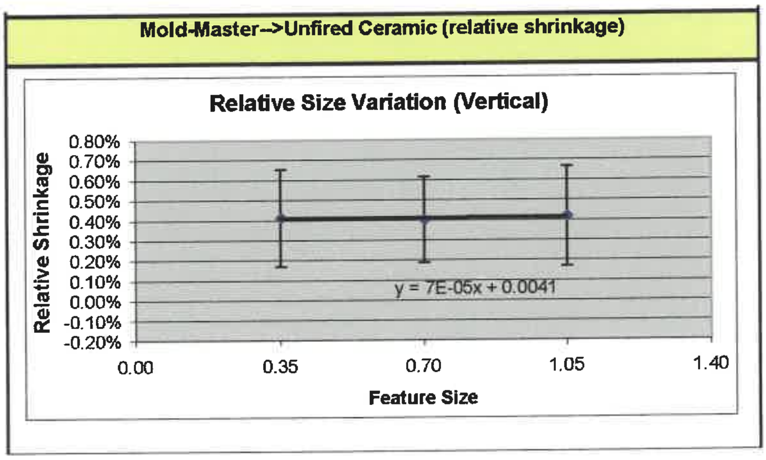
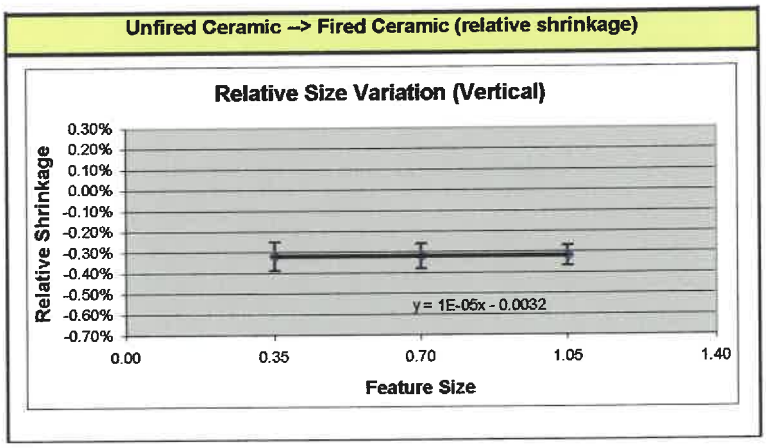


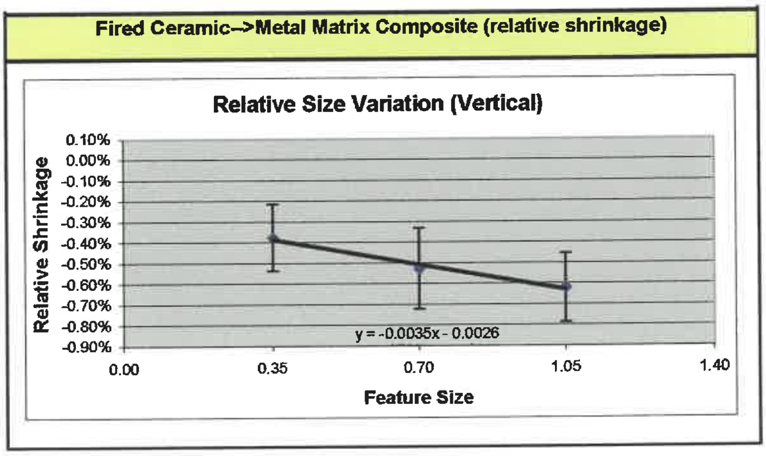
Figure E30. Uniform Probability Plot, Shrinkage, Master Pattern To MMC (1.5" Horizontal).

Appendix F. Plots, Relative Shrinkage

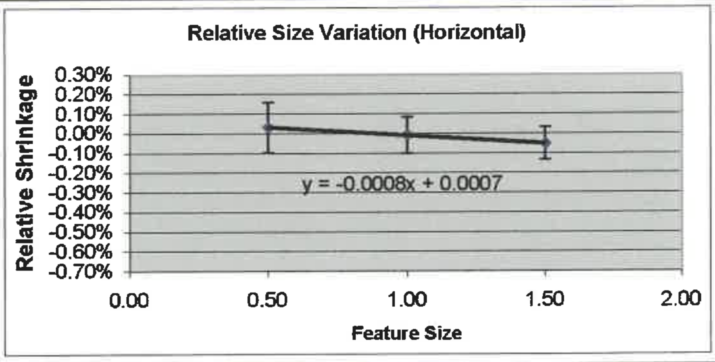




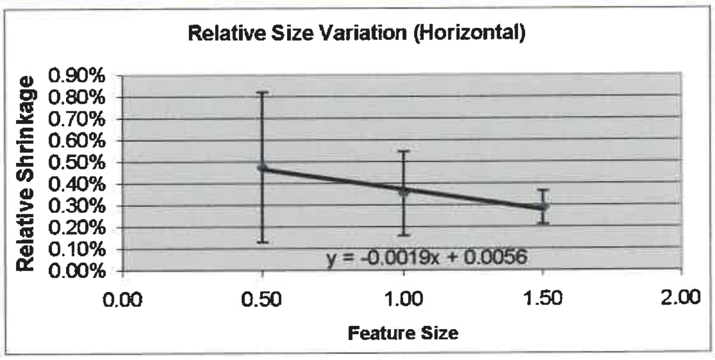




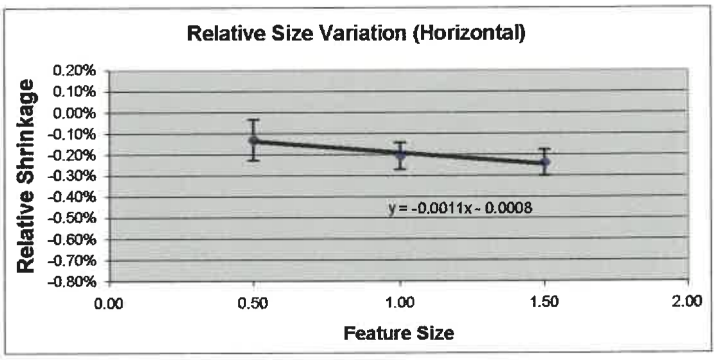
Master Pattern-->Mold-Master (relative shrinkage)



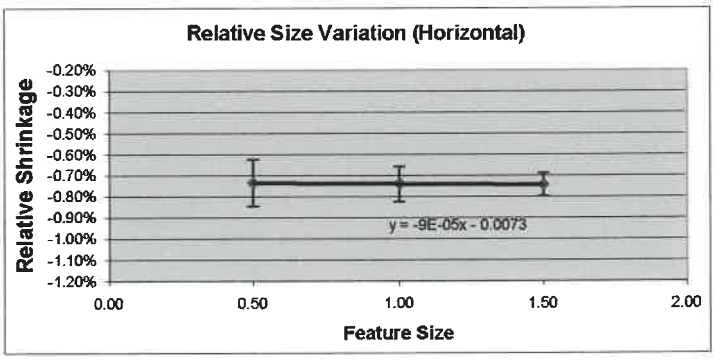
Mold-Master-->Unfired Ceramic (relative shrinkage)



Unfired Ceramic --> Fired Ceramic (relative shrinkage)



Fired Ceramic-->Metal Matrix Composite (relative shrinkage)



Engineering Project Approval Form

Master of Science –Engineering

Milwaukee School of Engineering

This report, for the project titled "Metrology Evaluation and Calibration Tool Kit for Rapid Tooling Processes," submitted by the student Vito R. Gervasi, has been approved by the following committee:

Faculty Advisor: ______________________ Date: 8-1-03

Faculty Member: ______________________ Date: 7-14-03

Faculty Member: ______________________ Date: 7-14-03